Equations and Inequalities

Unit Overview

Investigating patterns is a good foundation for studying Algebra 1. You will begin this unit by analyzing, describing, and generalizing patterns using tables, expressions, graphs, and words. You will then write and solve equations and inequalities in mathematical and real-world problems.

Key Terms

As you study this unit, add these and other terms to your math notebook. Include in your notes your prior knowledge of each word, as well as your experiences in using the word in different mathematical examples. If needed, ask for help in pronouncing new words and add information on pronunciation to your math notebook. It is important that you learn new terms and use them correctly in your class discussions and in your problem solutions.

Academic Vocabulary
- consecutive

Math Terms
- sequence
- common difference
- expression
- variable
- equilateral
- equation
- solution
- formula
- literal equation
- compound inequality
- conjunction
- disjunction
- absolute value
- absolute value notation
- absolute value equation
- absolute value inequality

ESSENTIAL QUESTIONS

? How can you represent patterns from everyday life by using tables, expressions, and graphs?

? How can you write and solve equations and inequalities?

EMBEDDED ASSESSMENTS

These assessments, following Activities 2 and 4, will give you an opportunity to demonstrate what you have learned about patterns, equations, and inequalities.

Embedded Assessment 1:

Patterns and Equations   p. 33

Embedded Assessment 2:

Inequalities and Absolute Value   p. 61

Write your answers on notebook paper.
Show your work.

1. What is $\frac{2}{3} + \frac{4}{5}$?

2. What condition must be met before you can add or subtract fractions?

3. Jennifer is checking Megan's homework. They disagree on the answer to this problem: $4^2 \times 2^2$. Jennifer says the product is 32 and Megan says it is 64. Who has the correct answer? Explain how she arrived at that correct product.

4. A piece of lumber $2\frac{1}{4}$ feet long is to be cut into 3 equal pieces. How long will each piece of cut wood be? Give the measurement in feet and in inches.

5. Arrange the following expressions in order of their value from least to greatest.
   a. $4 - 6$ b. $-4 + 6$
   c. $-4 - 6$

6. Which expression has the greater value? Justify your answer.
   A. $-8 + 3$ **B.** -8×3

7. Which of the following are equal to 14.95?
   A. 2.3×6.5 **B.** $21.45 - 6.5$
   C. $8.32 + 6.63$

8. Which equation has the least solution?
   A. $x + 5 = 13$ **B.** $-6x = -30$
   C. $\frac{x}{4} = 18$ **D.** $x - 2 = -11$

9. Combine like terms in the following expressions.
   a. $10x - 4x$ **b.** $-15n + 3n$
   c. $7.5y + 1.6y - 2$ **d.** $m + 4 - 2m$

10. The Venn diagram below provides a visual representation of the students in Mr. Griffin's class who participate in music programs after school. What does the diagram tell you about the musical involvement of Student B and Student G? Explain how you reached your conclusion.

Students who play piano   Students in the band

A   B   C
F   D   E
          G
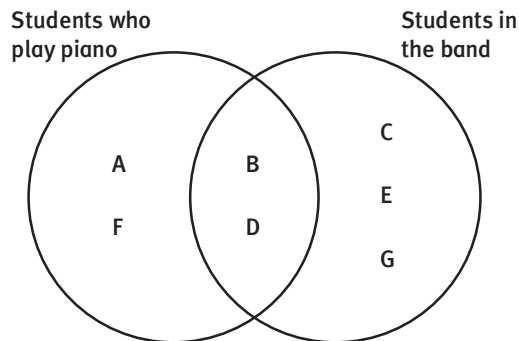
Investigating Patterns

Cross-Country Adventures
Lesson 1-1 Numeric and Graphic Representations of Data

Learning Targets:
- Identify patterns in data.
- Use tables, graphs, and expressions to model situations.
- Use expressions to make predictions.

SUGGESTED LEARNING STRATEGIES: Sharing and Responding, Create Representations, Discussion Groups, Look for a Pattern, Interactive Word Wall

Mizing spent his summer vacation traveling cross-country with his family. Their first stop was Yellowstone National Park in Wyoming and Montana. Yellowstone is famous for its geysers, especially one commonly referred to as Old Faithful. A geyser is a spring that erupts intermittently, forcing a fountain of water and steam from a hole in the ground. Old Faithful can have particularly long and fairly predictable eruptions. As a matter of fact, park rangers have observed the geyser over many years and have developed patterns they use to predict the timing of the next eruption.

Park rangers have recorded the information in the table below.

Length of Eruption (in minutes)	Approximate Time Until Next Eruption (in minutes)
1	46
2	58
3	70
4	82

1. Describe any patterns you see in the table.

2. Why might it be important for park rangers to be able to predict the timing of Old Faithful's eruptions?

3. If an eruption lasts 8 minutes, about how long must park visitors wait to see the next eruption? Explain your reasoning using the patterns you identified in the table.
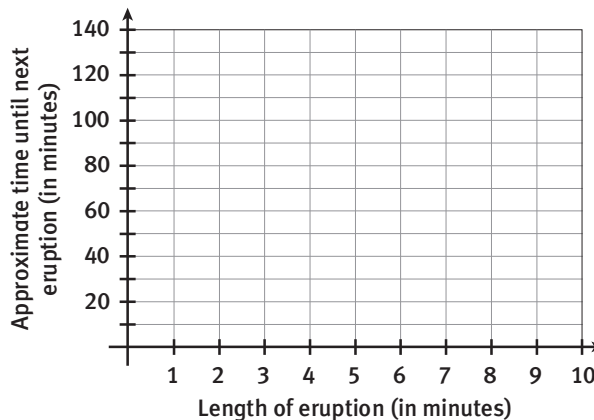
My Notes

CONNECT TO HISTORY

Yellowstone National Park was the first National Park. The park was established by Congress on March 1, 1872. President Woodrow Wilson signed the act creating the National Park Service on August 25, 1916.

My Notes

4. Graph the data from the table on the grid below.

5. **Reason quantitatively.** Mizing and his family arrived at Old Faithful to find a sign indicating they had just missed an eruption and that it would be approximately 2 hours before the next one. How long was the eruption they missed? Explain how you determined your answer.

Patterns can be written as **sequences.**

6. Using the table or graph above, write the approximate times until the next Old Faithful eruption as a sequence.

7. How would you describe this sequence of numbers?

MATH TERMS

A **sequence** is a list of numbers, and each number is called a *term* of the sequence. For example: 2, 4, 6, 8, … and 2, 5, 10, 17, … are sequences.

In the table below, 5 and 8 are *consecutive* terms. Some sequences have a **common difference** between consecutive terms. The common difference between the terms in the table below is 3.

Sequence: 5, 8, 11, 14…

Term number	Term
1	5
2	8
3	11
4	14

8. Identify two consecutive terms in the sequence of next eruption times that you created in Item 6.

9. The sequence of next eruption times has a common difference. Identify the common difference.

10. Each term in the sequence above can be written using the first term and repeated addition of the common difference. For example, the first term is 5, the second term is $5 + 3$, and the third term can be expressed as $5 + 3 + 3$ or $5 + 2(3)$. Similarly, the terms in the sequence of next eruption times can also be written using repeated addition of the common difference.

   a. Write the approximate waiting time for the next eruption after eruptions lasting 4 and 5 minutes using repeated addition of the common difference.

   b. **Model with mathematics.** Let n represent the number of minutes an eruption lasts. Write an **expression** using the **variable** n that could be used to determine the waiting time until the next eruption.

   c. Check the accuracy of your expression by evaluating it when $n = 2$.

   d. Use your expression to determine the number of minutes a visitor to the park must wait to see another eruption of Old Faithful after a 12-minute eruption.

My Notes

ACADEMIC VOCABULARY

Consecutive refers to items that follow each other in order.

MATH TIP

A common difference is also called a **constant difference**.

MATH TERMS

An **expression** may consist of numbers, variables, and operations. A **variable** is a letter or symbol used to represent an unknown quantity.

My Notes

Check Your Understanding

SB-Mobile charges $20 for each gigabyte of data used on any of its smartphone plans.

11. Copy and complete the table showing the charges for data based on the number of gigabytes used.

Number of Gigabytes Used	Total Data Charge
1	
2	
3	
4	
5	

12. Graph the data from the table. Be sure to label your axes.

13. Write a sequence to represent the total price of a data plan.

14. The sequence you wrote in Item 13 has a common difference. Identify the common difference.

15. Let n represent the number of gigabytes used. Write an expression that can be used to determine the total data charge for the phone plan.

16. Use your expression to calculate the total data charge if 10 gigabytes of data are used.

LESSON 1–1 PRACTICE

Travis owns stock in the SBO Company. After the first year of ownership the stock is worth $45 per share. Travis estimates that the value of a share will increase by $2.80 per year.

17. Copy and complete the table showing the value of the stock over the course of several years.

Year	Share Value
1	$45
2	
3	
4	
5	

18. Write a sequence to show the increase in the stock value over the course of several years.

19. **Make use of structure.** The sequence you wrote in Item 18 has a common difference. Identify the common difference.

20. Let *n* represent the number of years that have passed. Write an expression that can be used to determine the value of one share of SBO stock.

21. Use your expression to calculate the value of one share of stock after 20 years.

My Notes

Learning Targets:

- Use patterns to write expressions.
- Use tables, graphs, and expressions to model situations.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Create Representations, Think-Pair-Share, Discussion Groups, Sharing and Responding

Mizing and his family also visited Mesa Verde National Park in Colorado. As Mizing investigated the artifacts on display from the ancestral Pueblo people who once called the area home, Mizing began to notice that the patterns used to decorate pottery, baskets, and textiles were geometric.

Mizing found a pattern similar to the one below particularly interesting.
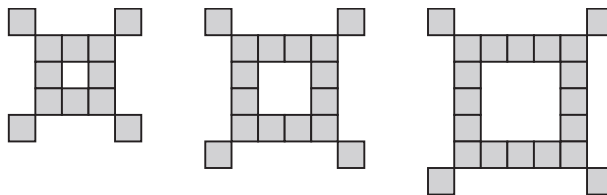
1. **Reason abstractly.** Draw the next two figures in the pattern.

2. Create a table showing the relationship between the figure number and the number of small squares in each figure.

Figure Number	Number of Squares

**My Notes**

3. Use the variable *n* to represent the figure number. Write an expression that could be used to determine the number of small squares in any figure number.

4. Use your expression to determine the number of small squares in the 12th figure.

Mizing noticed that many times the centers of the figures in the pattern were filled in with small squares of the same size as the outer squares but in a different color.

5. Fill in the centers of the diagrams with small colored squares.
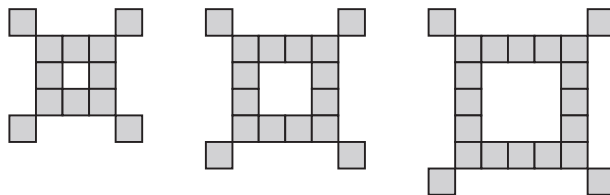
6. Draw the next two figures in the pattern. Be sure to include the inner colored squares.

7. Copy the first two columns of the table you created in Item 2 and add a column to show the relationship between the figure number and the number of inner colored squares.

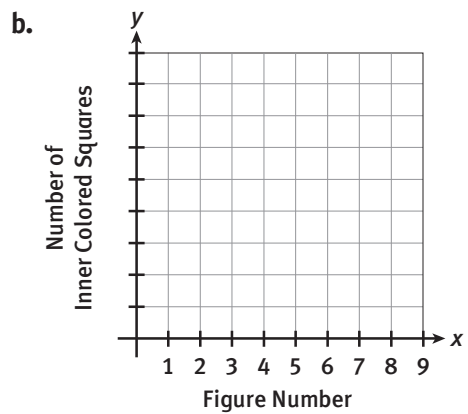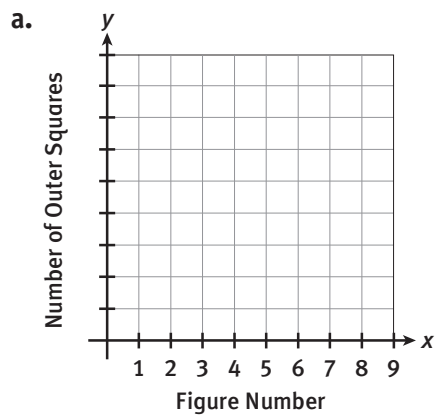| Figure Number	Number of Outer Squares	Number of Inner Colored Squares

My Notes

8. Describe any numerical patterns you see in the table.

9. Write the numbers of inner colored squares as a sequence.

10. Does the sequence of numbers of inner colored squares have a common difference? If so, identify it. If not, explain.

11. **Model with mathematics.** Graph the data from the table on the appropriate grid. Be sure to label an appropriate scale on the *y*-axis.

a.

Number of Outer Squares

y

1 2 3 4 5 6 7 8 9 → *x*

Figure Number

b.

Number of
Inner Colored Squares

y

1 2 3 4 5 6 7 8 9 → *x*

Figure Number

12. Compare the graphs.

13. Reason quantitatively. Use the patterns you have described to predict the number of inner colored squares in the 10th figure of the pattern.

14. How is the number of inner squares related to the figure number?

15. Use the variable *n* to represent the figure number. Write an expression that could be used to determine the number of inner colored squares in any figure number.

16. Use your expression to determine the number of inner colored squares in the 17th figure.

Mizing discovered another pattern in the artifacts. He noticed that when triangles were used, the triangles were all **equilateral** and often multicolored.
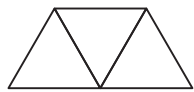

Figure 1   Figure 2   Figure 3
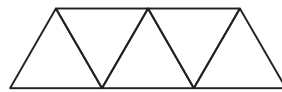
17. Attend to precision. Determine the perimeter of each figure in the pattern if each side of one triangle measures 1 cm.

> **MATH TERMS**
>
> In an **equilateral** triangle, all three sides have the same measure.

My Notes

My Notes

18. Mizing found that he could determine the perimeter of any figure in the pattern using the expression $2n + 1$. Use Mizing's expression to calculate the perimeters of the next three figures in the pattern. Use the table below to record your calculations.

Figure Number	Perimeter (cm)

19. Create a sequence to represent the perimeters of the figures in the pattern. Does the sequence have a common difference? If so, identify it. If not, explain.

20. Represent the relationship between the figures in the pattern and their perimeters as a graph. Be sure to label your axes and the scale on the y-axis.
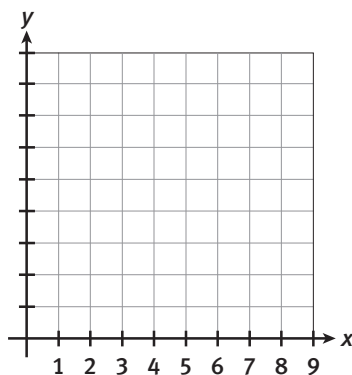
Lesson 1-2
Writing Expressions

Check Your Understanding

A pattern of small squares is shown below. Use the pattern to respond to the following questions.
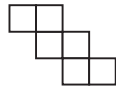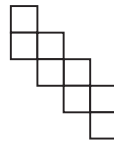

Figure 1   Figure 2   Figure 3

21. Create a table to show the number of small squares in the first through the fifth figures, assuming the pattern continues.

22. Write the number of small squares in each figure as a sequence. Does the sequence have a common difference? If so, identify it. If not, explain.

23. How many small squares would be in the 10th figure? Justify your response using the sequence or the table.

24. Use the variable n to write an expression that could be used to determine the number of small squares in any figure in the pattern.

25. Use your expression to determine the number of small squares in the 20th figure.

LESSON 1-2 PRACTICE

A toothpick pattern is shown below. Use the pattern for Items 26–29.

Figure 1   Figure 2   Figure 3

26. Create a table to show the number of toothpicks in the first through the fifth figures, assuming the pattern continues.

27. Write the number of toothpicks in each figure as a sequence. Does the sequence have a common difference? If so, identify it. If not, explain.

28. **Express regularity in repeated reasoning.** How many toothpicks would be in the 15th figure? Justify your response using the sequence or the table.

29. Use the variable n to write an expression that could be used to determine the number of toothpicks in any figure in the pattern.

ACTIVITY 1 PRACTICE

Write your answers on notebook paper.
Show your work.

Lesson 1-1

1	3	3	3
2	6	12	3
3	9	27	3
4	12	48	3

1. Describe any patterns you notice in the table of values.

2. Write the second column of the table as a sequence. Identify the common difference.

3. Create a graph to show the relationship between the first two columns of the table.

4. Write an expression to represent the relationship between the first two columns of the table.

5. Explain how you could use your expression to determine the value that would appear in the 25th row of the second column.

6. Draw the next three figures in the pattern.

7. Determine the perimeter of each of the first six figures. Assume each side of each pentagon measures 1 inch.

8. What is the perimeter of the 10th figure? Justify your response.

9. Use the variable n to write an expression that could be used to determine the perimeter of any figure in the pattern.

10. Use your expression to calculate the perimeter of the 50th figure. Be sure to include units.

11. Evaluate $3x - 12$ for $x = 5$.

12. Evaluate $45 + 15(n - 1)$ for $n = 6$.

MATHEMATICAL PRACTICES
Construct Viable Arguments and Critique the Reasoning of Others

13. Figure 1 of a mosaic pattern contains one tile and figure 2 contains four tiles. Nancy and Richard were asked to predict the number of tiles in figure 3. Nancy wrote 9, and Richard wrote 7. Who is correct? Explain your reasoning.

Lesson 1-2
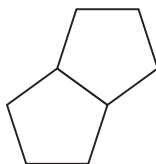Use the visual pattern below for Items 6–10.

Figure 1

Figure 2

Figure 3

Solving Equations

What's My Number?
## Lesson 2-1 Writing and Solving Equations

Learning Targets:

- Use the algebraic method to solve an equation.
- Write and solve an equation to model a real-world situation.

> **SUGGESTED LEARNING STRATEGIES:** Guess and Check, Create Representations, Discussion Groups, Identify a Subtask, Note Taking

Let's play "What's My Number?"

1. **Make sense of problems.** Determine the number, and explain how you came up with the solution.

> If you multiply me by 3 and increase that value by 5, I am 20! What's My Number?
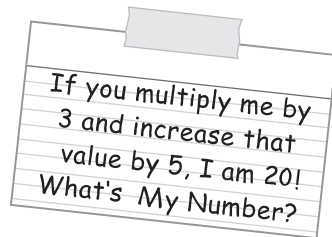
2. You may have used an **equation** to answer Item 1. Write an equation that could be used to represent the problem in Item 1.

MATH TERMS

An **equation** is a mathematical statement that shows that two expressions are equal. A **solution** is any value that makes an equation true when substituted for the variable.

3. For each equation, tell whether the given value of x is a **solution**. Explain.
   a. $2 + 4(x - 1) = 22$; $x = 6$

   b. $12 - \frac{x}{4} = 8$; $x = 24$

   c. $3.8 + 6x = 8.6$; $x = 0.9$

One way to solve an equation containing a variable is to use the algebraic method. This method is also called the *symbolic method* or *solving equations using symbols*.

MATH TIP

The **Properties of Equality** state that you can perform the same operation on both sides of an equation without affecting the solution.

Addition Property of Equality
If $a = b$, then $a + c = b + c$.

Subtraction Property of Equality
If $a = b$, then $a - c = b - c$.

Multiplication Property of Equality
If $a = b$, then $ac = bc$.

Division Property of Equality
If $a = b$, then $a \div c = b \div c$, when $c \neq 0$.

Other important properties are:

Symmetric Property of Equality
If $a = b$, then $b = a$.

Commutative Property of Addition/Multiplication
$a + b = b + a$
$ab = ba$

Associative Property of Addition/Multiplication
$(a + b) + c = a + (b + c)$
$(ab)c = a(bc)$

Distributive Property
$a(b + c) = ab + ac$

Example A

Solve the equation $3x + 90 + 2x = 360$ using the algebraic method, showing each step. List a property or provide an explanation for each step. Check your solution.

$3x + 90 + 2x = 360$	Original equation
$3x + 2x + 90 = 360$	Commutative Property of Addition
$5x + 90 = 360$	Combine like terms.
$5x + 90 - 90 = 360 - 90$	Subtraction Property of Equality
$5x = 270$	Combine like terms.
$\dfrac{5x}{5} = \dfrac{270}{5}$	Division Property of Equality
$x = 54$	Simplify.

Check by substitution:
$$3(54) + 90 + 2(54) = 360$$
$$162 + 90 + 108 = 360$$
$$360 = 360$$

Solution: $x = 54$

Try These A

a. Solve the "What's My Number?" problem using the equation you wrote in Item 2 and the algebraic method.

Solve each equation using the algebraic method, showing each step. List a property or provide an explanation for each step.

b. $-5x - 6 = 1$

c. $12d + 2 - 3d = 5$

d. $7.4p - 9.2p = -2.6 + 5.3$

e. $\dfrac{x}{4} + 7 = 12$

f. $20x - 3 + 5x = 22$

g. $8 = \dfrac{3}{2}w - 12 + 8$

Check Your Understanding

4. Is $x = 4$ a solution of $2(x - 3) + 7 = 23$? How do you know?

5. Which property of equality would you use to solve the equation $\dfrac{x}{5} = 13$? Explain your answer.

Julio has 5 more dollars than Dan. Altogether, Julio and Dan have 19 dollars. How much money does each young man have?

6. Let d represent the amount of money, in dollars, that Dan has. Use d to write an expression that represents the amount of money that Julio has.

7. Write an equation to represent the problem situation.

8. In the space below, solve your equation from Item 7, showing each step. State a property or provide an explanation for each step. Check your solution.

9. Interpret your solution to Item 8 within the context of the problem.

10. Verify the reasonableness of your solution by checking that your answer to Item 9 matches the information given in the original problem situation at the top of the page.

My Notes

MATH TIP

When using the Addition or Subtraction Property of Equality, you may find it helpful to add or subtract vertically on both sides of the equation.

$$2d + 5 = 19$$
$$\underline{ -5 \quad -5}$$
$$2d = 14$$

My Notes

Check Your Understanding

11. In Item 6, the variable j could have been defined as the amount of money, in dollars, that Julio has.
   a. Use j to write an expression to represent the amount of money, in dollars, that Dan has. Describe the similarities and differences between this expression and the one you wrote in Item 6.
   b. Write and solve an equation using the variable j to represent the problem situation. Interpret the solution within the context of the problem.
   c. Does the definition of the variable in a problem situation change the solution to the problem? Explain your reasoning.

12. Elaine is 8 years younger than her brother Tyler. The sum of their ages is 34. Define a variable and then write and solve an equation to find Elaine's and Tyler's ages.

LESSON 2-1 PRACTICE

13. Attend to precision. Justify each step in the solution of $5x + 15 = 0$ below by stating a property or providing an explanation for each step.

$$5x + 15 = 0$$
$$5x + 15 - 15 = 0 - 15$$
$$5x = -15$$
$$\frac{5x}{5} = \frac{-15}{5}$$
$$x = -3$$

Solve the equations below using the algebraic method. State a property or provide an explanation for each step. Check your solutions.

14. $3x - 24 = -6$ **15.** $\frac{4x + 1}{3} = 11$

Define a variable for each problem. Then write and solve an equation to answer the question. Check your solutions.

16. Last week, Donnell practiced the piano 3 hours longer than Marcus. Together, Marcus and Donnell practiced the piano for 11 hours. For how many hours did each young man practice the piano?

17. Olivia ordered 24 cupcakes and a layer cake. The layer cake cost $16, and the total cost of the order was $52. What was the price of each cupcake?

Learning Targets:

- Write and solve an equation to model a real-world situation.
- Interpret parts of an expression in terms of its context.

SUGGESTED LEARNING STRATEGIES: Close Reading, KWL Chart, Create Representations, Discussion Groups, Construct an Argument

The Future Engineers of America Club (FEA) wants to raise money for a field trip to the science museum. The club members will hold an engineering contest to raise money. They are deciding between two different contests, the Straw Bridge contest and the Card Tower contest.

The Straw Bridge contest will cost the club $5.50 per competitor plus $34.60 in extra expenses. The Card Tower contest will cost $4.25 per competitor plus $64.60 in extra expenses.

To help decide which contest to host, club members want to determine how many competitors they would need for the costs of the two contests to be the same.

1. Write an equation that sets the costs of the two contests equal.

2. Solve the equation from Item 1 by using the algebraic method, showing each step. List a property of equality or provide an explanation for each step.

Equations	Properties/Explanations
	Original equation
	Multiplication Property of Equality: Multiply each side by 100.

My Notes

3. **Model with mathematics.** Interpret the meaning of the value of x in the context of the problem.

4. The FEA club estimates they will have more than 30 competitors in their contest. Make a recommendation to the club explaining which contest would be the better choice and why.

5. The FEA club will charge each competitor $10 to enter the engineering contest. Write an expression for the club's revenue if x competitors enter the contest.

6. **a.** Write an equation to find the break-even point for the fundraiser using the contest you recommended to the FEA club.

   b. Solve the equation. State a property of equality or provide an explanation for each step. How many competitors does the club need to break even?

CONNECT TO BUSINESS

Revenue is the amount of money made selling a product or service. *Profit* is earnings after costs are subtracted from the revenue.

$$\text{Profit} = \text{Revenue} - \text{Cost}$$

The *break-even point* occurs when revenue equals cost.

$$\text{Revenue} = \text{Cost}$$

7. How much profit will the FEA club earn from 32 competitors if they use the contest you recommended?

8. The Future Engineers of America Club treasurer was going back through the fundraising records. On Monday, the club made revenue of $140 selling contest tickets at $10 each. One person sold 8 tickets, but the other person selling that day forgot to write down how many she sold. Write and solve an equation to determine the number of tickets the other person sold.

Check Your Understanding

9. How can you use the Multiplication Property of Equality to rewrite the equation $0.6x + 4.8 = 7.2$ so that the numbers in the problem are integers and not decimals?

10. When writing an expression or equation to represent a real-world situation, why is it important to be able to describe what each part of the expression or equation represents?

LESSON 2-2 PRACTICE

On-the-Go Phone Company has two monthly plans for their customers. The EZ Pay Plan costs $0.15 per minute. The 40 to Go Plan costs $40 per month plus $0.05 per minute.

11. **a.** Write an expression that represents the monthly bill for x minutes on the EZ Pay Plan.
    b. Write an expression that represents the monthly bill for x minutes on the 40 to Go Plan.

12. Write an equation to represent the point at which the monthly bills for the two plans are equal.

13. Solve the equation, showing each step. List a property of equality or provide an explanation for each step.

14. Interpret the solution of the equation within the context of the problem.

15. **Construct viable arguments.** Which plan should you choose if you want only 200 minutes per month? Justify your response.

My Notes

Learning Targets:
- Solve complex equations with variables on both sides and justify each step in the solution process.
- Write and solve an equation to model a real-world situation.

SUGGESTED LEARNING STRATEGIES: Create Representations, Construct an Argument, Think-Pair-Share, Create a Plan

Some equations require multiple steps to solve them efficiently.

1. The equation $3x - 2(x + 3) = 5 - 2x$ is solved in the table below. Complete the table by stating a property or providing an explanation for each step.

Equations	Properties/Explanations
$3x - 2(x + 3) = 5 - 2x$	
$3x - 2x - 6 = 5 - 2x$	
$x - 6 = 5 - 2x$	
$x + 2x - 6 = 5 - 2x + 2x$	
$3x - 6 = 5$	
$3x - 6 + 6 = 5 + 6$	
$3x = 11$	
$\dfrac{3x}{3} = \dfrac{11}{3}$	
$x = 3\dfrac{2}{3}$	

Solution: $x = 3\dfrac{2}{3}$

2. Solve the following equations. State a property of equality or provide an explanation for each step.

   a. $5x + 8 = 3x - 3$

   b. $2(4y + 3) = 16$

   c. $\dfrac{2}{3}p + \dfrac{1}{5} = \dfrac{4}{5}$

   d. $\dfrac{3}{4}a - \dfrac{1}{6} = \dfrac{2}{3}a + \dfrac{1}{4}$

MATH TIP

If an equation includes the product of a number and an expression in parentheses, you can simplify by applying the Distributive Property.

Distributive Property:
$a(b + c) = ab + ac$

MATH TIP

You can eliminate the fractions in an equation by multiplying both sides of the equation by the least common denominator of the fractions.

My Notes

3. **Model with mathematics.** Bags of maple granola cost $2 more than bags of apple granola. The owner of a restaurant ordered 6 bags of maple granola and 5 bags of apple granola. The total cost of the order was $56.

   a. Let *m* represent the cost of a bag of maple granola. Write an expression for the cost of 6 bags of maple granola.

   b. Use the variable *m* to write an expression for the cost of a bag of apple granola.

   c. Write an expression for the cost of 5 bags of apple granola.

   d. Write an equation to show that the cost of 6 bags of maple granola and 5 bags of apple granola was $56.

   e. Solve your equation to find the cost per bag of each type of granola.

Check Your Understanding

4. Suppose you are asked to solve the equation $\frac{3}{4}x - \frac{2}{3} = \frac{1}{6}x$.

   a. What number could you multiply both sides of the equation by so that the numbers in the problem are integers and not fractions?
   b. What property allows you to do this?

5. Explain how the Commutative Property of Addition could help you solve the equation $-6x + 10 + 8x = 12 - 4x$.

My Notes

LESSON 2-3 PRACTICE

Solve the following equations, and explain each step.

6. $6x + 3 = 5x + 10$

7. $6 + 0.10x = 0.15x + 8$

8. $5 - 4x = 6 + 2x$

9. $9 - 2x = 7x$

10. $2(x - 4) + 2x = -6x - 2$

11. $\frac{1}{2}x - 3 = \frac{3}{2}x + 4$

12. Ben bought 4 pairs of jeans and a T-shirt that cost $8. He had a coupon for $3 off the price of each pair of jeans and spent a total of $92 before tax.
  a. Define a variable and write an equation to represent the situation.
  b. What was the price of each pair of jeans?

13. Use appropriate tools strategically. A student solved the equation $3(2x - 6) = 4x + 8$ and found the solution to be $x = 2.6$. How could you use estimation and mental math to show that the student's solution is incorrect?

My Notes

Learning Targets:

- Identify equations that have no solution.
- Identify equations that have infinitely many solutions.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Think-Pair-Share, Predict and Confirm, Construct an Argument, Sharing and Responding

Remember that a solution of an equation with one variable is a value of the variable that makes the equation true.

1. The set of numbers $\left\{\frac{1}{2},\ 3,\ 6,\ 17,\ 0,\ 11\right\}$ contains possible solutions to the following equations. Determine which of these numbers are solutions to each of the following equations.

   a. $9x + 5 = 4(x + 2) + 5x$

   b. $7x - 10 = 3x + 14$

   c. $3x - 12 = 3(x + 1) - 15$

An equation has *no solution* if there is no value of the variable that will create a true mathematical statement. An equation has *infinitely many solutions* if there are an unlimited number of values of the variable that will create a true mathematical statement.

2. Laura, Nia, and Leo solved the following three equations as shown. Identify each of the equations as having *one solution, no solution,* or *infinitely many solutions.* Justify your responses.

MATH TIP

An equation is true when both sides of the equation have the same value. Otherwise, the equation is false.

$2 + 3 = 5$ is a true mathematical statement because $2 + 3$ has the same value as 5.

$2 + 3 = 6$ is a false mathematical statement because $2 + 3$ does not have the same value as 6.

Laura	**Nia**	**Leo**
$-(5x + 3) - 4x = 8 + 6x$	$3(x + 4) - 10 = 3x + 2$	$2(4x + 3) - 3x = 17 + 5x$
$-5x - 3 - 4x = 8 + 6x$	$3x + 12 - 10 = 3x + 2$	$8x + 6 - 3x = 17 + 5x$
$-9x - 3 = 8 + 6x$	$3x + 2 = 3x + 2$	$5x + 6 = 17 + 5x$
$\underline{+9x \qquad\quad +9x}$	$\underline{-3x \qquad\quad -3x}$	$\underline{-5x \qquad\qquad -5x}$
$-3 = 8 + 15x$	$2 = 2$	$6 = 17$
$\underline{-8 \qquad -8}$		
$-11 = 15x$		
$\dfrac{-11}{15} = \dfrac{15x}{15}$		
$-\dfrac{11}{15} = x$		

My Notes

Check Your Understanding

Determine solutions to each of the following equations.

3. $3(2z + 4) = 6(5z + 2)$

4. $3(x + 1) + 1 + 2x = 2(2x + 2) + x$

5. $8b + 3 - 10b = -2(b - 2) + 3$

Some equations are true for all values of the variable. This type of equation has all real numbers as solutions. This means the equation has *infinitely many solutions*. Other equations are false for all values of the variable. This type of equation has *no solutions*.

6. Which of the following equations has no solutions and which has all real numbers as solutions? Explain your reasoning.

   a. $3x + 5 = 3x$ **b.** $4r - 2 = 4r - 2$

7. Critique the reasoning of others. A student claims that the equation $2x + 6 = 4x + 4$ has no solutions because when you substitute 0 for x, the left side has a value of 6 and the right side has a value of 4. Is the student's reasoning correct? Explain.

8. Explain why the equation $x + 3 = x + 2$ has no solution.

9. Reason quantitatively. For what value of a does the equation $3x + 5 = ax + 5$ have infinitely many solutions? Explain.

10. Consider the equation $nx - 4 = 6$.
   a. What are the solutions if the value of n is 0? Explain.

   b. What if the value of n is 2? What is the solution of the equation? How do you know?

My Notes

Check Your Understanding

Make use of structure. Create an equation that will have each of the following as its solution.

11. One solution

12. No solution

13. Infinitely many solutions

14. A solution of zero

LESSON 2-4 PRACTICE

Solve each equation. If an equation has no solutions or if an equation has infinitely many solutions, explain how you know.

15. $3x - x - 5 = 2(x + 2) - 9$

16. $7x - 3x + 7 = 3(x - 4) + 20$

17. $-2(x - 2) - 4x = 3(x + 1) - 9x$

18. $5(x + 2) - 3 = 3x - 8x + 7$

19. $4(x + 3) - 4 = 8x + 10 - 4x$

20. $3(x + 2) + 4x - 5 = 7(x + 1) - 6$

21. Construct viable arguments. Justify your response for each of the following.
   a. Write an equation with no solutions that has the expression $3x + 6$ on the left side of the equal sign. Demonstrate that your equation has no solutions.
   b. Write an equation with infinitely many solutions that has the expression $3x + 6$ on the left side of the equal sign. Demonstrate that your equation has infinitely many solutions.

MATH TERMS

A **literal equation** has more than one variable, and the equation can be solved for a specific variable.

Formulas are examples of literal equations. A **formula** is an equation written using symbols that describes the relationship between different quantities.

Learning Targets:

- Solve literal equations for a specified variable.
- Use a formula that has been solved for a specified variable to determine an unknown quantity.

SUGGESTED LEARNING STRATEGIES: Identify a Subtask, Close Reading, Work Backward, Create Representations, Discussion Groups

A **formula** describes how two or more quantities are related. Formulas are important in many disciplines; geometry, physics, economics, sports, and medicine are just a few examples of fields in which formulas are widely used.

A formula is an example of a **literal equation**. A literal equation contains more than one variable. Literal equations and formulas can be solved for a specific variable using the same procedures as equations containing one variable.

Example A

Solve the equation $4x + b = 12$ for x.

Step 1: Isolate the term that contains x by subtracting b from both sides.

$$4x + b = 12 \qquad \text{Original equation}$$
$$4x + b - b = 12 - b \qquad \text{Subtraction Property of Equality}$$
$$4x = 12 - b \qquad \text{Combine like terms.}$$

Step 2: Isolate x by dividing both sides by 4.

$$\frac{4x}{4} = \frac{12 - b}{4} \qquad \text{Division Property of Equality}$$
$$x = \frac{12 - b}{4} \qquad \text{Simplify.}$$

Solution: $x = \frac{12 - b}{4}$, or $x = 3 - \frac{b}{4}$

Try These A

Solve each equation for x.

a. $ax + 7 = 3$

b. $cx - 10 = -5$

c. $-3x + d = -9$

Check Your Understanding

1. Is the equation $2x + 4 = 5x - 6$ a literal equation? Explain.

2. Describe the similarities and differences between solving an equation containing one variable and solving a literal equation for a variable.

Example B

The equation $v = v_0 + at$ gives the velocity in meters per second of an object after t seconds, where v_0 is the object's initial velocity in meters per second and a is its acceleration in meters per second squared.

a. Solve the equation for a.

b. Determine the acceleration for an object whose velocity after 15 seconds is 25 meters per second and whose initial velocity was 15 meters per second.

CONNECT TO PHYSICS

An object's *velocity* is its speed in a particular direction. Its *acceleration* is the rate of change in velocity. If an object has a positive acceleration, it is speeding up; if it has a negative acceleration, it is slowing down.

a.

$v = v_0 + at$	Original equation
$v - v_0 = v_0 - v_0 + at$	Subtraction Property of Equality: Subtract v_0 from both sides.
$v - v_0 = at$	Combine like terms.
$\dfrac{v - v_0}{t} = \dfrac{at}{t}$	Division Property of Equality: Divide both sides by t.
$\dfrac{v - v_0}{t} = a$	Simplify.
$a = \dfrac{v - v_0}{t}$	Symmetric Property of Equality

READING MATH

Sometimes a variable may include a subscript. A subscript is a small number or letter written to the lower right of a variable. For example, the variable v_0 has the subscript 0. The subscript 0 is often read "nought." (*Nought* is another word for 0.) A variable with a subscript of 0 usually indicates an initial value. So, v_0 indicates the initial value of the velocity, or the velocity when the time $t = 0$.

b. To determine the acceleration for an object whose velocity after 15 seconds is 25 meters per second and whose initial velocity was 15 meters per second, substitute 25 for v, 15 for v_0, and 15 for t.

$$a = \frac{25 - 15}{15} = \frac{10}{15} = \frac{2}{3} \text{ m/s}^2$$

Try These B

The equation $t = 13p + 108$ can be used to estimate the cooking time t in minutes for a stuffed turkey that weighs p pounds. Solve the equation for p. Then find the weight of a turkey that requires 285 minutes to cook.

My Notes

3. **Reason abstractly.** Solve for the indicated variable in each formula.

Name	Formula	Solve for
Distance	$d = rt$, where d is the distance an object travels, r is the average rate of speed, and t is the time traveled	r
Pressure	$p = \dfrac{F}{A}$, where p is the pressure on a surface, F is the force applied, and A is the area of the surface	F
Kinetic energy	$k = \dfrac{1}{2}mv^2$, where k is the kinetic energy of an object, m is its mass, and v is its velocity	m
Gravitational energy	$U = mgh$, where U is the gravitational energy of an object, m is its mass, g is the acceleration due to gravity, and h is the object's height	h
Boyle's Law	$p_1V_1 = p_2V_2$, where p_1 and V_1 are the initial pressure and volume of a gas and p_2 and V_2 are the final pressure and volume of the gas when the temperature is kept constant	V_2

Check Your Understanding

4. Solve the equation $w + i = \dfrac{s}{c}$ for c.

5. Why do you think being able to solve a literal equation for a variable would be useful in certain situations?

LESSON 2-5 PRACTICE

Solve each equation for the indicated variable.

6. $W = Fd$, for d

7. $P = \dfrac{W}{t}$, for W

8. $P = \dfrac{W}{t}$, for t

9. $ak - r = on$, for k

10. **Reason quantitatively.** In baseball, the equation $E = \dfrac{9R}{I}$ gives a pitcher's earned run average E, where R is the number of earned runs the player allowed and I is the number of innings pitched.

   a. Solve the equation for I. State a property or provide an explanation for each step.

   b. Last season, a pitcher had an earned run average of 2.80 and allowed 70 earned runs. How many innings did the pitcher pitch last season?

ACTIVITY 2 PRACTICE
Write your answers on notebook paper.
Show your work.

Lesson 2-1

1. Which of the following shows the Addition Property of Equality?
   A. $\frac{4x}{4} = \frac{10}{4}$
   $x = 2.5$
   B. $16x - 2 + 2 = 18 + 2$
   $16x = 20$
   C. $8x + 2x = 10x$
   D. $4(3x + 1) = 8$
   $12x + 4 = 8$

Solve each equation.

2. $3z - 2 = 13$

3. $-5y - 10 = -60$

4. $4x - 5 + 2x = -2$

5. $\frac{x - 8}{3} = 3$

6. Which equation has the solution $x = -4$?
   A. $2(x - 4) = 16$
   B. $3x + 6 - 2x = -2$
   C. $\frac{1}{4}x + 4 = 3$
   D. $(x + 6)3 = -6$

7. Is 7 a solution of $5x - 3 = 12$? Justify your answer.

8. The perimeter of triangle ABC is 54. The triangle has side lengths $AB = 3x$, $BC = 4x$, and $AC = 5x$. Find the length of each side.

A group of 19 students want to see the show at the planetarium. Tickets cost $11 for each student who is a member of the planetarium's frequent visitor program and $13 for each student who is not a member. The total cost of the students' tickets is $209. Use this information for Items 9–16.

9. Let x represent the number of students in the group who are members of the frequent visitor program. Write an expression in terms of x for the number of students in the group who are not members.

10. Write an equation to determine the value of x. Explain what each part of your equation represents.

11. Solve the equation you wrote in Item 10. List a property of equality or provide an explanation for each step.

12. How many students in the group are members of the frequent visitor program? How many are not members? Explain how you know.

Lesson 2-2

13. Joining the frequent visitor program at the planetarium costs $5 per year. Write an equation that can be used to determine n, the number of visits per year for which the cost of being a member of the frequent visitor program is equal to the cost of not being a member.

14. Solve your equation from Item 13. List a property of equality or provide an explanation for each step.

15. Explain the meaning of the solution of the equation.

16. Nash plans to visit the planetarium twice in the next year. Should he join the frequent visitor program? Explain.

For Items 17–24, solve the equations, and explain each step.

17. $8x + 5 = 3x + 15$

18. $3x + 11 = 2x - 5$

19. $6x - 9 = 8x + 11$

20. $0.5x - 3.5 = 0.2x - 0.5$

Lesson 2-3

21. $6 - 2(x + 6) = 3x + 4$

22. $3x + 2(x - 1) = 9x + 4$

23. $5(x - 2) + x = 6(x + 3) - 4x$

24. $2 - 3(4 - x) = 5(2 - x) + 4x$

25. Which equation has the greatest solution?
  A. $0.4x - 2.5 = 1.3x + 4.7$
  B. $2.4(x - 6) = -5.6x + 8$
  C. $\frac{3}{4}x + 5 = \frac{1}{2}x + 9$
  D. $\frac{1}{3}x + \frac{2}{3} = \frac{1}{4}(x + 3)$

26. Provide a reason for each step in solving the equation shown below.

$$2(x - 1) - 3(x + 2) = 8 - 4x$$
$$2x - 2 - 3x - 6 = 8 - 4x$$
$$-1x - 8 = 8 - 4x$$
$$-1x + 4x - 8 = 8 - 4x + 4x$$
$$3x - 8 = 8$$
$$3x - 8 + 8 = 8 + 8$$
$$3x = 16$$
$$\frac{3x}{3} = \frac{16}{3}$$
$$x = 5\frac{1}{3}$$

27. Tyrell exercised this week both by walking and by biking. He walked at a rate of 4 mi/h and biked at a rate of 12 mi/h. The total distance he covered both walking and biking was 36 miles, and Tyrell spent one more hour walking than biking.
  a. Define a variable and write an equation for this situation.
  b. How many hours did Tyrell spend on each activity?

Lesson 2-4

Solve each equation. If an equation has no solutions, or if an equation has infinitely many solutions, explain how you know.

28. $3(x + 4) - 4 = 2(x + 4) + x$

29. $4(2x + 6) = 5(x + 5) + 2$

30. $6x - 8x + 5 = -2(x + 2) + 7$

31. $0.3x + 1.8 = 0.4(x + 5) - 0.2$

32. $6(x + 2) + 2x - 4 = 8(x - 2) + 8$

33. $\frac{1}{2}(x + 1) + \frac{1}{4} = \frac{2}{3}x - \frac{1}{6}x + \frac{3}{4}$

Lesson 2-5

34. Which shows the equation $c = a\left(\frac{w}{150}\right)$ correctly solved for the variable w?
  A. $w = a(150c)$
  B. $w = a\left(\frac{c}{150}\right)$
  C. $w = \frac{150}{ac}$
  D. $w = \frac{150c}{a}$

Solve each equation for the indicated variable.

35. $w = gm$, for m

36. $Q = \frac{1}{2}P + 15$, for P

37. $I = \frac{V}{R}$, for R

38. $y = mx + b$, for m

39. The equation $f = d + e + t$ can be used to find an athlete's final score f in an Olympic trampoline event, where d is the difficulty score, e is the execution score, and t is the time of flight score.
  a. Solve the equation for t.
  b. An athlete's final score is 55.675. His difficulty score is 14.6, and his execution score is 24.9. What is the athlete's time of flight score?

MATHEMATICAL PRACTICES
Construct Viable Arguments and Critique the Reasoning of Others

40. A student solved the equation $2(x - 4) - 4x = -6x + 9x + 4$ as shown below. Did the student solve the equation correctly? If so, list a property or explanation for each step. If not, solve the equation correctly, and list a property or explanation for each step.

$$2(x - 4) - 4x = -6x + 9x + 4$$
$$2x - 8 - 4x = -6x + 9x + 4$$
$$-2x - 8 = 3x + 4$$
$$-2x + 2x - 8 = 3x + 2x + 4$$
$$-8 = 5x + 4$$
$$-8 - 4 = 5x + 4 - 4$$
$$-12 = 5x$$
$$-60 = x$$

Solving Inequalities

Physical Fitness Zones
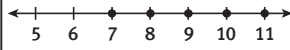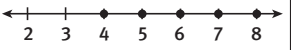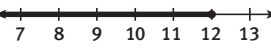## Lesson 3-1 Inequalities and Their Solutions

Learning Targets:

- Understand what is meant by a solution of an inequality.
- Graph solutions of inequalities on a number line.

SUGGESTED LEARNING STRATEGIES: Levels of Questions, Think-Pair-Share, Interactive Word Wall, Construct an Argument, Quickwrite

MATH TERMS

The **graph of an inequality** in one variable is all the points on a number line that make the inequality true.

Spartan Middle School students participate in Physical Education testing each semester. In order to pass, 12- and 13-year-old girls have to do at least 7 push-ups and 4 modified pull-ups. They also have to run one mile in 12 minutes or less.

You can use an inequality to express the passing marks in each test.

	Push-Ups, p	**Modified Pull-Ups, m**	**One-Mile Run, r**
Verbal	At least 7 push-ups	At least 4 pull-ups	12 minutes or less
Inequality	$p \geq 7$	$m \geq 4$	$r \leq 12$
Graph	5 6 7 8 9 10 11	2 3 4 5 6 7 8	7 8 9 10 11 12 13

1. Why do you think the graphs of push-ups and pull-ups are dotted but the graph of the mile run is a solid ray?

2. **Reason quantitatively.** Jamie ran one mile in 12 minutes 15 seconds, did 8 push-ups, and did 4 modified pull-ups. Did she pass the test? Explain.

3. Karen did 7 push-ups.
   a. Is this a passing number of push-ups? Which words in the verbal description indicate this? Explain.

   b. How is this represented in the inequality $p \geq 7$?

WRITING MATH

Other phrases that are equivalent to "at least" are "no less than" and "no fewer than." These phrases are also represented by the inequality symbol $\geq$.

The **solution of an inequality** in one variable is the set of numbers that make the inequality true. To verify a solution of an inequality, substitute the value into the inequality and simplify to see if the result is a true statement.

My Notes

4. Use the table below to figure out which *x*-values are solutions to the equation and which ones are solutions to the inequality. Show your work in the rows of the table.

x-values	Solution to the equation? $2x + 3 = 5$	Solution to the inequality? $2x + 3 > 5$
−1		
0		
1		
2		
8.5		

5. How many solutions are there to the equation $2x + 3 = 5$? Explain.

6. Which numbers in the table are solutions to the inequality $2x + 3 > 5$? Are these the only solutions to the inequality? Explain.

7. Would 1 be a solution to the inequality $2x + 3 \geq 5$? Explain.

Here are the number line graphs of two different inequalities.

$x < 3$

$-3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3$

$x \geq -2$

$-3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3$
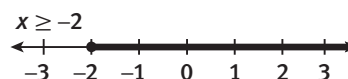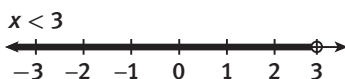
8. Use the graphic organizer to compare and contrast the two inequalities and graphs that are shown above.

Similarities	Differences

9. Think about why the graphs are different.
   a. Why is one of the graphs showing a solid ray going to the left and the other graph showing a solid ray going to the right?

   b. Why does one graph have an open circle and the other graph a filled-in circle?

WRITING MATH

An open circle represents $<$ or $>$ inequalities, and a filled-in circle represents $\leq$ or $\geq$ inequalities.

Check Your Understanding

10. Write an inequality to represent each statement.
    a. x is less than 12.
    b. m is no greater than 35.
    c. Your height h must be at least 42 inches for you to ride a theme park ride.
    d. A child's age a can be at most 12 for the child to order from the children's menu.

11. How are the graphs of $x > -4$ and $x \geq -4$ alike and how are they different?

LESSON 3-1 PRACTICE

Graph each inequality on a number line.

12. $x < -2$

13. $x \geq 5$

14. $x < 4$

15. $x > -2$

16. $x \geq \frac{1}{2}$

17. Write a real-world statement that could be represented by the inequality $x \leq 6$.

18. **Attend to precision.** Consider the inequalities $x \leq -3$ and $x \geq -3$.
    a. Graph $x \leq -3$ and $x \geq -3$ on the same number line.
    b. Describe any overlap in the two graphs.
    c. Describe the combined graphs.

My Notes

Learning Targets:

● Write inequalities to represent real-world situations.
● Solve multi-step inequalities.

> **SUGGESTED LEARNING STRATEGIES:** Create Representations, Guess and Check, Look for a Pattern, Think-Pair-Share, Identify a Subtask

1. **Make sense of problems.** Chloe and Charlie are taking a trip to the pet store to buy some things for their new puppy. They know that they need a bag of food that costs $7, and they also want to buy some new toys for the puppy. They find a bargain barrel containing toys that cost $2 each.
   a. Write an expression for the amount of money they will spend if the number of toys they buy is t.

   b. Chloe has $30 and Charlie has one-third of this amount with him. Use this information and the expression you wrote in Part (a) to write an inequality for finding the number of toys they can buy.

There are different methods for solving the inequality you wrote in the previous question. Chloe suggests that they guess and check to find the number of new toys that they could buy.

2. Use Chloe's suggestion to find the number of new puppy toys that Chloe and Charlie can buy with their combined money.

Charlie remembered that they could use algebra to solve inequalities. He imagined that the inequality symbol was an equal sign. Then he used equation-solving steps to solve the inequality.

3. Use Charlie's method to solve the inequality you wrote in Item 1b.

My Notes

4. Did you get the same answer using Charlie's method as you did using Chloe's method? Explain.

Check Your Understanding

5. How would you graph the solution to Charlie and Chloe's inequality?

6. Jaden solved an inequality as shown below. Describe and correct any errors in his work.

$$3x + 5 + 6x > 23$$
$$9x + 5 > 23$$
$$9x > 28$$
$$x > 7\frac{1}{9}$$

Chloe liked the fact that Charlie's method for solving inequalities did not involve guess and check, so she asked him to show her the method for the inequality $-2x - 4 > 8$.

Charlie showed Chloe the work below to solve $-2x - 4 > 8$.

$$-2x - 4 > 8$$
$$-2x - 4 + 4 > 8 + 4$$
$$-2x > 12$$
$$\frac{-2x}{-2} > \frac{12}{-2}$$
$$x > -6$$

When Chloe went back to check the solution by substituting a value for x back into the original inequality, she found that something was wrong.

7. Confirm or disprove Chloe's conclusion by substituting values for x into the original inequality.

My Notes

Chloe tried the problem again but used a few different steps.

$$-2x - 4 > 8$$
$$-2x + 2x - 4 > 8 + 2x$$
$$-4 > 8 + 2x$$
$$-4 - 8 > 8 - 8 + 2x$$
$$-12 > 2x$$
$$\frac{-12}{2} > \frac{2x}{2}$$
$$-6 > x$$

Chloe concluded that $x < -6$.

8. Is Chloe's conclusion correct? Explain.

9. Explain what Chloe did to solve the inequality.

Charlie looked back at his work. He said that he could easily fix his work by simply switching the inequality sign.

10. Critique the reasoning of others. What do you think about Charlie's plan? Explain.

Although all of these methods worked, Charlie and Chloe wanted to know why they were working.

Here is an experiment to discover what went wrong with Charlie's first method. Look at what happens when you multiply or divide by a negative number.

Directions	Numbers	Inequality
Pick two different numbers.	2 and 4	$2 < 4$
Multiply both numbers by 3.	2(3) and 4(3)	$6 < 12$
Multiply both numbers by −3.	2(−3) and 4(−3)	$-6 > -12$
Divide both numbers by 2.	2 ÷ 2 and 4 ÷ 2	$1 < 2$
Divide both numbers by −2.	2 ÷ (−2) and 4 ÷ (−2)	$-1 > -2$

11. Try this experiment again with two different numbers. Record your results in the *My Notes* section of this page. Compare your results to the rest of your class.

My Notes

12. **Express regularity in repeated reasoning.** What happens each time you multiply each side of an inequality by a negative number? What happens each time you divide each side of an inequality by a negative number?

13. How does this affect how you solve an inequality?

Example A

Solve and graph: $-5x + 8 \leq -2x + 23$

Step 1: Subtract 8 from both sides.

$$-5x + 8 - 8 \leq -2x + 23 - 8$$
$$-5x \leq -2x + 15$$

Step 2: Add $2x$ to both sides.

$$-5x + 2x \leq -2x + 2x + 15$$
$$-3x \leq 15$$

Step 3: Divide both sides by -3.
Remember to reverse the inequality sign.

$$\frac{-3x}{-3} \geq \frac{15}{-3}$$
$$x \geq -5$$

Solution: $x \geq -5$
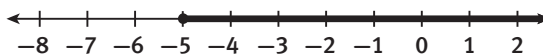
Try These A

Solve and graph each inequality.

a. $3 - 4x \leq 11$

b. $6 - 3(x + 2) > 15$

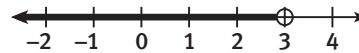**c.** $2(x + 5) < 8(x - 3)$

MATH TIP

Substitute some sample answers back into the original inequality to check your work.

My Notes

Check Your Understanding

14. Write two different inequalities that have the solution graphed on the number line below.

15. Explain why you reverse the inequality sign when you multiply or divide both sides of an inequality by a negative number.

MATH TIP

Remember to substitute some sample answers back into the original inequality to check your work.

LESSON 3-2 PRACTICE

Solve and graph each inequality.

16. $5 < 3x + 8$

17. $5 < -3x + 8$

18. $3x - 8 + 4x > 6$

19. $-5x + 2 \geq -8x$

20. $4 - 2(x + 1) < 18$

21. $-6x - 3 \leq -4x + 1$

22. $3(x + 7) \geq 2(2x + 8)$

23. $-2x - 3 + 8 < -3(3x + 5)$

24. Model with mathematics. Riley and Rhoda plan to buy several bags of dog food and a dog collar. Each bag of dog food costs $7, and the dog collar costs $5.
   a. Use the information above to write an expression for the amount of money they will spend if they buy b bags of food.
   b. Riley and Rhoda have $30. Use this information and the expression you wrote for Part (a) to write an inequality for finding the number of bags of food they could buy.
   c. Solve the inequality and graph the solutions. Check your answer in the original situation.

25. In Example A, $-5x + 8 \leq -2x + 23$ was solved by dividing each side of the inequality by -3 in the last step. Is there another way to solve this inequality so that you can avoid dividing by a negative number? Explain.

Learning Targets:
- Graph compound inequalities.
- Solve compound inequalities.

SUGGESTED LEARNING STRATEGIES: Vocabulary Organizer, Look for a Pattern, Create Representations, Think-Pair-Share, Note Taking

Compound inequalities are two inequalities joined by the word *and* or by the word *or*. Inequalities joined by the word *and* are called **conjunctions**. Inequalities joined by the word *or* are **disjunctions**. You can represent compound inequalities using words, symbols, or graphs.

1. Complete the table. The first two rows have been done for you.

Verbal Description	Some Possible Solutions	Inequality	Graph
all numbers from 3 to 8, inclusive	$3.5, 4, 4\frac{1}{3}, 5, 6, 7.9, 8$	$x \geq 3$ and $x \leq 8$	
all numbers less than 5 or greater than 10	$-2, 0, 3, 4, 4.8, 10\frac{3}{4}, 11$	$x < 5$ or $x > 10$	
all numbers greater than -1 and less than or equal to 4			
all numbers less than or equal to 3 or greater than 6			

2. Use the graphic organizer below to compare and contrast the graphs for conjunctions and disjunctions.

Similarities	Differences

Example A

Spartan Middle School distributes this chart to students each year to show what students must be able to do to pass the fitness test.
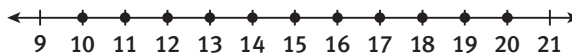
Age	Mile Run (min:sec)		Push-Ups		Modified Pull-Ups	
	Boys	Girls	Boys	Girls	Boys	Girls
12	8:00–10:30	9:00–12:00	10–20	7–15	7–20	4–13
13	7:30–10:00	9:00–12:00	12–25	7–15	8–22	4–13

Write and graph a compound inequality that describes the push-up range for 12-year-old boys.

Step 1: Choose a variable.
Let p represent the number of push-ups for 12-year-old boys.

Step 2: Determine the range and write an inequality.
The range is whole numbers p such that $p \geq 10$ and $p \leq 20$.

Solution: The compound inequality is $10 \leq p \leq 20$.

$$\leftarrow\!+\!\!\!\!\stackrel{\bullet}{}\!\!\!\!\stackrel{\bullet}{}\!\!\!\!\stackrel{\bullet}{}\!\!\!\!\stackrel{\bullet}{}\!\!\!\!\stackrel{\bullet}{}\!\!\!\!\stackrel{\bullet}{}\!\!\!\!\stackrel{\bullet}{}\!\!\!\!\stackrel{\bullet}{}\!\!\!\!\stackrel{\bullet}{}\!\!\!\!\stackrel{\bullet}{}\!\!\!\!\stackrel{\bullet}{}\!+\!\rightarrow$$
$$9 \quad 10 \quad 11 \quad 12 \quad 13 \quad 14 \quad 15 \quad 16 \quad 17 \quad 18 \quad 19 \quad 20 \quad 21$$

WRITING MATH

The inequality $p \geq 10$ is equivalent to the inequality $10 \leq p$. So the compound inequality "$p \geq 10$ and $p \leq 20$" can also be written as "$10 \leq p \leq 20$."

CONNECT TO AP

In upper-level mathematics classes, inequalities are expressed in interval notation. The interval notation for $x > 3$ is $(3, \infty)$.

Try These A

Write and graph a compound inequality for each range or score.

a. the push-up range for 13-year-old boys

b. the pull-up range for 13-year-old girls

c. the mile run range for 12-year-old girls

d. the mile run range for 13-year-old boys

e. a score outside the healthy fitness zone for girls' push-ups

3. **Attend to precision.** Why are individual points used in the graphs for Example A and some of the graphs in Try These A?

The solution of the conjunction will be *the solutions that are common to both parts.*

Example B
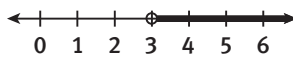
Solve and graph the conjunction: $3 < 3x - 6 < 8$

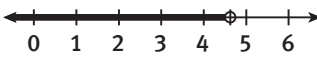**Step 1:** Break the compound inequality into two parts.
$3 < 3x - 6$ and $3x - 6 < 8$

Step 2: Solve and graph $3 < 3x - 6$.
$$3 < 3x - 6$$
$$3 + 6 < 3x - 6 + 6$$
$$9 < 3x$$
$$3 < x \text{ or } x > 3$$

Step 3: Solve and graph $3x - 6 < 8$.
$$3x - 6 < 8$$
$$3x - 6 + 6 < 8 + 6$$
$$3x < 14$$
$$x < 4\frac{2}{3}$$

Step 4: Determine what is common to the solutions of each part. In the inequalities and graphs in Steps 2 and 3, the points between 3 and $4\frac{2}{3}$ are in common.

Solution: $3 < x < 4\frac{2}{3}$

Try These B

Solve and graph each conjunction.

a. $-1 < 3x + 5 < 6$

b. $2 < \frac{x}{3} - 5 < 6$

c. $3 < 2(x + 2) - 7 \leq 13$

d. $-2 < 3(x + 6) < 18$

MATH TIP

Remember to substitute some sample answers back into the original inequality to check your work.
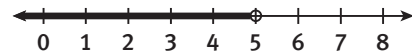
My Notes

The solution of a disjunction will be *all the solutions from both its parts.*

Example C
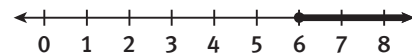Solve and graph the compound inequality: $2x - 3 < 7$ or $4x - 4 \geq 20$.

Step 1: Solve and graph $2x - 3 < 7$.
$$2x - 3 + 3 < 7 + 3$$
$$2x < 10$$
$$x < 5$$

Step 2: Solve and graph $4x - 4 \geq 20$.
$$4x - 4 \geq 20$$
$$4x - 4 + 4 \geq 20 + 4$$
$$4x \geq 24$$
$$x \geq 6$$

Step 3: Combine the solutions.

Solution: $x < 5$ or $x \geq 6$
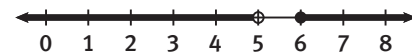
Try These C
Solve and graph each compound inequality.

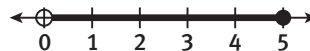**a.** $5x + 1 > 11$ or $x - 1 < -4$ **b.** $-5x > 20$ or $x - 2 \geq -7$

Check Your Understanding

4. The solutions of a conjunction are graphed below. What is the inequality?

5. Describe the difference in the graph of the conjunction "$x > 2$ and $x < 10$" and the graph of the disjunction "$x > 2$ or $x > 10$."

LESSON 3-3 PRACTICE

6. **Reason quantitatively.** A Category 2 hurricane has wind speeds of at least 96 miles per hour and at most 110 miles per hour. Write the wind speed of a Category 2 hurricane as two inequalities joined by the word *or* or *and*.

Solve and graph each compound inequality on a number line.

7. $-2x + 3 < 8$ and $3(x + 4) - 11 < 10$

8. $-3x + 5 > -1$ or $2(x + 4) > 14$

9. Write a real-world statement that could be represented by the compound inequality $\$7.50 < p \leq \$18.50$.

ACTIVITY 3 PRACTICE

Write your answers on notebook paper.
Show your work.

Lesson 3-1

1. Describe the similarities and differences in the solutions of $2x - 7 = 15$ and $2x - 7 \leq 15$.

2. For the equation $-3x + 2 = 8$ and the inequality $-3x + 2 > 8$, which x-values indicated below are solutions of the equation and which are solutions of the inequality?
   A. -3
   B. -2
   C. -1
   D. 0

3. Describe the graph of $x > -2$.

4. Describe the graph of $x \leq -2$.

5. Graph $x < 1\frac{1}{2}$ on a number line.

6. Graph $x \geq -3$ on a number line.

Lesson 3-2

7. Mayumi plans to buy pencils and a notebook at the school store. A pencil costs $0.15, and a notebook costs $1.59. Mayumi has $5.00. Which inequality could she use to find the number of pencils she can buy?
   A. $5.00 < 0.15x + 1.59$
   B. $5.00 \geq 0.15x + 1.59$
   C. $5.00 < 0.15x - 1.59$
   D. $5.00 \geq 0.15x - 1.59$

8. Which values of a and b disprove the statement below?

$$\text{If } a > b, \text{ then } a^2 > b^2.$$

   A. $a = 2, b = 0$
   B. $a = 4, b = 1$
   C. $a = 3, b = -5$
   D. $a = \frac{3}{4}, b = \frac{1}{2}$

Solve the inequality and graph the solutions on a number line. Check your answers.

9. $5x - 4 > -4$

10. $8 > 6 + \frac{2}{5}x$

11. $5 - 3x \leq 8$

12. $3x - 4 \geq 6x + 11$

13. $x - 2 \geq -8x + 16$

14. $3x - 7 < 2(2x - 1)$

15. $2\left(\frac{1}{2}x - 4\right) < -(x - 5)$

16. $\frac{2x - 11}{3} \geq -x - 2$

Write an inequality that requires more than one step to solve and that has the given solution.

17. $x < -3$

18. $x > 1$

19. Roy is attending his cousin's graduation ceremony in another town. Roy has already driven 30 miles and the ceremony starts in 2 hours.
   a. Let r represent Roy's driving speed in miles per hour. Write an expression to show the total distance Roy will have driven in 2 hours.
   b. The graduation is 150 miles from Roy's home. Use this information and your expression from Part (a) to write an inequality showing the possible speeds Roy could drive to make it to the ceremony on time.
   c. Solve your inequality and graph the solutions. What does your solution mean in the context of the problem?

Lesson 3-3

Use the table for Items 20 and 21.

Average High and Low Monthly Temperatures		
	January	**August**
Austin, TX	41–62°F	75–97°F
Columbus, OH	20–36°F	63–83°F

20. Write a compound inequality for the range of temperatures for Austin, TX, in August.

21. Write a compound inequality for the range of temperatures for Columbus, OH, in January.

22. The sum of the lengths of any two sides of a triangle must be greater than the length of the third side.
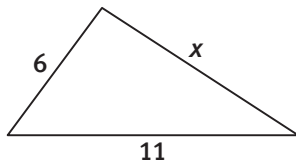
a. For the triangle shown, Anna said that any value of x greater than 5 is possible. Explain Anna's error.

b. Write a compound inequality that represents all possible values of x.

23. Find a value for n so that the compound inequality $-n < x < n$ has no solutions.

Solve each compound inequality and graph the solutions on a number line. Check your answers.

24. $-2 < 3x + 4 < 31$

25. $1 < 5x - 9 < 6$

26. $-2x + 7 > 1$ and $4x + 3 \geq -13$

27. $0 \leq \frac{6-x}{9}$ and $-2x \leq -10$

28. $5x - 2 < -7$ or $2x + 1 > 5$

29. $7x - 2 < -30$ or $4x + 5 > 13$

30. $\frac{1}{2} < \frac{2x-9}{2} \leq 3$

31. $3 \leq 2(x + 4) - 3 < 15$

32. $-2(x + 2) - 7 > 9$ or $3(x + 3) > -6$

MATHEMATICAL PRACTICES
Construct Viable Arguments and Critique the Reasoning of Others

33. The inequality $x + 5 < x + 4$ has no solutions. Explain why.

Learning Targets:

- Understand what is meant by a solution of an absolute value equation.
- Solve absolute value equations.

SUGGESTED LEARNING STRATEGIES: Paraphrasing, Create Representations, Think-Pair-Share, Note Taking, Identify a Subtask

Ms. Patel is preparing the school marching band for the homecoming show. She has the first row of band members stand in positions along a number line on the floor of the band room. The students' positions match the points on a number line as shown.
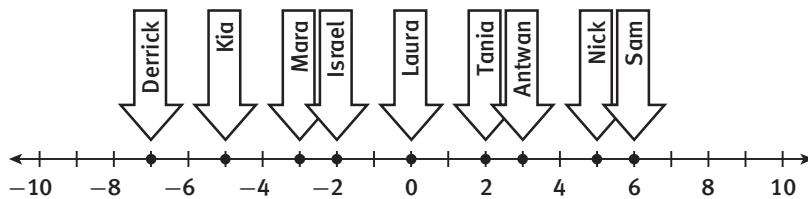
1. Use the number line to write each student's distance from 0 next to their name. For example, Tania is 2 units away from 0. Israel's distance from 0 is also 2 units even though he is at −2.

Derrick	Laura
Kia	Israel
Mara	Antwan
Tania	Nick
Sam	

The **absolute value** of a number is the distance from 0 to the number on a number line. Using **absolute value notation,** Mara's distance is $|-3|$ and Antwan's distance is $|3|$. Since Mara and Antwan are each 3 units from 0, $|-3| = 3$ and $|3| = 3$.

2. **Attend to precision.** Write each person's distance from 0 using absolute value notation.

Absolute value equations can represent distances on a number line.

3. The locations of the two students who are 5 units away from 0 are the solutions of the absolute value equation $|x| = 5$. Which two students represent the solutions to the equation $|x| = 5$?
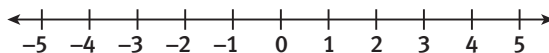
READING MATH

Read $|-3|$ as "the absolute value of negative three."

MATH TERMS

An **absolute value equation** is an equation involving the absolute value of a variable expression.
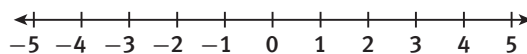
My Notes

4. You can create a graph on a number line to represent the solutions of an absolute value equation. Graph the solutions of the equation $|x| = 5$ on the number line below. Then use the graph to help you explain why it makes sense that the equation $|x| = 5$ has two solutions.

$$\xleftarrow{\quad} \overset{-5}{|} \; \overset{-4}{|} \; \overset{-3}{|} \; \overset{-2}{|} \; \overset{-1}{|} \; \overset{0}{|} \; \overset{1}{|} \; \overset{2}{|} \; \overset{3}{|} \; \overset{4}{|} \; \overset{5}{|} \xrightarrow{\quad}$$

Absolute value equations can also represent distances between two points on a number line.

5. In the student line, which two people are 4 units away from 1? Mark their location on the number line below.

$$\xleftarrow{\quad} \overset{-5}{|} \; \overset{-4}{|} \; \overset{-3}{|} \; \overset{-2}{|} \; \overset{-1}{|} \; \overset{0}{|} \; \overset{1}{|} \; \overset{2}{|} \; \overset{3}{|} \; \overset{4}{|} \; \overset{5}{|} \xrightarrow{\quad}$$

The equation $|x| = 4$ represents the numbers located 4 units away from 0. So the equation $|x| = 4$ can also be written as $|x - 0| = 4$, which shows the distance (4) away from the point 0. In Item 5, you were looking for the numbers located 4 units away from 1. So you can write the absolute value equation $|x - 1| = 4$ to represent that situation.

6. What are two possible values for $x - 1$ given that $|x - 1| = 4$? Explain.

7. Use the two values you found in Item 6 to write two equations showing what $x - 1$ could equal.

8. Solve each of the two equations that you wrote in Item 7.

The solutions in Item 8 represent the two points on the number line that are 4 units from 1.

9. How do the solutions relate to your answer for Item 5?

My Notes

10. Draw a number line to show the answer to each question. Then write an absolute value equation to represent the points described.

   a. Which two points are 2 units away from 0?

   b. Which two points are 5 units away from -2?

   c. Which two points are 3 units away from 4?

11. Solve these absolute value equations.

   a. $|x| = 10$ **b.** $|x| = -3$

   c. $|x + 2| = 7$ **d.** $|2x - 1| = 5$

MATH TIP

If there are no values of x that make an equation true, the equation has **no solution**.

In the equation $|x - 3| = 7$, the 7 indicates that the distance between x and 3 is 7 units. There are two points that are 7 units from 3. These can be found by solving the equations $x - 3 = 7$ and $x - 3 = -7$. Rewriting an absolute value equation as two equations allows you to solve the absolute value equation using algebra.

Example A
Solve the equation $|x - 3| = 7$.

Step 1: Rewrite the equation as two equations that do not have absolute value symbols.
$$x - 3 = 7 \qquad\qquad x - 3 = -7$$

Step 2: Solve $x - 3 = 7$.
$$x - 3 + 3 = 7 + 3$$
$$x = 10$$

Step 3: Solve $x - 3 = -7$.
$$x - 3 + 3 = -7 + 3$$
$$x = -4$$

Solution: $10, -4$

My Notes

Try These A

Solve each absolute value equation.

a. $|x - 5| = 1$

b. $|x + 5| = 2$

c. $|2x + 3| = 11$

d. $|3x - 4| = 8$

You may need to first isolate the absolute value expression to solve an absolute value equation.

Example B

Solve the equation $6|x + 2| = 18$.

Step 1: Divide both sides of the equation by 6 so that the absolute value expression is alone on one side of the equation.

$$\frac{6|x+2|}{6} = \frac{18}{6}$$

$$|x + 2| = 3$$

Step 2: Rewrite the equation as two equations that do not have absolute value symbols.

$$x + 2 = 3 \text{ and } x + 2 = -3$$

Step 3: Solve $x + 2 = 3$.

$$x + 2 - 2 = 3 - 2$$
$$x = 1$$

Step 4: Solve $x + 2 = -3$.

$$x + 2 - 2 = -3 - 2$$
$$x = -5$$

Solution: $1, -5$

MATH TIP

Check your answers by substituting the solutions in the original equation.

$$6|1 + 2| = 18$$
$$6|3| = 18$$
$$6(3) = 18$$
$$18 = 18$$

$$6|-5 + 2| = 18$$
$$6|-3| = 18$$
$$6(3) = 18$$
$$18 = 18$$

Try These B

Solve each absolute value equation.

a. $3|x - 1| = 12$

b. $|x| - 14 = 6$

c. $|x + 4| + 5 = 8$

d. $3|x + 6| - 7 = 20$

Check Your Understanding

12. Tell whether each statement is true or false. Explain your answers.
   a. For $x > 0$, $|x| = x$.
   b. For $x < 0$, $|x| = -x$.

13. Kate says that the opposite of $|-6|$ is 6. Is she correct? Explain.

LESSON 4–1 PRACTICE

Draw a number line to show the answer for each question. Then write an absolute value equation that has the numbers described as solutions.

14. Which two numbers are 3 units away from 0?

15. Which two numbers are 4 units away from -1?

16. Which two numbers are 3 units away from 3?

Solve each equation. Check your answers.

17. $|x - 5| = 8$

18. $|-2(x + 2)| = 1$

19. $|-(x - 5)| = 8.5$

20. $|3(x + 1)| = 15$

21. $2|x - 7| = -4$

22. $-2|x - 7| = -4$

23. **Make sense of problems.** Use the equations $|x - 3| = 7$ and $|x| - 3 = 7$ to answer the following questions.
   a. Describe the similarities and differences between the equations.
   b. Which of the following values are solutions of each equation: $-10, -4, 10$?
   c. Are the equations $|x - 3| = 7$ and $|x| - 3 = 7$ equivalent? Explain.
   d. Are the equations $|x| - 3 = 7$ and $|x| = 10$ equivalent? Explain.
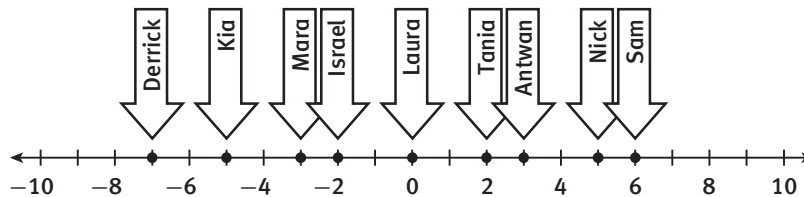
My Notes

Learning Targets:

- Solve absolute value inequalities.
- Graph solutions of absolute value inequalities.

> **SUGGESTED LEARNING STRATEGIES:** Role Play, Visualization, Create Representations, Guess and Check, Think-Pair-Share

Here is the marching band line-up once again.

1. Which people in the line up are 3 or fewer units from 0?

2. Show the portion of the number line that includes numbers that are 3 or fewer units from 0.

The graph you created in Item 2 can be represented with an **absolute value inequality**. The inequality $|x| \leq 3$ represents the numbers on a number line that are 3 or fewer units from 0.

3. Circle the numbers below that are solutions of $|x| \leq 3$. Explain why you chose those numbers.

$$-1 \qquad 3 \qquad 0.5 \qquad 4 \qquad -3.1$$

4. **Reason abstractly.** How many solutions does the inequality $|x| \leq 3$ have?

5. If you were to write a compound inequality for the graph of $|x| \leq 3$ that you sketched in Item 2, would it be a conjunction ("and" inequality) or a disjunction ("or" inequality)? Explain.
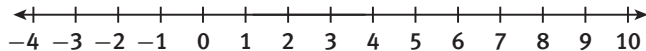
My Notes

6. Write a compound inequality to represent the solutions to $|x| \leq 3$.

7. What numbers are more than 4 units away from 3 on a number line? Show the answer to this question on the number line.

The absolute value inequality $|x - 3| > 4$ represents the situation in Item 7. A "greater than" symbol indicates that the distances are greater than 4.

8. Circle the numbers below that are solutions to the inequality $|x - 3| > 4$. Explain why you chose those numbers.

   7.1   7   0   −2   6.9   100

9. **Construct viable arguments.** If you were to write a compound inequality for the graph of $|x - 3| > 4$ that you sketched in Item 7, would it be a conjunction or disjunction? Explain.

10. To solve $|x - 3| > 4$ for x, you need to write the absolute value inequality as a compound inequality.
    a. Based on the graph from Item 7, the expression $x - 3$ is either greater than 4 or less than −4. Write this statement as a compound inequality.

    b. Solve each of the inequalities you wrote in Part (a). Graph the solution.

My Notes

11. Make a graph that represents the answer to each question. Then write an absolute value inequality that has the solutions that are graphed. Finally, write each absolute value inequality as a compound inequality.

   a. What numbers are less than 2 units from 0?

   b. What numbers are 4 or more units away from 0?

   c. What numbers are 4 or fewer units away from –2?

12. Describe the absolute value inequalities $|x| < 3$ and $|x| > 3$ as conjunctions or disjunctions and justify your choice in each case.

You can solve absolute value inequalities algebraically.

Example A

Solve the inequality $|2x| + 3 > 9$. Graph the solutions.

Step 1: Subtract 3 from both sides of the inequality so that the absolute value expression is alone on one side.
$$|2x| + 3 - 3 > 9 - 3$$
$$|2x| > 6$$

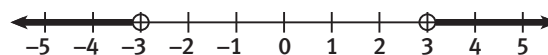**Step 2:** Rewrite the equation as a compound inequality. Determine if the relationship is "and" or "or."
$$2x > 6 \text{ or } 2x < -6$$

Step 3: Solve the inequalities.
$$2x > 6 \text{ or } 2x < -6$$
$$x > 3 \text{ or } x < -3$$

Solution: $x > 3$ or $x < -3$

My Notes

Try These A

Solve each absolute value inequality and graph the solutions.

a. $|2x - 7| > 3$

b. $|3x + 8| < 5$

c. $|4x| - 3 \geq 5$

d. $2|x| + 7 \leq 11$

e. $-3|x - 9| \geq -21$

> **MATH TIP**
>
> Remember that when multiplying or dividing each side of an inequality by a negative number, you must reverse the inequality symbol.

f. $-5|2x - 8| < -20$

My Notes

Check Your Understanding

13. Describe the similarities and differences between solving an absolute value equation and solving an absolute value inequality.

LESSON 4-2 PRACTICE

14. Graph the following and then write an absolute value inequality that represents each question.
  a. What numbers are 5 units or more away from -2 on a number line?
  b. What numbers are 5 units or fewer from -2 on a number line?

Solve each inequality and graph the solutions.

15. $|x - 4| \leq 2$

16. $|x - 5| > 3$

17. $\left|\dfrac{2}{3}x + 5\right| > 4$

18. $\left|\dfrac{3x - 5}{2}\right| \leq 7$

19. $|x| - 3 \leq 4$

20. $-3|x| < -12$

21. $6 \leq |2x - 9|$

22. $-5|x + 12| > -35$

23. Critique the reasoning of others. Isabelle was asked to write an absolute value inequality to represent the numbers that are less than 1 unit away from 7 on a number line. Isabelle wrote $|x - 1| < 7$. Explain and correct Isabelle's error.
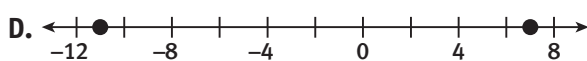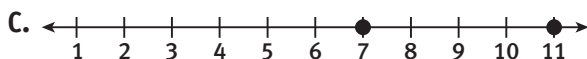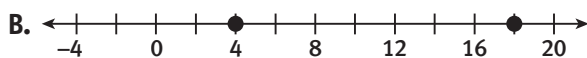
Absolute Value Equations and Inequalities
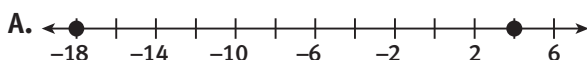**Student Distances**

ACTIVITY 4 PRACTICE
Write your answers on notebook paper.
Show your work.

Lesson 4-1

1. Use a number line to show the numbers that are 3 units from -1 on a number line. Then write an absolute value equation to describe the graph.

2. Explain why $|x| = -5$ does not have a solution.

3. Which graph shows the solutions of $|x - 11| = 7$?

   A.
   $-18 \quad -14 \quad -10 \quad -6 \quad -2 \quad 2 \quad 6$

   B.
   $-4 \quad 0 \quad 4 \quad 8 \quad 12 \quad 16 \quad 20$

   C.
   $1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10 \quad 11$

   D.
   $-12 \quad -8 \quad -4 \quad 0 \quad 4 \quad 8$

4. Suppose that x is negative and $|x| = n$. What conclusion can you draw?
   A. x and n are opposites.
   B. x and n are equal.
   C. x is greater than n.
   D. None of the above

For Items 5–17, solve each absolute value equation. Check your answers.

5. $|x| = 7$

6. $|x - 2| = -2$

7. $|x - (-2)| = 5$

8. $|3(x - 1)| = 15$

9. $\left|\frac{2}{5}x\right| = 4$

10. $|2(x - 3)| = 10$

11. $|3(x - 2)| = x$

12. $|4(x + 2)| + 9 = 15$

13. $|-2x + 3| = 7$

14. $|-3(x - 7)| = 21$

15. $-5|x - 2| = -20$

16. $-3|x + 5| + 7 = 4$

17. $\left|\frac{2x - 5}{7}\right| = 3$

Lesson 4-2

For Items 18–21, graph the solutions. Then write an absolute value inequality that represents each question.

18. What numbers are more than 3 units from -1 on a number line?

19. What numbers are less than 3 units from -1 on a number line?

20. What numbers are 5 or fewer units away from 3?

21. What numbers are 3 or more units away from 5?

For Items 22–25, graph the solutions of each absolute value inequality and write compound inequalities for the solutions.

22. $|x| > 3$

23. $|x| < 3$

24. $|x - 4| \geq 7$

25. $|x - 4| \leq 7$

26. Which describes the solutions of $|6x - 3| > 21$?
    A. all numbers greater than 4
    B. all numbers greater than -3 and greater than 4
    C. all numbers between -3 and 4
    D. all numbers less than -3 and greater than 4

27. Without solving, match each absolute value equation or inequality with its number of solutions. Justify your answers.

    $|x - 7| < -2$ \quad one solution

    $|x| = 0$ \quad no solutions

    $|x + 1| > -5$ \quad infinitely many solutions

28. The solutions to which absolute value inequality are shown in the graph below?

$-5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5$

A. $|x+1| < 1$
B. $|x+1| > 1$
C. $|x-1| < 1$
D. $|x-1| > 1$

29. Create a graphic organizer that compares and contrasts the following equation and inequalities.

$|x-3| = 6$

$|x-3| \le 6$

$|x| - 3 > 6$

For Items 30–40, solve each absolute value inequality and graph the solutions.

30. $|x-2| > 3$

31. $|x-5| < 2$

32. $|2x+7| \ge 5$

33. $|3x+2| \le 11$

34. $\left|\dfrac{5x-3}{2}\right| < 6$

35. $|4(x-1)| > 16$

36. $|x-7| + 3 < 2$

37. $|x+5| - 2 < 3$

38. $|2(x+1)| - 7 \le 1$

39. $\left|\dfrac{3x-1}{4}\right| \ge 5$

40. $-2|3x-4| \le -6$

41. Marty said that all absolute value inequalities that contain the symbol $<$ are conjunctions. Give a counterexample to show that Marty's statement is incorrect.

MATHEMATICAL PRACTICES
Model with Mathematics

42. According to some medical websites, normal body temperature can be as much as one degree above or below 98.6°F. Write a compound inequality that shows the range of normal body temperatures, t. Then write an absolute value inequality that shows the same information. Explain how you wrote the absolute value inequality, and include a number line graph in your explanation.