Linear Models
Stacking Boxes
Lesson 10-1 Direct Variation

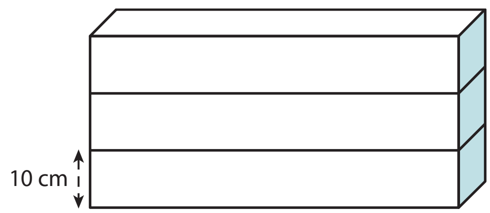
ACTIVITY 10

My Notes

Learning Targets:
- Write and graph direct variation.
- Identify the constant of variation.

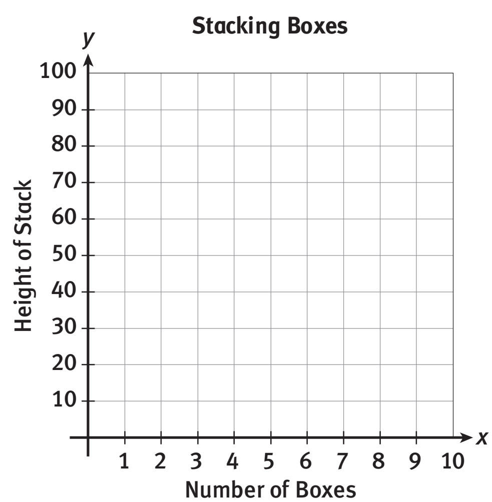
SUGGESTED LEARNING STRATEGIES: Create Representations, Interactive Word Wall, Marking the Text, Sharing and Responding, Discussion Groups

You work for a packaging and shipping company. As part of your job there, you are part of a package design team deciding how to stack boxes for packaging and shipping. Each box is 10 cm high.



1. Complete the table and make a graph of the data points (number of boxes, height of the stack).

Number of Boxes	Height of the Stack (cm)
0	0
1	10
2	20
3	
4	
5	
6	
7	



2. Write a function to represent the data in the table and graph above.

3. What is a reasonable and realistic domain for the function? Explain.

4. What is a reasonable and realistic range for the function? Explain.

WRITING MATH

Either y or $f(x)$ can be used to represent the output of a function.

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5. What do $f(x)$, or y, and x represent in your equation from Item 2?

6. Describe any patterns that you notice in the table and graph representing your function.

7. The number of boxes is **directly proportional** to the height of the stack. Use a proportion to determine the height of a stack of 12 boxes.

MATH TERMS

A **direct proportion** is a relationship in which the ratio of one quantity to another remains constant.

When two values are directly proportional, there is a **direct variation**. In terms of stacking boxes, the height of the stack *varies directly* as the number of boxes.

8. Using variables x and y to represent the two values, you can say that y varies directly as x. Use your answer to Item 6 to explain this statement.

MATH TERMS

direct variation
constant of variation

9. Direct variation is defined as $y = kx$, where $k \neq 0$ and the coefficient k is the **constant of variation**.

a. Consider your answer to Item 2. What is the constant of variation in your function?

b. Why do you think the coefficient is called the constant of variation?

c. **Reason quantitatively.** Explain why the value of k cannot be equal to 0.

d. Write an equation for finding the constant of variation by solving the equation $y = kx$ for k.

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10. a. Interpret the meaning of the point (0, 0) in your table and graph.

b. True or False? Explain your answer. "The graphs of all direct variations are lines that pass through the point (0, 0)."

c. Identify the slope and y-intercept in the graph of the stacking boxes.

d. Describe the relationship between the constant of variation and the slope.

Direct variation can be used to answer questions about stacking and shipping your boxes.

11. The height y of a different stack of boxes varies directly as the number of boxes x. For this type of box, 25 boxes are 500 cm high.

a. Find the value of k. Explain how you found your answer.

b. Write a direct variation equation that relates y, the height of the stack, to x, the number of boxes in the stack.

c. How high is a stack of 20 boxes? Explain how you would use your direct variation equation to find the height of the stack.

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My Notes

12. At the packaging and shipping company, you get paid each week. One week you earned \$48 for 8 hours of work. Another week you earned \$30 for 5 hours of work.

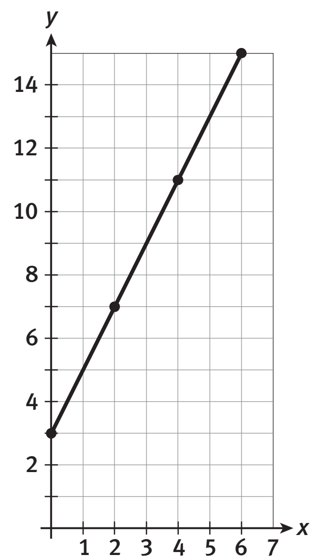
a. Write a direct variation equation that relates your wages to the number of hours you worked each week. Explain the meaning of each variable and identify the constant of variation.

b. How much would you earn if you worked 3.5 hours in one week?

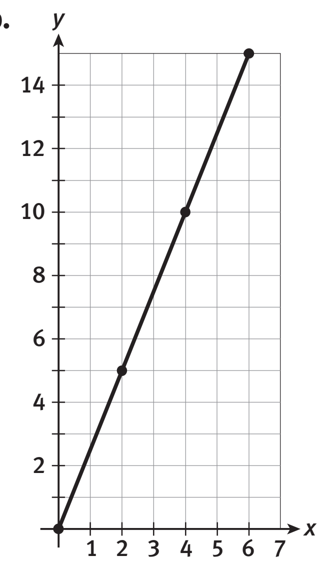
Check Your Understanding

13. Tell whether the tables, graphs, and equations below represent direct variations. Justify your answers.

a.



b.



c.

x	y
2	12
4	24
6	36

d.

x	y
2	8
4	12
6	16

e. $y = 20x$

f. $y = 3x + 2$

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My Notes

LESSON 10-1 PRACTICE

14. In the equation $y = 15x$, what is the constant of variation?

15. In the equation $y = 8x$, what is the constant of variation?

16. The value of y varies directly with x and the constant of variation is 7. What is the value of x when $y = 63$?

17. The value of y varies directly with x and the constant of variation is 12. What is the value of y when $x = 5$?

18. **Model with mathematics.** The height of a stack of boxes varies directly with the number of boxes. A stack of 12 boxes is 15 feet high. How tall is a stack of 16 boxes?

19. Jan's pay is in direct variation to the hours she works. Jan earns \$54 for 12 hours of work. How much will she earn for 18 hours work?

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Lesson 10-2
Indirect Variation

My Notes

Learning Targets:

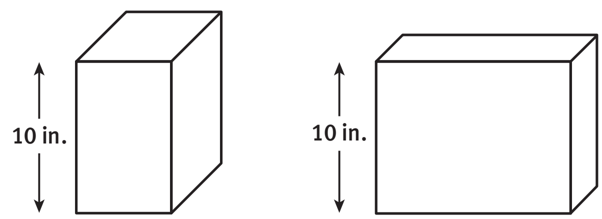
- Write and graph indirect variations.
- Distinguish between direct and indirect variation.

SUGGESTED LEARNING STRATEGIES: Create Representations, Marking the Text, Sharing and Responding, Think-Pair-Share, Discussion Groups

When packaging a different product, your team at the packaging and shipping company determines that all boxes for this product will have a volume of 400 cubic inches and a height of 10 inches. The lengths and the widths will vary.

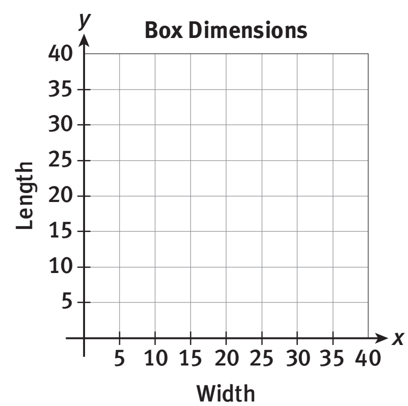
MATH TIP

The volume of a rectangular prism is found by multiplying length, width, and height: $V = lwh$.



1. To explore the relationship between length and width, complete the table and make a graph of the points.

Width (x)	Length (y)
1	40
2	20
4	10
5	
8	
10	
20	



2. How are the lengths and widths in Item 1 related? Write an equation that shows this relationship.

3. Use the equation you wrote in Item 2 to write a function to represent the data in the table and graph above.

4. Describe any patterns that you notice in the table and graph representing your function.

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My Notes

In terms of box dimensions, the length of the box varies indirectly as the width of the box. Therefore, this function is called an **indirect variation**.

5. Recall that direct variation is defined as $y = kx$, where $k \neq 0$ and the coefficient k is the constant of variation.

a. How would you define indirect variation in terms of y, k, and x?

b. Are there any limitations on these variables as there are on k in direct variation? Explain.

c. Write an equation for finding the constant of variation by solving for k in your answer to Part (a).

6. Reason abstractly. Compare and contrast the equations of direct and indirect variation.

7. Compare and contrast the graphs of direct and indirect variation.

8. Use your function in Item 3 to determine the following measurements for your company.

a. Find the length of a box whose width is 80 inches.

b. Find the length of a box whose width is 0.4 inches.

MATH TIP

Indirect variation is also known as **inverse variation.**

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My Notes

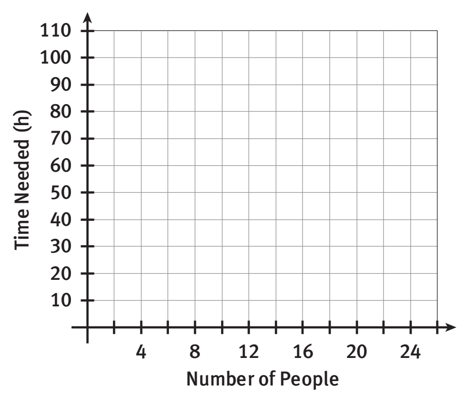
9. The time, y, needed to load the boxes on a truck for shipping varies indirectly as the number of people, x, working. If 10 people work, the job is completed in 20 hours.

a. Explain how to find the constant of variation. Then find it.

b. Write an indirect variation equation that relates the time to load the boxes to the number of people working.

c. How long does it take 8 people to finish loading the boxes? Use your equation to answer this question.

d. On the grid below, make a graph to show the time needed for 2, 4, 5, 8, 10, and 25 people to load the boxes on the truck.



10. The cost for the company to ship the boxes varies indirectly with the number of boxes being shipped. If 25 boxes are shipped at once, it will cost $10 per box. If 50 boxes are shipped at once, the cost will be $5 per box.

a. Write an indirect variation equation that relates the cost per box to the number of boxes being shipped.

b. How much would it cost to ship only 10 boxes?

11. Is an indirect variation function a linear function? Explain.

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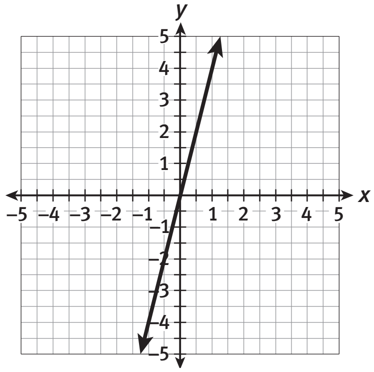
My Notes

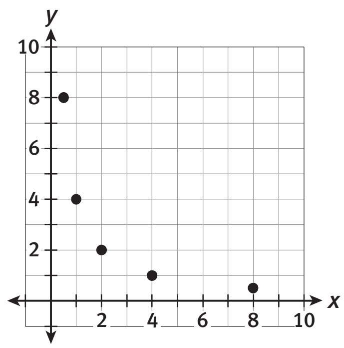
Check Your Understanding

12. Identify the following graphs as direct variation, indirect variation, neither, or both.

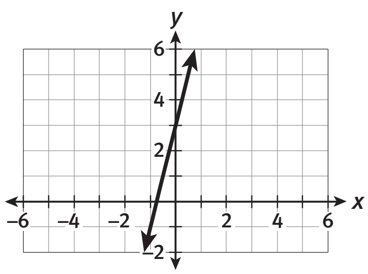
a.



b.



c.



13. Which equations are examples of indirect variation? Justify your answers.

A. $y = 2x$ **B.** $y = \frac{x}{2}$

C. $y = \frac{2}{x}$ **D.** $xy = 2$

14. In the equation $y = \frac{80}{x}$, what is the constant of variation?

LESSON 10-2 PRACTICE

15. Graph each function. Identify whether the function is an indirect variation.

a.

x	-4	-2	-1	1	2	4
y	$-\frac{3}{4}$	$-\frac{3}{2}$	-3	3	$\frac{3}{2}$	$\frac{3}{4}$

b.

x	-4	-2	-1	1	2	4
y	11	5	3	-4	-7	-13

16. Make sense of problems. For Parts (a) and (b) below, y varies indirectly as x.

a. If $y = 6$ when $x = 24$, find y when $x = 16$.

b. If $y = 8$ when $x = 20$, find the value of k.

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My Notes

CONNECT TO GEOMETRY

The carton will be a right rectangular prism. A **rectangular prism** is a closed, three-dimensional figure with three pairs of opposite parallel faces that are congruent rectangles.

Learning Targets:

- Write, graph, and analyze a linear model for a real-world situation.
- Interpret aspects of a model in terms of the real-world situation.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Discussion Groups, Create Representations, Guess and Check, Use Manipulatives

Your design team at the packaging and shipping company has been asked to design a cardboard box to use when packaging paper cups for sale. Your supervisor has given you the following requirements.

- All lateral faces of the container must be rectangular.
- The base of the container must be a square, just large enough to accommodate one cup.
- The height of the container must be given as a function of the number of cups the container will hold.
- All measurements must be in centimeters.

To help discover which features of the cup affect the height of the stack, collect data on two types of cups found around the office.

1. **Use appropriate tools strategically.** Use two different types of cups to complete the tables below.

CUP 1	
Number of Cups	Height of Stack
1	
2	
3	
4	
5	
6	

CUP 2	
Number of Cups	Height of Stack
1	
2	
3	
4	
5	
6	

2. **Express regularity in repeated reasoning.** What patterns do you notice that might help you figure out the relationship between the height of the stack and the number of cups in that stack?

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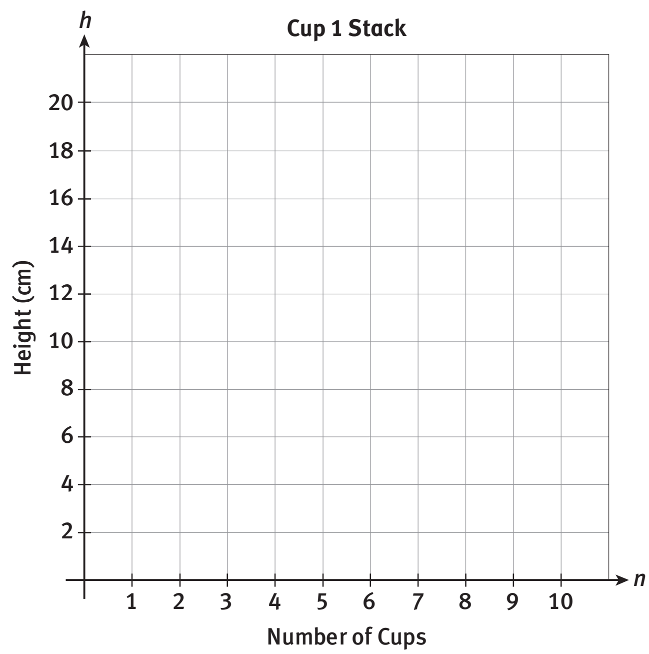
Lesson 10-3
Another Linear Model

My Notes

Use your data for Cup 1 to complete Items 3–13.

3. Make a graph of the data you collected.



4. Predict, without measuring, the height of a stack of 16 cups. Explain how you arrived at your prediction.

5. Predict, without measuring, the height of a stack of 50 cups. Explain how you arrived at your prediction.

6. Write an equation that gives the height of a stack of cups, h, in terms of n, the number of cups in the stack.

7. Use your equation from Item 6 to find h when $n = 16$ and when $n = 50$. Do your answers to this question agree with your predictions in Items 4 and 5?

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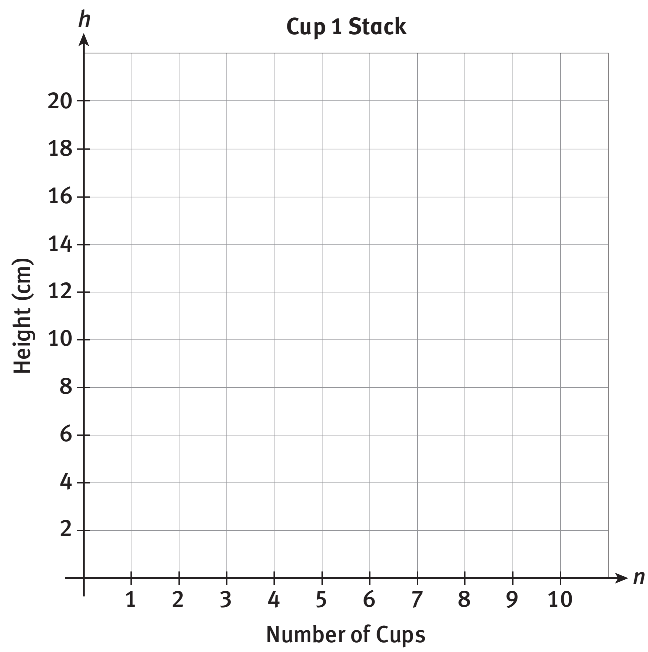
My Notes

8. Sketch the graph of your equation from Item 6.



9. How are the graphs you made in Items 3 and 8 the same? How are they different?

10. Do the graphs in Items 3 and 8 represent direct variation, indirect variation, or neither? Explain.

11. Remember that you are designing a container with a square base. What dimension(s), other than the height of the stack, do you need to design your cup container? Use Cup 1 to find this/these dimension(s).

12. Find the dimensions of a container that will hold a stack of 25 cups.

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13. Your team has been asked to communicate its findings to your supervisor. Write a report to her that summarizes your findings about the cup container design. Include the following information in your report.
- The equation your team discovered to find the height of the stack of Cup 1 style cups
- A description of how your team discovered the equation and the minimum number of cups needed to find it
- An explanation of how the numbers in the equation relate to the physical features of the cup
- An equation that could be used to find the height of the stack of Cup 2 style cups

My Notes

MATH TIP

When writing your answer to Item 13, you can use a RAFT.

- Role—team leader
- Audience—your boss
- Format—a letter
- Topic—stacks of cups

Check Your Understanding

14. A group of students performed the cup activity described in this lesson. For their Cup 1, they found the equation $h = 0.25n + 8.5$, where h is the height in inches of a stack of cups and n is the number of cups.

a. What would be the height of 25 cups? Of 50 cups?

b. Graph this equation. Describe your graph.

LESSON 10-3 PRACTICE

15. Reason quantitatively. A group of students performed the cup activity in this lesson using plastic drinking cups. Their data is shown below.

CUP 1	
Number of Cups	Height of Stack
1	14.5 cm
2	16 cm
3	17.5 cm
4	19 cm
5	20.5 cm

CUP 2	
Number of Cups	Height of Stack
1	10.5 cm
2	11.75 cm
3	13 cm
4	14.25 cm
5	15.5 cm

For each cup, write and graph an equation. Describe your graphs.

16. A consultant earns a flat fee of \$75 plus \$50 per hour for a contracted job. The table shows the consultant's earnings for the first four hours she works.

Hours	0	1	2	3	4
Earnings	\$75	\$125	\$175	\$225	\$275

The consultant has a 36-hour contract. How much will she earn?

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My Notes

Learning Targets:

- Write the inverse function for a linear function.
- Determine the domain and range of an inverse function.

SUGGESTED LEARNING STRATEGIES: Visualization, Create Representations, Think-Pair-Share, Discussion Groups, Construct an Argument

After reading your report, your supervisor was able to determine the equation for the height of the stack for the specific cup that the company will manufacture. The company will use the function $S(n) = 0.5n + 12.5$.

1. What do S, n, and $S(n)$ represent?

2. What do the numbers 0.5 and the 12.5 in the function S tell you about the physical features of the cup?

3. Evaluate $S(1)$ to find the height of a single cup.

4. How tall is a stack of 35 cups? Show your work using function notation.

5. If you add 2 cups to a stack, by how much does the height of the stack increase?

6. If you add 20 cups to a stack, by how much does the height of the stack increase?

7. **Critique the reasoning of others.** A member of one of the teams stated: “If you double the number of cups in a stack, then the height of the stack is also doubled.” Is this statement correct? Explain.

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My Notes

8. If you were to graph the function $S(n) = 0.5n + 12.5$, you would see that the points lie on a line.

a. What is the slope of this line?

b. Interpret the slope of the line as a rate of change that relates a change in height to a change in the number of cups.

Check Your Understanding

Use this table for Items 9 and 10.

x	1	2	3	4	5
y	1	5	9	13	17

9. Write an equation for y in terms of x.

10. Explain how the numbers in your equation relate to the numbers in the table.

11. Evaluate the function you wrote in Item 9 for each of the following values of x.

a. $x = 8$ **b.** $x = 12$ **c.** $x = 15$ **d.** $x = 0$

12. a. The supervisor wanted to increase the height of a container by 5 cm. How many more cups would fit in the container?

b. If the supervisor wanted to increase the height of a container by 6.4 cm, how many more cups would fit in the container?

c. How many cups fit in a container that is 36 cm tall?

d. How many cups fit in a container that is 50 cm tall?

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My Notes

13. The function $S(n) = 0.5n + 12.5$ describes the height S in terms of the number of cups n.

a. Solve this equation for n to describe the number of cups n in terms of the height S.

b. How many cups fit in a carton that is 85 cm tall? Compare your method of answering this question to your method used in Items 12c and 12d.

c. What is the slope of the line represented by your equation in Part (a)? Interpret it as a rate of change and compare it to the rate of change found in Item 8b.

READING MATH

$f^{-1}(x)$ is read as "f inverse of x". It does **not** mean "f to the negative one power."

An **inverse function** is a function that interchanges the independent and dependent variables of another function. In Item 13, you found the inverse function for $S(n)$. In general, the inverse function for $f(x)$ is $f^{-1}(x)$.

Example A

Use the table below to fill in the steps to find the inverse function for $f(x) = 2x + 3$.

Write the function, replacing $f(x)$ with y.	
Switch x and y.	
Solve for y in terms of x.	
Replace y with $f^{-1}(x)$.	

Try These A

Determine the inverses of each of the following of functions.

a. $f(x) = -4x - 5$ **b.** $f(x) = \frac{2}{3}x + 2$ **c.** $f(x) = -\frac{1}{2}x + 4$

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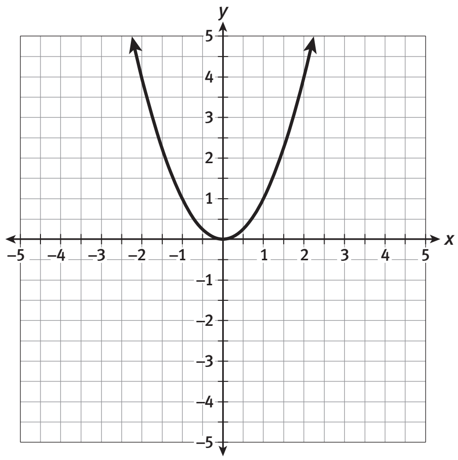
Lesson 10-4
Inverse Functions

Only those functions that are **one-to-one** functions have an inverse function. Functions that are not one-to-one must have their domain restricted for an inverse function to exist.

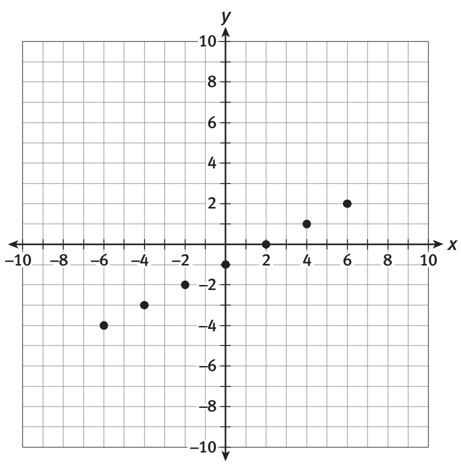
14. Is $S(n) = 0.5n + 12.5$ a one-to-one function? Explain.

15. Do the following graphs of functions show one-to-one functions? Justify your answers.

a.



b.



A visual test for a one-to-one function is the horizontal line test. If you can draw a horizontal line that intersects the graph of a function in more than one place, that function is not one-to-one.

16. Construct viable arguments. Are linear functions one-to-one functions? Justify your response.

My Notes

MATH TERMS

For a function to be **one-to-one** means that no two values of *x* are paired with the same value of *y*.

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My Notes

17. A function is defined by the ordered pairs {(−3, −1), (−1, 0), (1, 1), (3, 2), (5, 3)}. What are the domain and range of the function?

Because inputs and outputs are switched when writing the inverse of a function, the domain of a function is the range of its inverse function, and the range of a function is the domain of its inverse function.

18. What are the domain and range of the inverse function for the function in Item 17?

Check Your Understanding

The function $f(x) = 2.5x + 3.5$ gives the cost $f(x)$ of a cab ride of x miles.

19. What is the cost of a 6-mile ride?

20. What are the reasonable domain and range of the function?

21. Write the inverse function, $f^{-1}(x)$. What are the domain and range of $f^{-1}(x)$?

22. What does x represent in the inverse function?

23. A cab ride costs \$46. Show how to use the inverse function to find the distance of the cab ride in miles.

LESSON 10-4 PRACTICE

Make use of structure. Find the inverse function, $f^{-1}(x)$, for the functions in Items 24–26.

24. $f(x) = 3x - 5$

25. $f(x) = -2x + 10$

26. $f(x) = \frac{7x}{3} - \frac{1}{6}$

27. The yearly membership fee for the Art Museum is \$75. After paying the membership fee, the cost to enter each exhibit is \$7.50.

- **a.** Write a function for the total cost of a member for one year of attending the art museum.
- **b.** What is the total cost for a member who sees 12 exhibits?
- **c.** What are the domain and range for the function?
- **d.** What is $f^{-1}(x)$? What are the domain and range for $f^{-1}(x)$?
- **e.** What does x represent in $f^{-1}(x)$?
- **f.** How many exhibits can a member see in a year for a total of \$210, including the membership fee?

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ACTIVITY 10 PRACTICE

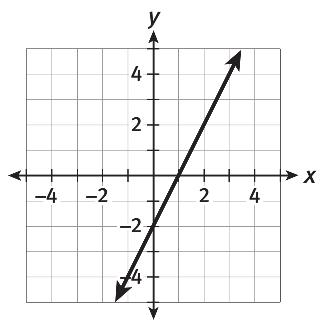
Write your answers on notebook paper.
Show your work.

Lesson 10-1

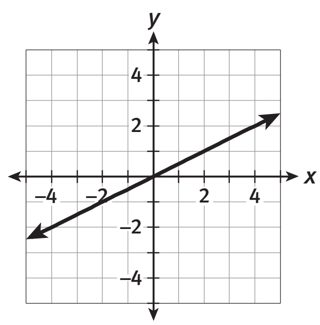
1. The value of y varies directly as x and $y = 125$ when $x = 25$. What is the value of y when $x = 2$?

2. Which is the graph of a direct variation?

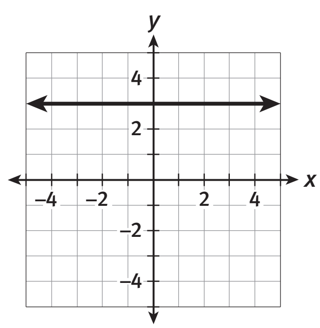
A.



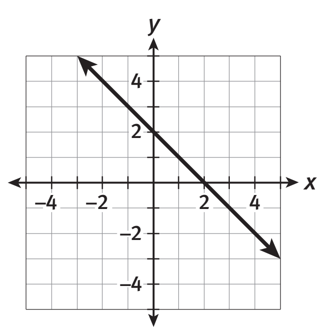
B.



C.



D.



3. Which equation **does not** represent a direct variation?
   A. $y = \frac{x}{3}$
   B. $y = \frac{2}{5}x$
   C. $y = \frac{3}{x}$
   D. $y = \frac{5x}{2}$

4. The value of y varies directly as x and $y = 9$ when $x = 6$. What is the value of y when $x = 15$?

5. The tailor determines that the cost of material varies directly with the amount of material. The cost is \$42 for 14 yards of material. What is the cost for 70 yards of material?

Lesson 10-2

6. The value of y varies indirectly as x and $y = 4$ when $x = 20$. What is the value of y when $x = 40$?
   A. $y = 2$
   B. $y = 8$
   C. $y = 50$
   D. $y = 80$

7. The temperature varies indirectly as the distance from the city. The temperature equals 3°C when the distance from the city is 40 miles. What is the temperature when the distance is 20 miles from the city?

8. The amount of gas left in the gas tank of a car varies indirectly to the number of miles driven. There are 9 gallons of gas left after 24 miles. How much gas is left after the car is driven 120 miles?

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Lesson 10-3

The Pete's Pets chain of pet stores is growing. The table below shows the number of stores in business each month. Use the table for Items 9–12.

Month	Stores
January	0
February	3
March	6
April	9
May	12
June	15

9. According to the table, how many new stores open per month?
10. How many stores will be in business by December?
11. Are the Pete's Pets data an example of indirect variation, direct variation, or neither? Explain your reasoning.
12. What is the slope of this function? Interpret the meaning of the slope.

Jeremy collected the following data on stacking chairs. Use the data for Items 13 and 14.

Number of Chairs	Height (in.)
1	30
2	33
3	36
4	39
5	42
6	45

13. Write a linear function that models the data.
14. Chairs cannot be stacked higher than 5 feet. What is the maximum number of chairs Jeremy can stack? Justify your answer.

Lesson 10-4

Write the inverse function for each of the following.

15. $f(x) = -8x + 4$
16. $f(x) = \frac{1}{4}x - 3$
17. $f(x) = 8x - 15$
18. $f(x) = x + 1$
19. $f(x) = -x + 1$

The formula to convert degrees Celsius C to degrees Fahrenheit F is $F = \frac{9}{5}C + 32$. Use this formula for Items 20–23.

20. Use the formula to convert 100°C to degrees Fahrenheit.
21. What is the slope?
22. The temperature is 50°F. What is the temperature in degrees Celsius?
23. Solve for C to derive the formula that converts degrees Fahrenheit to degrees Celsius.

MATHEMATICAL PRACTICES
Look for and Make Use of Structure

24. Describe the similarities and differences between finding the inverse of a function and working backward to solve a problem.

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Arithmetic Sequences

ACTIVITY 11

Picky Patterns

Lesson 11-1 Identifying Arithmetic Sequences

My Notes

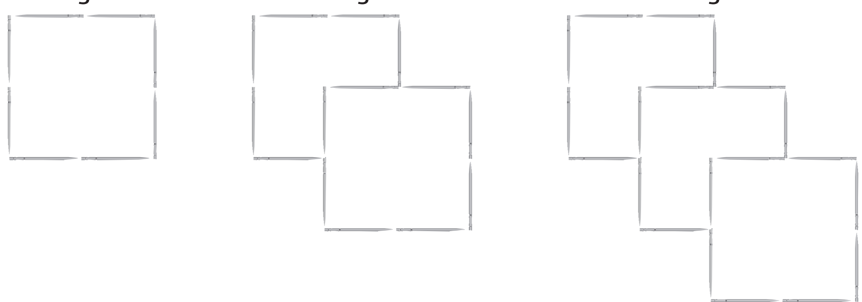
Learning Targets:

- Identify sequences that are arithmetic sequences.
- Use the common difference to determine a specified term of an arithmetic sequence.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Create Representations, Discussion Groups, Marking the Text, Use Manipulatives

1. Use toothpicks to make the following models.

Stage 1 Stage 2 Stage 3



2. Continue to Stage 4 and Stage 5. Draw your models below.

Stage 4 Stage 5

3. Complete the table for the number of toothpicks used for each stage of the models up through Stage 5.

Stage	Number of Toothpicks
1	8
2	
3	
4	
5	

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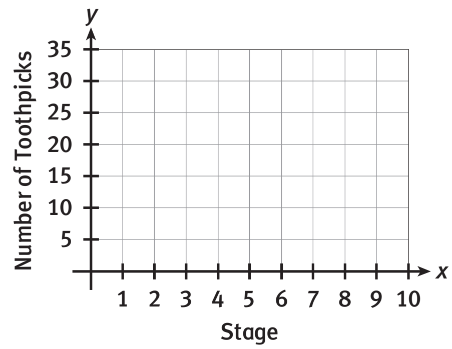
My Notes

4. **Model with mathematics.** Use the following grid to make a graph of the data in the table.



a. Is your graph discrete or continuous? Explain your answer.

b. Is your graph the graph of a linear function? Explain your answer.

An ordered list of numbers is called a **sequence**. The numbers in a sequence are **terms**. To refer to the *n*th term in a sequence, you can use either function notation, $f(n)$, or the indexed variable a_n.

READING MATH

Read a_n as "*a* sub *n*."

The toothpick data form a sequence. The numbers of toothpicks at each stage are the terms of the sequence.

5. What are the first four terms of the toothpick sequence?

$a_1 =$ $a_2 =$ $a_3 =$ $a_4 =$

6. In a sequence, what is the distinction between a term and a term number?

Check Your Understanding

7. For the sequence 7, −5, −3, 1, 1, . . . , what is a_4?

8. For the sequence 1, 5, 9, 13, 17, . . . , what is a_5?

An **arithmetic sequence** is a sequence in which the difference between terms is constant. The difference between consecutive terms in an arithmetic sequence is called the **common difference**.

READING MATH

A common difference may also be called a **constant difference**.

9. Explain why the toothpick sequence is an arithmetic sequence.

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My Notes

10. What is the rate of change for the toothpick data?

11. Look back at the graph in Item 4.

a. Determine the slope between any two points on the graph.

b. Describe the connections between the slope, the rate of change, and the common difference.

Check Your Understanding

Tell whether each sequence is an arithmetic sequence. For each arithmetic sequence, find the common difference.

12. 9, 16, 23, 30, 37, . . .

13. $-24, -20, -14, -10, -4, 0, \ldots$

14. $-2.8, -2.2, -1.6, -1.0, \ldots$

15. 3, 5, 8, 12, 17, . . .

16. $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, \ldots$

17. Reason abstractly. Can the common difference in an arithmetic sequence be negative? If so, give an example. If not, explain why not.

LESSON 11-1 PRACTICE

Tell whether each sequence is arithmetic. If the sequence is arithmetic, identify the common difference and find the indicated term.

18. $-9, -4, 1, 6, 11, \ldots; a_7 = ?$

19. $2, 4, 7, 11, 16, 22, \ldots; a_9 = ?$

20. $-7, -1, 5, 11, 17, \ldots; a_6 = ?$

21. $1.2, 1.9, 2.6, 3.3, 4.0, \ldots; a_8 = ?$

22. $3, \frac{5}{2}, \frac{3}{2}, -\frac{3}{2}, \ldots; a_7 = ?$

23. Write an arithmetic sequence in which the last digit of each term is 4. What is the common difference for your sequence?

24. Critique the reasoning of others. Jim said that the terms in an arithmetic sequence must always increase, because you must add the common difference to each term to get the next term. Is Jim correct? Justify your reasoning.

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My Notes

Learning Targets:

- Develop an explicit formula for the *n*th term of an arithmetic sequence.
- Use an explicit formula to find any term of an arithmetic sequence.
- Write a formula for an arithmetic sequence given two terms or a graph.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Create Representations, Interactive Word Wall, Predict and Confirm, Think-Pair-Share

1. Rewrite the terms of the toothpick sequence and identify the common difference.

2. Find the next three terms in the sequence without building toothpick models. Explain how you found your answers.

3. Why might it be difficult to find the 100th term of the toothpick sequence using repeated addition of the common difference?

MATH TERMS

An **explicit formula** for an arithmetic sequence describes any term in the sequence using the first term and the common difference.

An **explicit formula** for a sequence allows you to compute any term in a sequence without computing all of the terms before it.

4. Develop an explicit formula for the toothpick sequence using the first term and the common difference.

The first term of the sequence is $a_1 = 8$.

The second term is $a_2 = 8 + 6$.

The third term is $a_3 = 8 + 6 + 6$, or $a_3 = 8 + 2(6)$.

a. Write an expression for the fourth term using the value of a_1 and the common difference.
$a_4 =$

b. Express regularity in repeated reasoning. Use the patterns you have observed to determine the 15th term. Justify your reasoning.
$a_{15} =$

c. Write an expression that can be used to find the *n*th term of the toothpick sequence.
$a_n =$

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Lesson 11-2
A Formula for Arithmetic Sequences

My Notes

The formula you wrote in Item 4c is the explicit formula for the toothpick sequence.

For any arithmetic sequence, a_1 refers to the first term and d refers to the common difference.

5. Write an explicit formula for finding the nth term of any arithmetic sequence.

Example A

Write the explicit formula for the arithmetic sequence 3, −3, −9, −15, −21, ... Then use the formula to find the value of a_{10}.

Step 1: Find the common difference.

$$3 \xrightarrow{-6} -3 \xrightarrow{-6} -9 \xrightarrow{-6} -15 \xrightarrow{-6} -21$$

The common difference is −6.

Step 2: Write the explicit formula and simplify.

$a_n = a_1 + (n - 1)d$

$a_n = 3 + (n - 1)(-6) = 9 - 6n$

Step 3: Use the formula to find a_{10} by substituting for n.

$a_{10} = 9 - 6(10) = -51$

Solution: The explicit formula is $a_n = 9 - 6n$ and $a_{10} = -51$.

Try These A

For the following arithmetic sequences, find the explicit formula and the value of the indicated term.

a. 2, 6, 10, 14, 18, . . . ; a_{21}

b. −0.6, −1.0, −1.4, −1.8, −2.2, . . . ; a_{15}

c. $\frac{1}{3}, 1, \frac{5}{3}, \frac{7}{3}, \ldots; a_{37}$

An arithmetic sequence can be graphed on a coordinate plane. In the ordered pairs the term numbers (1, 2, 3, . . .) are the x-values and the terms of the sequence are the y-values.

6. Look back at the sequence in Example A. Make a prediction about its graph.

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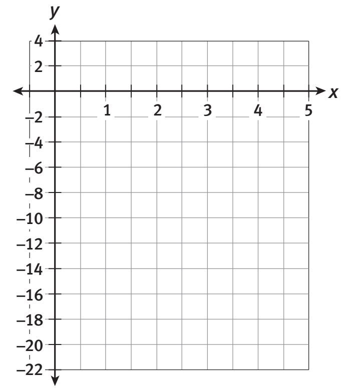
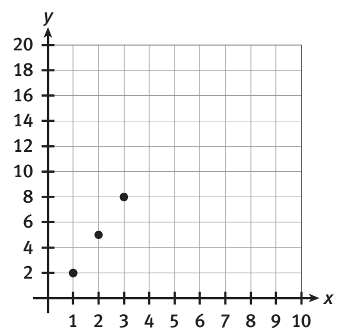
My Notes

7. On the grid below, create a graph of the arithmetic sequence in Example A. Revise your prediction in Item 6 if necessary.



8. Determine the slope between any two points on your graph in Item 7. How does the slope compare to the common difference of the sequence?

9. The first three terms of the arithmetic sequence 2, 5, 8, . . . are graphed below. Determine the common difference. Then graph the next three terms.



10. Determine the slope between any two points you graphed in Item 9. How does the slope compare to the common difference of the sequence?

11. Write the explicit formula for the sequence graphed in Item 9.

12. If you are given a graph of an arithmetic sequence, how do you find the explicit formula?

The 11th term of an arithmetic sequence is 59 and the 14th term is 74.

$$\begin{array}{cccc} a_{11} & a_{12} & a_{13} & a_{14} \\ 59 & ? & ? & 74 \end{array}$$

$$59 \xrightarrow{+d} ? \xrightarrow{+d} ? \xrightarrow{+d} 74$$

13. **Reason quantitatively.** How could you determine the value of *d*? What is the value of *d*?

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My Notes

14. How could you determine the value of a_1? What is this value?

15. Write the explicit formula for the nth term of the sequence.

16. Determine a_{30}, the 30th term of the sequence.

Check Your Understanding

17. The explicit formula for the nth term of an arithmetic sequence is $a_n = a_1 + (n - 1)d$. What does each variable in the explicit formula represent?

18. What is a_{50} of the arithmetic sequence 45, 40, 35, 30, . . . ?

19. The 8th term of an arithmetic sequence is 12.5 and the 13th term is 20. What is a_{25}?

20. **Construct viable arguments.** Could an arithmetic sequence also be a direct variation? Justify your answer.

21. Why is the graph of an arithmetic sequence made up of discrete points?

LESSON 11-2 PRACTICE

For the following arithmetic sequences, find the explicit formula and the value of the term indicated.

22. 2, 11, 20, 29, . . . ; a_{30}

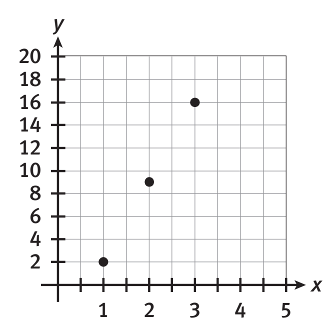
23. 0.5, 0.75, 1, . . . ; a_{18}

24. $\frac{1}{6}, 0, -\frac{1}{6}, -\frac{1}{3}, \ldots; a_{42}$

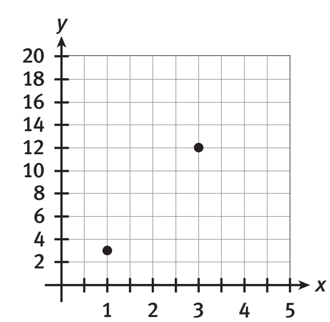
25. The 3rd term of an arithmetic sequence is -1 and the 7th term is -13. Find the explicit formula for this sequence. What is a_{22}?

26. **Make use of structure.** Write the explicit formula for each arithmetic sequence graphed below. Then find the 25th term.

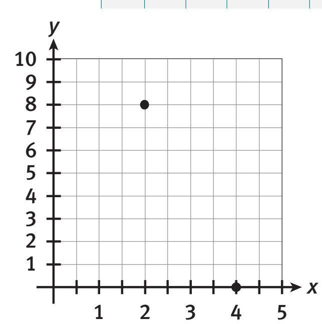
a.



b.



c.



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My Notes

Learning Targets:

- Use function notation to write a general formula for the *n*th term of an arithmetic sequence.
- Find any term of an arithmetic sequence written as a function.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Create Representations, Discussion Groups, Sharing and Responding, Group Presentation

An arithmetic sequence is a special case of a linear function. The terms of the sequence are the functional values $f(1), f(2), f(3), \ldots, f(n)$ for some n.

1. Fill in the next three terms of the arithmetic sequence.

$a_1 = 7 = f(1)$

$a_2 = 10 = f(2)$

$a_3 = 13 = f(3)$

$a_4 =$ ___________ $= f(4)$

$a_5 =$ ___________ $= f(5)$

$a_6 =$ ___________ $= f(6)$

2. What is the *n*th term of the sequence?

3. What function *f* could be used to describe the sequence?

4. What is the common difference of the sequence? How is the common difference related to the function you wrote in Item 3?

5. **Attend to precision.** Describe the domain of *f* using set notation. (*Hint*: What values are used as inputs for *f*?)

6. What ordered pair represents the *n*th term of the sequence?

7. Describe the graph of *f*. How is the common difference related to the graph?

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Check Your Understanding

For Items 8–11, use the arithmetic sequence –5, 1, 7, 13, ...

8. What is $f(1)$?

9. What is $f(4)$?

10. Write a function to describe the sequence.

11. Use your function to find $f(14)$.

LESSON 11-3 PRACTICE

Write a function to describe each arithmetic sequence.

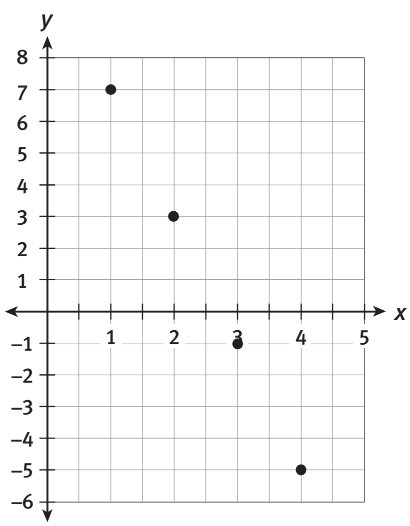
12. 10, 14, 18, 22, …

13. 8.5, 10.3, 12.1, 13.9, …

14. $\frac{1}{3}, \frac{7}{12}, \frac{5}{6}, \frac{13}{12}, \ldots$

15. −7, −4.5, −2, 0.5, …

16.



The 1st term of an arithmetic sequence is 5, and the common difference is 1.5. Use this information for Items 17–19.

17. What is $f(3)$?

18. Write a function to describe this arithmetic sequence.

19. Determine the 25th term of the sequence. Use function notation in your answer.

20. **Make sense of problems.** The 3rd term of an arithmetic sequence is −2, and the 8th term is −32. Write a function to describe this sequence.

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My Notes

MATH TIP

In a sequence, $f(n-1) = a_{n-1}$ refers to the term before $f(n) = a_n$. Item 1 is asking "For any value of n, how can you find the term before $f(n) = a_n$?"

Learning Targets:

- Write a recursive formula for a given arithmetic sequence.
- Use a recursive formula to find the terms of an arithmetic sequence.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Create Representations, Close Reading, Marking the Text, Discussion Groups

In a sequence, the term before $f(n) = a_n$ is $f(n-1) = a_{n-1}$.

The first four terms of the toothpick sequence can be written as

$a_1 = 8$	$f(1) = 8$
$a_2 = 14 = a_1 + 6$	$f(2) = 14 = f(1) + 6$
$a_3 = 20 = a_2 + 6$	$f(3) = 20 = f(2) + 6$
$a_4 = 26 = a_3 + 6$	$f(4) = 26 = f(3) + 6$

1. For any value of n, how can you find the value of $f(n-1) = a_n - 1$?

A **recursive formula** can be used to represent an arithmetic sequence. Recursion is the process of choosing a starting term and repeatedly applying the same process to each term to arrive at the following term.

A recursive formula for an arithmetic sequence looks like this:

$$\begin{cases} a_1 = \text{ 1st term} \\ a_n = a_{n-1} + d \end{cases}, \text{ or in function notation: } \begin{cases} f(1) = \text{ 1st term} \\ f(n) = f(n-1) + d \end{cases}$$

2. The recursive formulas for the toothpick sequence are partially given below. Complete them by writing the expressions for a_n and $f(n)$.

$$\begin{cases} a_1 = 8 \\ a_n = \end{cases} \quad \text{and} \quad \begin{cases} f(1) = 8 \\ f(n) = \end{cases}$$

Check Your Understanding

Write the recursive formula for the following arithmetic sequences. Include the recursive formula in function notation.

3. 2, 4, 6, 8, . . .

4. $-2, -5, -8, -11, \ldots$

5. $-3, -\frac{3}{2}, 0, \frac{3}{2}, \ldots$

6. Suppose that $a_{n-1} = -4$.

a. Find the value of a_n for the arithmetic sequence with the recursive formula $\begin{cases} a_1 = 6 \\ a_n = a_{n-1} + (-5) \end{cases}$.

b. What term did you find? (In other words, what is n equal to?)

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My Notes

7. An arithmetic sequence has the recursive formula below.

$$\begin{cases} f(1) = \frac{1}{2} \\ f(n) = f(n-1) + 2 \end{cases}$$

a. Determine the first five terms of the sequence.

b. Write the explicit formula for the sequence using function notation.

8. An arithmetic sequence has the explicit formula $a_n = 3n - 8$.

a. What are the values of a_1 and a_2?

b. How can you use the values of a_1 and a_2 to find d? What is d?

c. Use your answers to Parts (a) and (b) to write the recursive formula for the sequence.

In the 12th century, Leonardo of Pisa, also known as Fibonacci, first described a sequence known as the **Fibonacci sequence**. The sequence can be described by the recursive formula below.

$$\begin{cases} a_1 = 1 \\ a_2 = 1 \\ a_n = a_{n-1} + a_{n-2}, \text{ for } n > 2 \end{cases}$$

Notice that the first two terms of the sequence are 1 and that the expression describing a_n applies to those terms after the 2nd term.

9. Use the recursive formula to determine the first 10 terms of the Fibonacci sequence.

10. Is the Fibonacci sequence an arithmetic sequence? Justify your response.

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My Notes

Check Your Understanding

11. **Attend to precision.** Compare and contrast the explicit and recursive formulas for an arithmetic sequence.

12. Explain how to find any term of the Fibonacci sequence.

LESSON 11-4 PRACTICE

Write the recursive formula for each arithmetic sequence. Include the recursive formula in function notation.

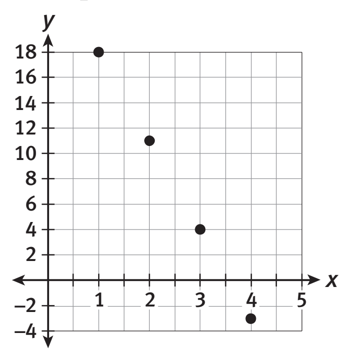
13. 1, 6, 11, 16, . . .

14. 1, 4, 7, 10, 13, . . .

15. $a_n = 11 - 3n$

16. $a_n = \frac{1}{4} + \frac{3}{20}n$

17.



18. Given $f(n-1) = 1.2$, use the recursive formula below to find $f(n)$.

$$\begin{cases} f(1) = -0.6 \\ f(n) = f(n-1) + 0.3 \end{cases}$$

19. **Reason quantitatively.** Describe how each sequence is similar to the Fibonacci sequence. Then find the next two terms.

a. 4, 4, 8, 12, 20, 32, . . .

b. 2, 2, 4, 6, 10, 16, . . .

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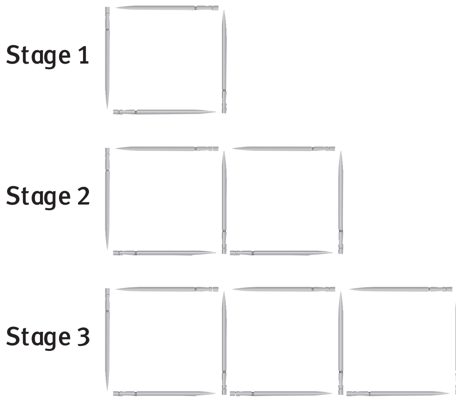
ACTIVITY 11 PRACTICE

**Write your answers on notebook paper.
Show your work.**

Lesson 11-1

For Items 1–3, refer to the toothpick pattern shown below.



1. Copy and complete the table below.

Stage	Number of Toothpicks
1	
2	
3	
4	
5	

2. Write the number of toothpicks as a sequence.
3. Is the sequence you wrote in Item 2 an arithmetic sequence? If so, determine the common difference. If not, explain why not.
4. Which of the following is **not** an arithmetic sequence?
   A. $\frac{1}{2}, 1, \frac{3}{2}, 2 \ldots$
   B. 11, 14, 17, 20, …
   C. 2, 4, 8, 16, …
   D. 5, 2, −1, −4, …

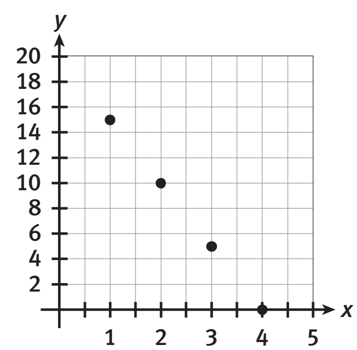
For Items 5 and 6, find the common difference for each arithmetic sequence.

5. −2.3, −1.1, 0.1, 1.3, …
6. −9, −13, −18, −23, …
7. What are the next three terms in the arithmetic sequence −6, −10, −14, … ?
8. Write an arithmetic sequence in which some of the terms are whole numbers and the common difference is $\frac{3}{4}$.

Lesson 11-2

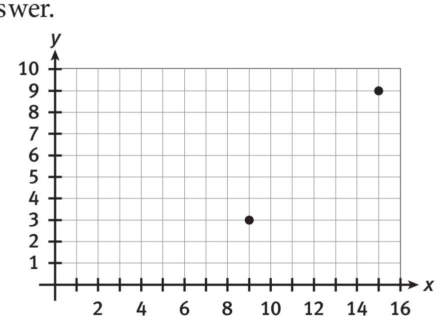
For Items 9 and 10, determine the explicit formula for each arithmetic sequence.

9. 3, −3, −9, −15, …
10. 1, 6, 11, 16, …
11. What is the 20th term of the arithmetic sequence: 7, 4, 1, … ?
12. The 9th and 10th terms of an arithmetic sequence are −24 and −30, respectively. What is the 30th term?
13. The 15th and 21st terms of an arithmetic sequence are −67 and −97, respectively. What is the 30th term?
14. The 9th and 14th terms of an arithmetic sequence are 23 and 33, respectively. What is the 1st term?
15. For the sequence graphed below, write the explicit formula. Then find the 57th term of the sequence.



Lesson 11-3

16. Write a function to describe the arithmetic sequence graphed below. Then find the 5th term of the sequence. Use function notation in your answer.



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17. For an arithmetic sequence, $f(1) = \frac{4}{5}$ and the common difference is -1. What is $f(20)$?

A. $18\frac{1}{5}$

B. $-\frac{1}{5}$

C. $-20\frac{1}{5}$

D. $-18\frac{1}{5}$

18. An arithmetic sequence is described by the function $f(n) = -3n + 7$. Determine the first five terms of this sequence.

Lesson 11-4

19. What are the first five terms in the arithmetic sequence with the recursive formula below?

$$\begin{cases} a_1 = 5 \\ a_n = a_{n-1} + 4 \end{cases}$$

20. What is the recursive formula for the arithmetic sequence described by the function below?

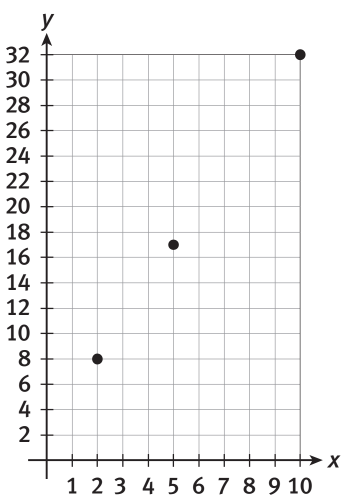
$$f(n) = \frac{1}{2}n - 2$$

21. What is the explicit formula for the arithmetic sequence that has the recursive formula below?

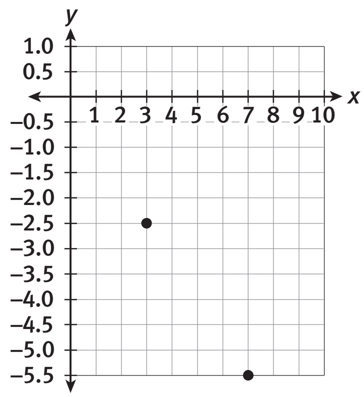
$$\begin{cases} f(1) = -2.5 \\ f(n) = f(n-1) - 4 \end{cases}$$

For Items 22 and 23, write the recursive formula for the arithmetic sequence that is graphed. Include the recursive formula in function notation.

22.



23.



MATHEMATICAL PRACTICES
Make Sense of Problems and Persevere in Solving Them

24. The Lucas sequence is related to the Fibonacci sequence. The first five terms of the Lucas sequence are given below.

$a_1 = 1$

$a_2 = 1 + 2 = 3$

$a_3 = 1 + 3 = 4$

$a_4 = 2 + 5 = 7$

$a_5 = 3 + 8 = 11$

a. Beginning with a_2, what do you observe about the first addends in each sum? What do you observe about the second addends?

b. What are the next two terms of the Lucas sequence? Explain how you determined your answer.

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Forms of Linear Functions

ACTIVITY 12

Under Pressure
Lesson 12-1 Slope-Intercept Form

My Notes

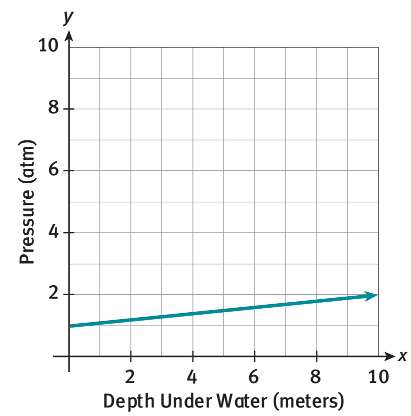
Learning Targets:

- Write the equation of a line in slope-intercept form.
- Use slope-intercept form to solve problems.

SUGGESTED LEARNING STRATEGIES: Create Representations, Think-Pair-Share, Marking the Text, Discussion Groups

When a diver descends in a lake or ocean, pressure is produced by the weight of the water on the diver. As a diver swims deeper into the water, the pressure on the diver's body increases at a rate of about 1 *atmosphere of pressure* per 10 meters of depth. The table and graph below represent the total pressure, y, on a diver given the depth, x, under water in meters.

x	y
0	1
1	1.1
2	1.2
3	1.3
4	1.4
5	1.5
6	1.6



CONNECT TO SCIENCE

Pressure is the measure of a force against a surface, and is usually expressed as a force per unit area. *Atmospheric pressure* is defined using the unit atmosphere. 1 atm is 14.6956 pounds per square inch.

1. Write an equation describing the relationship between the pressure exerted on a diver and the diver's depth under water.

2. What is the slope of the line? What are the units of the slope?

3. What is the y-intercept? Explain its meaning in this context.

MATH TERMS

A **linear equation** is an equation that can be written in standard form $Ax + By = C$ where A, B, and C are constants and A and B cannot both be zero.

Slope-Intercept Form of a Linear Equation

$$y = mx + b$$

where m is the slope of the line and $(0, b)$ is the y-intercept.

MATH TIP

Linear equations can be written in several forms.

4. Identify the slope and y-intercept of the line described by the equation $y = -2x + 9$.

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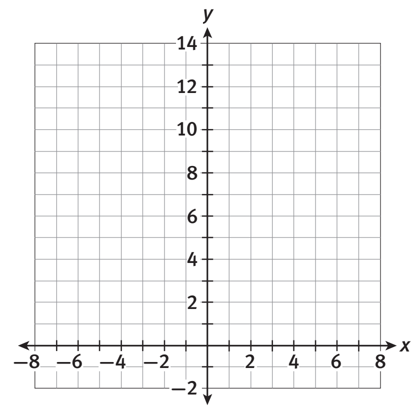
My Notes

5. Create a table of values for the equation $y = -2x + 9$. Then plot the points and graph the line.

x	y
-2	
0	
2	
4	
5	



6. Explain how to find the value of the slope from the table. What is the value of the slope of the line?

7. Explain how to find the y-intercept from the table. What is the y-intercept?

8. Explain how to find the value of the slope from the graph. What is the value of the slope?

9. Explain how to find the y-intercept from the graph. What is the y-intercept?

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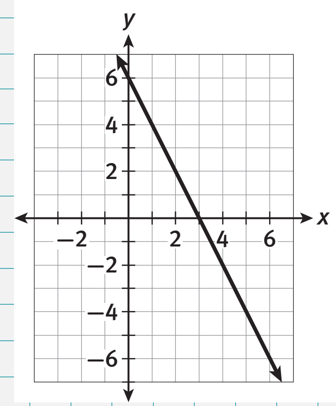
My Notes



Check Your Understanding

10. What are the slope and y-intercept of the line described by the equation $y = -\frac{4}{5}x - 10$?

11. Write the equation in slope-intercept form of the line that is represented by the data in the table.

x	−2	−1	0	1	2	3
y	9	7	5	3	1	−1

12. Write the equation, in slope-intercept form, of the line with a slope of 4 and a y-intercept of (0, 5).

13. Write an equation of the line graphed in the *My Notes* section of this page.

Monica gets on an elevator in a skyscraper. The elevator starts to move at a rate of −20 ft/s. After 6 seconds on the elevator, Monica is 350 feet from the ground floor of the building.

14. The rate of the elevator is negative. What does this mean in the situation? What value in the slope-intercept form of an equation does this rate represent?

15. a. How many feet was Monica above the ground when she got on the elevator? Show how you determined your answer.

b. What value in the slope-intercept form does your answer to Part (a) represent?

16. Model with mathematics. Write an equation in slope-intercept form for the motion of the elevator since it started to move. What do x and y represent?

a. What does the y-intercept represent?

b. Use the equation you wrote to determine, at this rate, how long it will take after Monica enters the elevator for her to exit the elevator on the ground floor. Explain how you found your answer.

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My Notes

Check Your Understanding

17. Write the equation $3x - 2y = 16$ in slope-intercept form. Explain your steps.

18. A flowering plant stands 6.5 inches tall when it is placed under a growing light. Its growth is 0.25 inches per day. Today the plant is 11.25 inches tall.

a. Write an equation in slope-intercept form for the height of the plant since it was placed under the growing light.

b. In your equation, what do x and y represent?

c. Use the equation to determine how many days ago the plant was placed under the light.

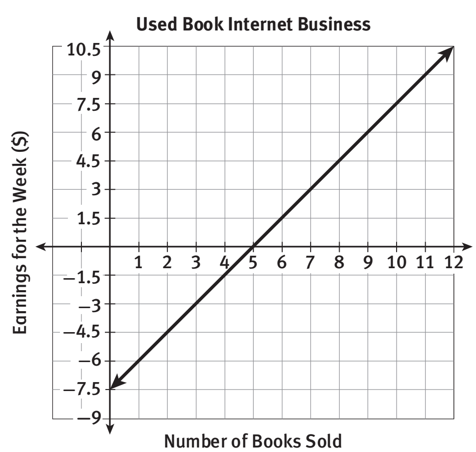
LESSON 12-1 PRACTICE

19. What are the slope, m, and y-intercept, $(0, b)$, of the line described by the equation $3x + 6y = 12$?

20. Write an equation in slope-intercept form for the line that has a slope of $\frac{2}{3}$ and y-intercept of $(0, -5)$.

21. Write an equation in slope-intercept form for the line that passes through the points $(6, -3)$ and $(0, 2)$.

22. Matt sells used books on the Internet. He has a weekly fee he has to pay for his website. He has graphed his possible weekly earnings, as shown.



a. What is the weekly fee that Matt pays for his website? How do you know?

b. How much does Matt make for each book sold? How do you know?

c. Write the equation in slope-intercept form for the line in Matt's graph.

d. How many books does Matt have to sell to make \$30 for the week? Explain.

23. **Make use of structure.** Without graphing, describe the graph of each equation below. Tell whether the line is ascending or descending from left to right and where the line crosses the y-axis.

a. $y = 3x$ **b.** $y = 5x + 2$ **c.** $y = -2x - 5$ **d.** $y = -6x + 4$

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My Notes

Learning Targets:

- Write the equation of a line in point-slope form.
- Use point-slope form to solve problems.

SUGGESTED LEARNING STRATEGIES: Create Representations, Marking the Text, Note Taking, Think-Pair-Share, Critique Reasoning, Sharing and Responding

Another form of the equation of a line is the point-slope form. The point-slope form of the equation is found by solving the slope formula $m = \frac{y - y_1}{x - x_1}$ for $y - y_1$, by multiplying both sides by $x - x_1$. You may use this form when you know a point on the line and the slope.

Point-Slope Form of a Linear Equation

$$y - y_1 = m(x - x_1)$$

where m is the slope of the line and (x_1, y_1) is a point on the line.

CONNECT TO AP

In calculus, the point-slope form of a line is used to write the equation of the line tangent to a curve at a given point.

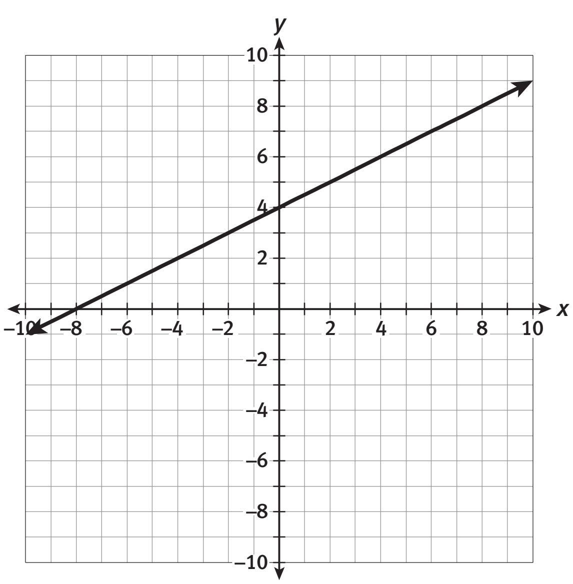
Example A

Write an equation of the line with a slope of $\frac{1}{2}$ that passes through the point (2, 5). Graph the line.

Step 1: Substitute the given values into point-slope form.

$$y - y_1 = m(x - x_1)$$
$$y - 5 = \frac{1}{2}(x - 2)$$

Step 2: Graph $y - 5 = \frac{1}{2}(x - 2)$. Plot the point (2, 5) and use the slope to find another point.



MATH TIP

If you needed to express the solution to Example A in slope-intercept form, you could apply the Distributive Property and combine like terms.

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My Notes

Try These A

Find an equation of the line given a point and the slope.

a. $(-2, 7), m = \frac{2}{3}$ **b.** $(6, -1), m = -\frac{5}{4}$

The town of San Simon charges its residents for trash pickup and water usage on the same bill. Each month the city charges a flat fee for trash pickup and a fee of $0.25 per gallon for water used. In January, one resident used 44 gallons of water, and received a bill for $16.

1. If x is the number of gallons of water used during a month, and y represents the bill amount in dollars, write a point (x_1, y_1).

2. What does $0.25 per gallon represent?

3. **Reason abstractly.** Use point-slope form to write an equation that represents the bill cost y in terms of the number of gallons of water x used in a month.

4. Write the equation in Item 3 in slope-intercept form. What does the y-intercept represent?

Check Your Understanding

5. Determine the equation of the line given the point (86, 125) and the slope $m = -18$.

6. Violet has an Internet business selling paint sets. After an initial website fee each week, she makes a profit of $0.75 on each set she sells. If she sells 8 sets, she makes $2.25. Write an equation representing her weekly possible earnings.

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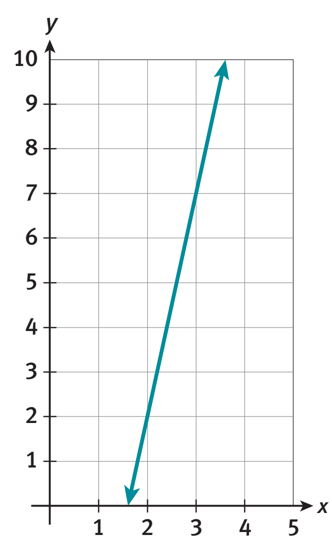
My Notes

7. Critique the reasoning of others. Jamilla and Ryan were asked to write the equation of the line through the points (6, 4) and (3, 5). Both Jamilla and Ryan determined that the slope was $-\frac{1}{3}$. Jamilla wrote the equation of the line as $y-4=-\frac{1}{3}(x-6)$. Ryan wrote the equation of the line as $y-5=-\frac{1}{3}(x-3)$.

a. Rewrite each student's equation in slope-intercept form and compare the results.

b. Whose equation was correct? Justify your response.

8. Find the equation in point-slope form of the line shown in the graph.



9. Write the equation of the line in slope-intercept form.

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Check Your Understanding

10. Explain the process you would use to write an equation of a line in point-slope form when given two points on the line.

11. Describe the similarities and differences between point-slope form and slope-intercept form.

My Notes

LESSON 12-2 PRACTICE

12. Write an equation of the line with a slope of 0.25 that passes through the point $(-1, -8)$.

13. Find the slope and a point on the line for the line whose equation is $y = 3 - \frac{2}{3}(x + 3)$.

14. Write the equation of the line through the points $(-3, 3)$ and $(7, 5)$ in slope-intercept form. What is the y-intercept?

15. Jay pays a flat fee each month for basic cable service. He also pays $3.50 for each movie he orders during the month. Last month, he ordered 5 movies and his total bill came to $54.

a. Write an equation in point-slope form that represents the total bill, y, in terms of the number of movies, x.

b. Write the equation in slope-intercept form.

c. What is the monthly fee for basic cable service? How do you know?

d. Next month, Jay plans to order 7 movies. What will be his total bill for the month?

e. This month, Jay's total bill is $78.50. How many movies did he order this month?

16. **Attend to precision.** The equation $y - 160 = 40(x - 1)$ represents the height in feet, y, of a hot-air balloon x minutes after the pilot started her stopwatch.

a. Is the hot-air balloon rising or descending? Justify your answer.

b. At what rate is the hot-air balloon rising or descending? Be sure to use appropriate units.

c. What was the height of the balloon when the pilot started her stopwatch?

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My Notes

Learning Targets:

- Write the equation of a line in standard form.
- Use the standard form of a linear equation to solve problems.

SUGGESTED LEARNING STRATEGIES: Create Representations, Note Taking, Discussion Groups, Think-Pair-Share, Identify a Subtask

A **linear equation** can be written in the form $Ax + By = C$ where A, B, and C are constants and A and B are not both zero.

> **Standard Form of a Linear Equation**
> $$Ax + By = C$$
> where $A \geq 0$, A and B are not both zero, and A, B, and C are integers whose **greatest common factor** is 1.

MATH TERMS

The **greatest common factor** of two or more integers is the greatest integer that is a divisor of all the integers.

1. **Reason abstractly.** You can use the coefficients of this form of an equation to find the x-intercept, y-intercept, and slope.
   a. Determine the x-intercept.

   b. Determine the y-intercept.

   c. Write $Ax + By = C$ in slope-intercept form to find the slope.

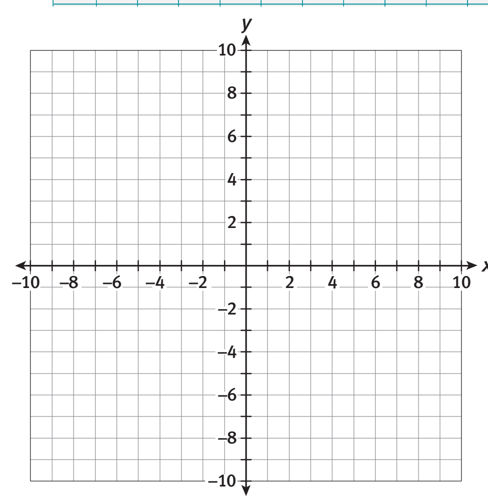
The definition of standard form states that both A and B are not 0. However, one of A or B may be equal to 0.

2. Write the standard form if $A = 0$.
   a. Suppose $A = 0$, $B = -1$, and $C = 3$. Write the equation of the line in standard form.

   b. Graph the line on the grid in the *My Notes* section. Describe the graph. What is the slope?

3. Write the standard form if $B = 0$.
   a. Suppose $A = 1$, $B = 0$, and $C = -6$. Write the equation of the line in standard form.

   b. Graph the equation on the grid in the *My Notes* section. Describe the graph. What is the slope?



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My Notes

4. Write $3x + 2y = 8$ in slope-intercept form.

5. Write the equation $y - 7 = 2(x + 1)$ in standard form.

Check Your Understanding

6. Write the equation $2x + 3y = 18$ in slope-intercept form.

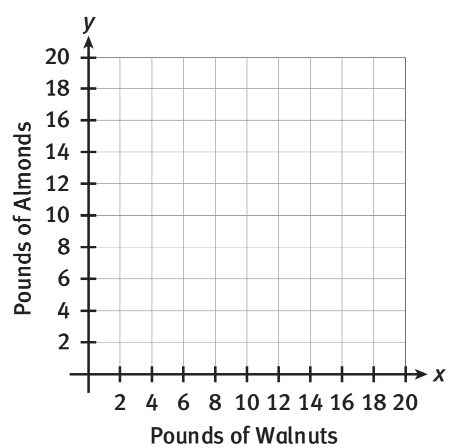
7. Write the equation $y = -\frac{6}{5}x - 4$ in standard form.

8. Describe the graph of any line whose equation, when written in standard form, has $A = 0$.

9. Susheila is making a large batch of granola to sell at a school fundraiser. She needs to buy walnuts and almonds to make the granola. Walnuts cost \$3 per pound and almonds cost \$2 per pound. She has \$30 to spend on these ingredients.

a. Write an equation that represents the different amounts of walnuts, x, and almonds, y, that Susheila can buy.

b. Graph the x- and y-intercepts on the coordinate plane below. Use these to help you graph the line.



c. If Susheila buys 4 pounds of walnuts, how many pounds of almonds can she buy?

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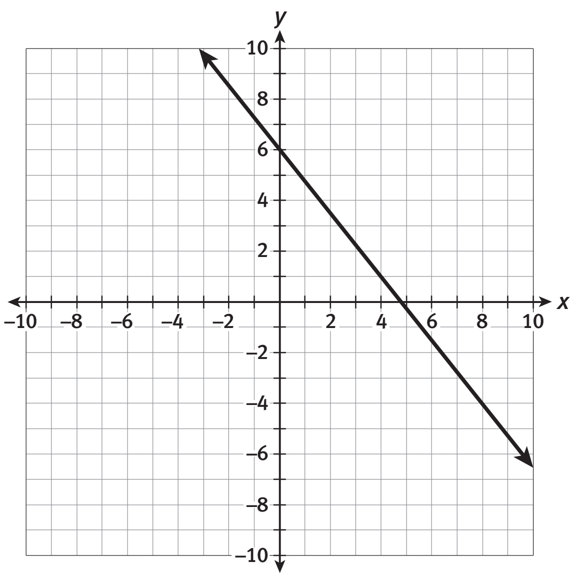
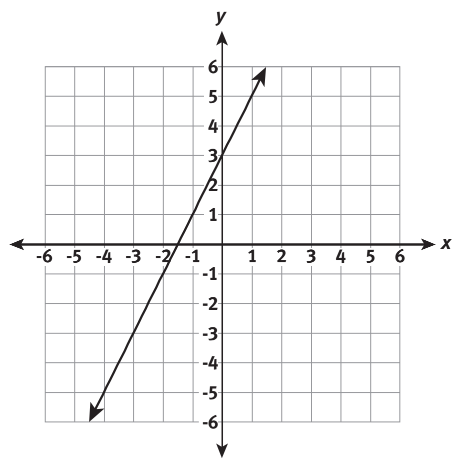
10. Refer to the graph you made in Item 9b. What is the x-intercept? What does it represent?

11. Write an equation in standard form for the line shown.



12. Make use of structure. The equation $2x - 5y = 20$, the table below, and the graph below represent three different linear functions.

x	y
-3	1
-2	4
-1	7
0	10
1	13
2	16
3	19



Which function represents the line with the greatest slope? Explain your reasoning.

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My Notes

Check Your Understanding

13. Write an equation in standard form for the line that is represented by the data in the table.

x	−2	−1	0	1	2	3
y	9	7	5	3	1	−1

14. Write an equation in standard form for the line with a slope of 7 that passes through the point (1, 2).

LESSON 12-3 PRACTICE

15. Determine the x-intercept, y-intercept, and slope of the line described by $-3x + 7y = -21$.

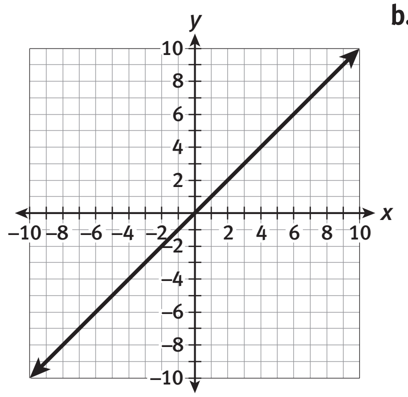
16. Write each equation in standard form.

a. $8x = 26 + 14y$

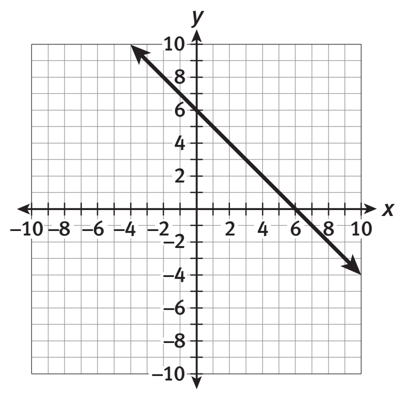
b. $y = -\frac{6}{7}x + 12$

17. Write an equation in standard form for each line below.

a.



b.



18. Pedro walks at a rate of 4 miles per hour and runs at a rate of 8 miles per hour. Each week, his exercise program requires him to cover a total distance of 20 miles with some combination of walking and/or running.

a. Write an equation that represents the different amounts of time Pedro can walk, x, and run, y, each week.

b. Graph the equation.

c. What is the y-intercept? What does this tell you?

19. **Make sense of problems.** Keisha bought a discount pass at a movie theater. It entitles her to a special discounted admission price for every movie she sees. Keisha wrote an equation that gives the total cost y of seeing x movies. In standard form, the equation is $7x - 2y = -31$.

a. What was the cost of the pass?

b. What is the discounted admission price for each movie?

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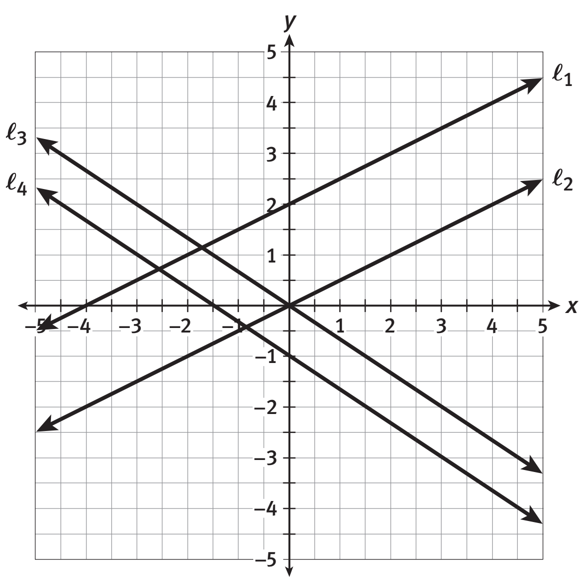
My Notes

Learning Targets:

- Describe the relationship among the slopes of parallel lines and perpendicular lines.
- Write an equation of a line that contains a given point and is parallel or perpendicular to a given line.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Predict and Confirm, Create Representations, Look for a Pattern, Discussion Groups

Parallel lines and perpendicular lines are pairs of lines that have special relationships.
Parallel lines in a plane are equidistant from each other at all points.



1. Consider lines l_1, l_2, l_3, and l_4 on the graph above. Determine the slope of each line.

2. Reason quantitatively. In the graph above, l_1 is parallel to l_2 and l_3 is parallel to l_4. Write a conjecture about the slopes of parallel lines.

3. Determine the slope of a line that is parallel to the line whose equation is $y = -3x + 4$.

4. Write the equation of a line that is parallel to the line $y = \frac{3}{4}x - 1$ and has a y-intercept of (0, 5).

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My Notes

5. Horizontal lines are described by equations of the form $y = \textit{number}$. For example, the equation of the x-axis is $y = 0$, because all points on the x-axis have y-coordinate 0. Explain why any two horizontal lines are parallel.

6. Vertical lines are described by equations of the form $x = \textit{number}$. For example, the equation of the y-axis is $x = 0$, because all points on the y-axis have x-coordinate 0. Do you think that any two vertical lines are parallel? Explain why or why not.

7. Use the information in Items 5 and 6 to write the equation of a line that is

a. parallel to the x-axis.

b. parallel to the y-axis.

8. A line is parallel to $y = 3x + 2$ and passes through the point $(1, 4)$.

a. What is the slope of the line? Explain how you know.

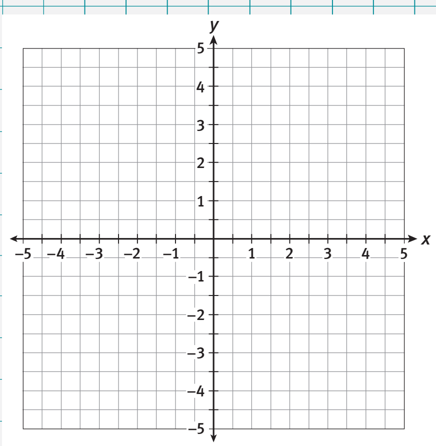
b. Write an equation of the line.

MATH TIP

Perpendicular lines intersect to form right angles.

9. Graph and label each line described below on the grid in the *My Notes* section. Which lines appear to be perpendicular?

- l_5 has slope $-\frac{4}{3}$ and contains the point $(0, 2)$.
- l_6 has slope $-\frac{3}{4}$ and contains the point $(0, 0)$.
- l_7 has slope $\frac{3}{4}$ and contains the point $(-2, -1)$.



10. Write a conjecture about the slopes of perpendicular lines.

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My Notes

11. Use your prediction from Item 10 to write the equations of two lines that are perpendicular. On the grid in the *My Notes* section on the previous page, graph both lines and confirm that they are perpendicular.

12. In the coordinate plane, what is true about a line that is perpendicular to a horizontal line?

13. Line l_1 contains the points $(0, -1)$ and $(3, 1)$. It is perpendicular to line l_2 that contains the point $(-1, 2)$.

a. What is the slope of each line? Explain how you know.

b. Write the equation of each line.

Check Your Understanding

14. Determine whether the lines with the given slopes are parallel, perpendicular, or neither.

a. $m_1 = -4,\ m_2 = \frac{1}{4}$ **b.** $m_1 = -3, m_2 = 3$

c. $m_1 = \frac{10}{12},\ m_2 = -1\frac{1}{5}$ **d.** $m_1 = \frac{1}{2}, m_2 = \frac{1}{2}$

15. The equation of line l_1 is $y = \frac{1}{3}x - 2$.

a. Write the equation of a line parallel to l_1. Explain.

b. Write the equation of a line perpendicular to l_1. Explain.

16. Write the equation of a line that is parallel to the line $3x + 4y = 4$ and contains the point $(8, 1)$.

17. Write an equation of a line that is perpendicular to the line $y = 5x + 1$ and contains the point $(-10, 2)$.

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My Notes

LESSON 12-4 PRACTICE

18. Determine whether the lines with the given slopes are parallel, perpendicular, or neither.

a. $m_1 = 5, m_2 = \frac{1}{5}$

b. $m_1 = -6, m_2 = \frac{1}{6}$

c. $m_1 = -\frac{2}{3}, m_2 = -\frac{2}{3}$

19. The slopes of three lines are given below.

$m_1 = -\frac{1}{2}$ $\quad m_2 = 3$ $\quad m_3 = 0$

a. Determine the slope of a line that is parallel to a line with each given slope.

b. Determine the slope of a line that is perpendicular to a line with each given slope.

20. Determine the slope of any line that is parallel to the line described by $y = -\frac{1}{2}x + 5$.

21. Write the equation of a line that is parallel to the line described by $x - 4y = 8$. Explain how you know the lines are parallel.

22. Determine the slope of any line that is perpendicular to the line described by $y = \frac{3}{4}x - 9$.

23. Write an equation of the line that is perpendicular to the line $2x + 5y = -15$ and contains the point $(-8, 3)$.

24. Determine the equation of a line perpendicular to the x-axis that passes through the point $(4, -1)$.

25. **Construct viable arguments.** A line a passes through points with coordinates $(-3, 5)$ and $(0, 0)$ and a line b passes through points with coordinates $(3, 5)$ and $(0, 0)$. Are lines a and b parallel, perpendicular, or neither? Explain your answer.

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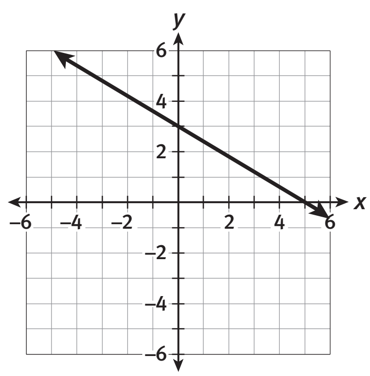
ACTIVITY 12 PRACTICE

Write your answers on notebook paper. Show your work.

Lesson 12-1

1. Write the equation of a line in slope-intercept form that has a slope of -8 and a y-intercept of $(0, 3)$.
2. Write the equation of a line in slope-intercept form that passes through point $(0, -7)$ and has a slope of $\frac{3}{4}$.
3. Find the slope and the y-intercept of the line whose equation is $-5x + 3y - 8 = 0$.
4. Which of the following is the slope-intercept form of the equation of the line in the graph?



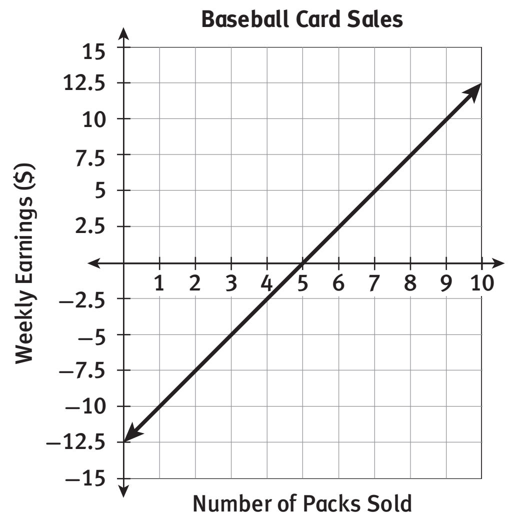
A. $y = -\frac{5}{3}x + 3$

B. $y = -\frac{3}{5}x + 5$

C. $y = -\frac{3}{5}x + 3$

D. $y = -\frac{5}{3}x + 5$

After paying an initial fee each week, Mike can sell packs of baseball cards in a sports shop. He displays his possible earnings for one week on the following graph. Use the graph for Items 5–9.



5. What is the initial fee Mike pays each week?
6. How many packs does Mike have to sell to break even?
7. What is the price of one pack of cards?
8. What is the equation in slope-intercept form for the line shown in graph?
9. How many packs of cards must Mike sell to make $40? Explain.

Lesson 12-2

10. What is the equation in point-slope form of the line that passes through $(-9, 12)$ with a slope of $\frac{5}{6}$?
11. What is the equation in slope-intercept form of the line that has a slope of 0.25 and passes through the point $(6, -8)$?
12. What is the equation in point-slope form of the line that passes through the points $(2, -3)$ and $(-5, 8)$?

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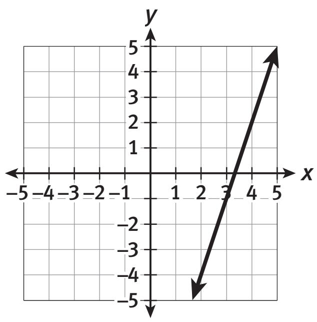
13. Write an equation in slope-intercept form of the line that passes through the points (4, 2) and (1, −7).

14. What is the equation in slope-intercept form of the line that passes through the points (2, 7) and (6, 7)? Describe the line.

15. What is the point-slope form of the line in the graph?



Lesson 12-3

16. Write the equation of the line in the graph from Item 15 in standard form.

17. David is ordering tea from an online store. Black tea costs \$0.80 per ounce and green tea costs \$1.20 per ounce. He plans to spend a total of \$12 on the two types of tea.
- **a.** Write an equation that represents the different amounts of black tea, x, and green tea, y, that David can buy.
- **b.** Graph the equation.
- **c.** What is the x-intercept? What does it represent?
- **d.** Suppose David decides to buy 10 ounces of black tea. How many ounces of green tea will he buy?

18. Is the equation $6x - 15y = -12$ in standard form? Why or why not?

19. Which is a true statement about the line $x - 4y = 8$?
- **A.** The x-intercept of the line is (2, 0).
- **B.** The y-intercept of the line is (0, 2).
- **C.** The slope of the line is $\frac{1}{4}$.
- **D.** The line passes through the origin.

20. Write the equation of a line in standard form that has an x-intercept of (3, 0) and a y-intercept of (0, 5).

Lesson 12-4

21. What is the slope of a line parallel to a line whose equation is $3x + 5y = 12$?

22. What is the slope of a line perpendicular to a line whose equation is $-4x - 2y + 18 = 0$?

23. Which is the slope of a line that is perpendicular to the line whose equation is $5x - 3y = -10$?
- **A.** $\frac{3}{5}$
- **B.** $-\frac{3}{5}$
- **C.** $\frac{5}{3}$
- **D.** $-\frac{5}{3}$

24. What is the equation of the line that is perpendicular to $2x + 4y = 1$ and that passes through the point (6, 8)?

25. What is the slope of any line that is perpendicular to the line that contains the points (8, 8) and (12, 12)?

MATHEMATICAL PRACTICES
Construct Viable Arguments and Critique the Reasoning of Others

26. Aidan stated that for any value of b, the line $y = 2x + b$ is parallel to the line that passes through (2, 5) and (−1, −1). Do you agree with Aidan? Explain why or why not.

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Equations From Data

ACTIVITY 13

Pass the Book

Lesson 13-1 Scatter Plots and Trend Lines

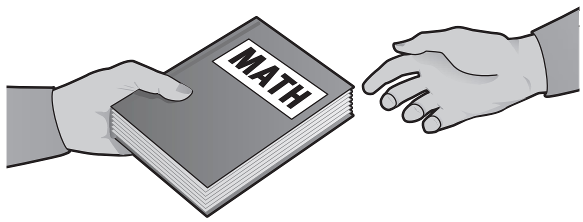
My Notes

Learning Targets:

- Use collected data to make a scatter plot.
- Determine the equation of a trend line.

SUGGESTED LEARNING STRATEGIES: Predict and Confirm, Sharing and Responding, Create Representations, Look for a Pattern, Interactive Word Wall

How fast can you and your classmates pass a textbook from one person to the next until the book has been relayed through each person in class?



1. Suppose your entire class lined up in a row. Estimate the length of time you think it would take to pass a book from the first student in the row to the last. Assume that the book starts on a table and the last person must place the book on another table at the end of the row.

   Estimated time to pass the book: ______________

2. As a class, experiment with the actual time it takes to pass the book using small groups of students in your class. Use the table below to record the times.

Number of students passing the book	3	6	9	11	13	15
Time to pass the book (nearest tenth of a second)						

3. **Reason quantitatively.** Based on the data you recorded in the table above, would you revise your estimated time from Item 1? Explain the reasoning behind your answer.

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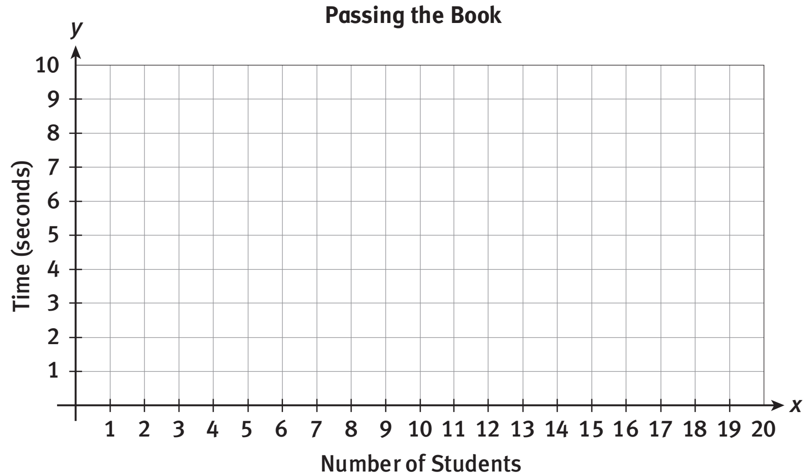
My Notes

MATH TERMS

A **scatter plot** displays the relationship between two sets of numerical data. It can reveal trends in data.

4. Graph the data in your table from Item 2 as a **scatter plot** on the coordinate grid.



5. Are the data that you collected linear data?

a. Explain your answer using the scatter plot.

b. Explain your answer using the table of data.

6. Describe how the time to pass the book changes as the number of students increases.

7. Work as a group to predict the number of seconds it will take to pass the book through the whole class.

a. Place a **trend line** on the scatter plot in Item 4 in a position that your group feels best models the data. Then, mark two points on the line.

MATH TERMS

A **trend line** is a line drawn on a scatter plot to show the **correlation**, or association, between two sets of data.

b. In the spaces provided below, enter the coordinates of the two points identified in Part (a).

Point 1: (________, ________) Point 2: (________, ________)

c. Why does your group think that this line gives the best position for modeling the scatter plot data?

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My Notes

8. Use the coordinate pairs you recorded in Item 7b to write the equation for your trend line (or linear model) of the scatter plot.

9. Explain what the variables in the equation of your linear model represent.

10. **Reason abstractly.** Interpret the meaning of the slope in your linear model.

11. Use your model to predict how long it would take to pass the book through all the students in your class.

Predicted time to pass the book: ________________

12. Using all of the students in your class, find the actual time it takes to pass the book.

Actual time to pass the book: ________________

13. How do your estimate from Item 1 and your predicted time from Item 11 compare to the actual time that it took to pass the book through the entire class?

14. **Attend to precision.** Suppose that another class took 1 minute and 47 seconds to pass the book through all of the students in the class. Use your linear model to estimate the number of students in the class.

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My Notes

Check Your Understanding

The table shows the number of days absent and the grades for several students in Ms. Reynoso's Algebra 1 class.

Days Absent	0	3	6	1	2	2	4
Grade (percent)	98	88	69	89	90	86	77

15. Create a scatter plot of the data using Days Absent as the independent variable.

16. Are the data linear? Explain using the scatter plot and the table of data.

17. Based on the data, how do grades change as the number of days absent increases?

18. Draw a trend line on your scatter plot. Identify two points on the trend line and write an equation for the line containing those two points.

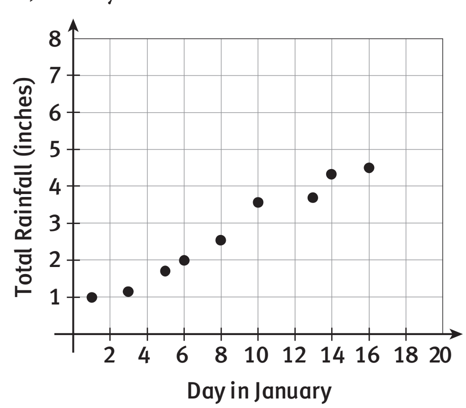
19. What is the meaning of the x and y variables in the equation you wrote?

20. Interpret the meaning of the slope and the y-intercept of your trend line.

21. Use your equation to predict the grade of a student who is absent for 5 days.

LESSON 13-1 PRACTICE

Model with mathematics. The scatter plot shows the day of the month and total rainfall for January.



22. Copy the scatter plot and draw a trend line on the scatter plot. Identify two points on the trend line and write a linear equation to model the data containing those two points.

23. Explain the meaning of x and y in your equation.

24. Interpret the meaning of the slope and the y-intercept of your trend line.

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My Notes

Learning Targets:

- Use a linear model to make predictions.
- Use technology to perform a linear regression.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Interactive Word Wall, Look for a Pattern, Think-Pair-Share, Quickwrite

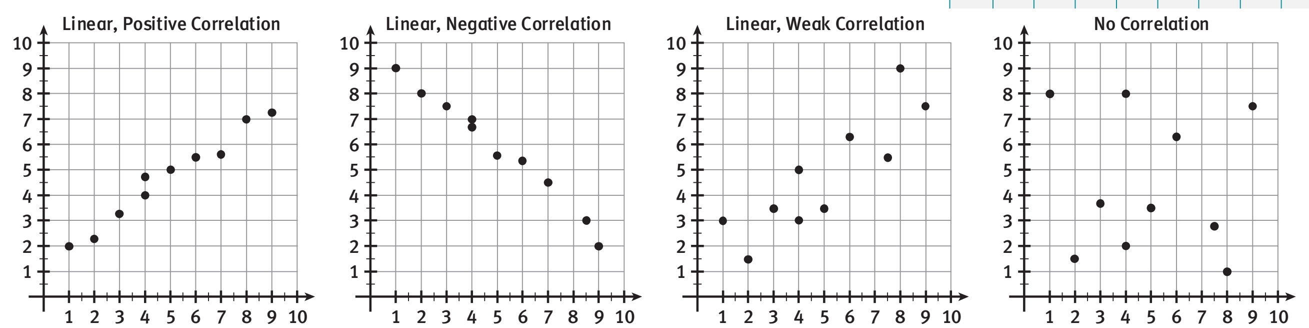
There is a *correlation* between two variables if they share some kind of relationship.

1. Is there a correlation between the variables of your linear model in Item 4 in Lesson 13-1? Explain.

MATH TERMS

A scatter plot will show a **positive correlation** if y tends to increase as x increases. Other data may have a **negative correlation**, where y tends to decrease as x increases, or **no correlation**. A correlation is sometimes called an association.

Examples of data with two variables that illustrate a **positive correlation**, a **negative correlation**, and **no correlation** are shown below. The more closely the data resemble a line, the stronger the linear correlation.



2. Look back at your linear model in Item 4 in Lesson 13-1. Does your linear model represent a positive correlation, a negative correlation, or no correlation? Explain.

There is ***causation*** between two variables if a change in one variable causes the other variable to change. For example, doing more exercise causes a greater number of calories to be burned.

3. Does there seem to be causation between the variables of your linear model in Item 4 in Lesson 13-1? Explain.

ACADEMIC VOCABULARY

The idea of ***causation*** is important in physics. For example, a cause can be represented by a force acting on an object.

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My Notes

Correlation does not imply causation. Just because there is a correlation between two variables does not mean that there is causation between them; there may be other factors affecting the situation.

Check Your Understanding

4. Consider the following two variables: your shoe size each year since you were born and the average price of a movie ticket each year since you were born.
   a. Is there a correlation between the variables? Explain.
   b. Is there causation between the variables? Explain.

5. Give an example of two variables for which there is both correlation and causation.

A scatter plot and a **line of best fit**, the most accurate trend line, can be created using a graphing calculator, a spreadsheet program, or other Computer Algebra Systems (CAS).

Linear regression is a method used to find the line of best fit. A line found using linear regression is more accurate than a trend line that has been visually estimated. You can perform linear regression using a graphing calculator.

6. Use appropriate tools strategically. Enter the book-passing data you collected in Item 2 in Lesson 13-1 into your graphing calculator. Enter the numbers of students as x-values and the corresponding times to pass the book as y-values.

   a. To find the equation of the line of best fit, use the linear regression feature of your calculator.

   The calculator should return values for a and b. Write these values below.

   $a =$

   $b =$

   b. The value of a is the slope of the line of best fit, and $(0, b)$ is the y-intercept. Round a and b to the nearest hundredth and write the equation of the line of best fit in the form $y = ax + b$. Describe how this equation is different from or similar to your equation in Item 8 in Lesson 13-1.

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My Notes

Check Your Understanding

7. Enter the following data into your graphing calculator. Make sure that any data have been cleared.

(6, 1), (9, 0), (12, −3), (3, 3), (0, 5), (−3, 7), (−5, 9), (−7, 13)

a. Find the equation of the line of best fit. Round values to the nearest hundredth.

b. Use the equation of the line of best fit to predict the value of y when $x = 20$.

8. Compare using a graphing calculator to using paper and pencil when plotting data and finding a trend line.

LESSON 13-2 PRACTICE

The owner of a café kept records on the daily high temperature and the number of hot apple ciders sold on that day. Some of the owner's data are shown below.

Daily High Temperature (°F)	32	75	80	48	15
Number of Hot Apple Ciders Sold	51	22	12	40	70

9. Create a scatter plot of the data.

10. Is there a correlation between the variables? If so, what type?

11. Determine the equation of the line of best fit. Round values to the nearest hundredth.

12. What is the slope? What does the slope represent?

13. Identify the y-intercept. What does the y-intercept represent?

14. **Model with mathematics.** Use your model to predict the number of hot apple ciders the café would sell on a day when the high temperature is 92°F. Explain.

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My Notes

Learning Targets:

- Use technology to perform quadratic and exponential regressions, and then make predictions.
- Compare and contrast linear, quadratic, and exponential regressions.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Create Representations, Quickwrite, Think-Pair-Share, Discussion Groups

Online shopping has experienced tremendous growth since the year 2000. One way to measure the growth is to track the average number of daily hits at the websites of online stores. The tables show the average number of daily hits for three different online stores in various years since the year 2000.

Nile River Retail					
Years Since 2000	0	2	4	6	8
Daily Hits (thousands)	52.1	56.2	60.0	64.1	68.0

eBuy					
Years Since 2000	0	2	4	6	8
Daily Hits (thousands)	1.0	4.9	17.2	37.2	64.9

Spendco					
Years Since 2000	0	2	4	6	8
Daily Hits (thousands)	2.0	6.6	20.9	68.1	220.4

1. **Make sense of problems.** Compare and contrast the growth of the three online stores based on the data in the tables.

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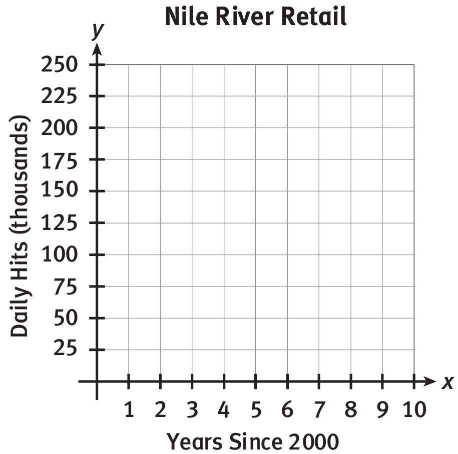
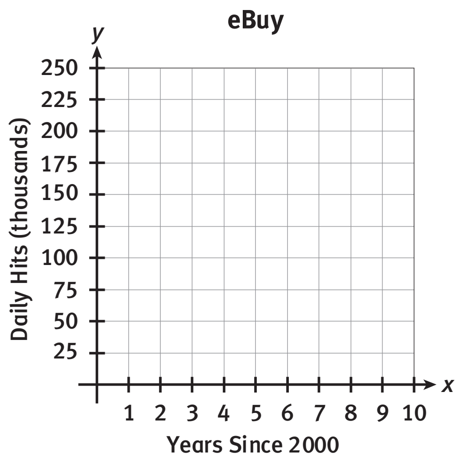
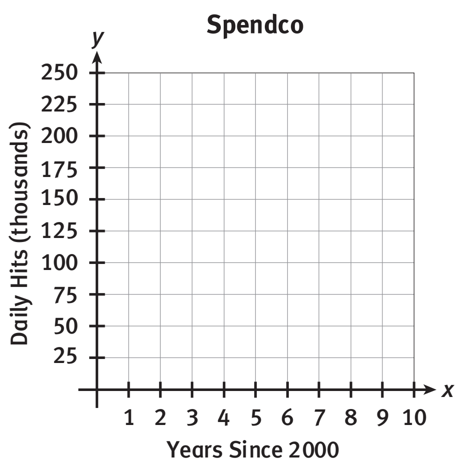
My Notes

2. Plot the data for the three online stores on the graphs below.







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My Notes

3. Which online store's growth could best be modeled by a linear function? Explain.

4. For the online store you identified in Item 3, determine the equation of the line of best fit. Round values to the nearest hundredth.

5. Predict the number of daily hits for this online store in 2015.

When a line does not appear to be a good fit for a set of data, you may want to model the data using a nonlinear model.

Quadratic regression is a method used to find a **quadratic function** that models a set of data. You can perform quadratic regression using a graphing calculator.

MATH TERMS

A **quadratic function** is a nonlinear function that can be written in the form $y = ax^2 + bx + c$, where $a \neq 0$. You will study quadratic functions in more detail later in this book.

6. Enter the data for eBuy into your graphing calculator. Enter the years since 2000 as the x-values and the corresponding daily hits in thousands as the y-values.
   a. To find the quadratic equation that models the data, use the quadratic regression feature of your calculator.

      The calculator should return values for a, b, and c. Write these values below, rounding to the nearest hundredth.

      $a =$

      $b =$

      $c =$

   b. Write the quadratic equation in the form $y = ax^2 + bx + c$.

   c. Use the quadratic equation to predict the number of daily hits for eBuy in 2015.

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Lesson 13-3
Quadratic and Exponential Regressions

My Notes

When a set of data shows very rapid growth or decay, an exponential model may be the best choice for modeling the data.

Exponential regression is a method used to find an **exponential function** that models a set of data. You can perform exponential regression using a graphing calculator.

7. Enter the data for Spendco into your graphing calculator. Enter the years since 2000 as the x-values and the corresponding daily hits in thousands as the y-values.

a. To find the exponential equation that models the data, use the exponential regression feature of your calculator.

The calculator should return values for a and b. Write these values below, rounding to the nearest hundredth.

$a =$

$b =$

b. Write the exponential equation in the form $y = ab^x$.

c. Use the exponential equation to predict the number of daily hits for Spendco in 2015.

8. Construct viable arguments. Based on your predictions for the number of daily hits for each online store in 2015, which type of function has the fastest growth: linear, quadratic, or exponential? Explain.

MATH TERMS

An **exponential function** is a nonlinear function that can be written in the form $y = ab^x$. You will study exponential functions in more detail later in this book.

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My Notes

Check Your Understanding

9. How are quadratic regression and exponential regression similar to and different from linear regression?

10. Do you think the exponential model would be appropriate for predicting the number of daily hits for Spendco in any future year? Explain your reasoning.

LESSON 13-3 PRACTICE

The population of Williston, North Dakota, has grown rapidly over the past decade due to an oil boom. The table gives the population of the town in 2007, 2009, and 2011.

Years Since 2000	7	9	11
Population (thousands)	12.4	13.0	16.0

11. Use your calculator to find the equation of the line of best fit for the data.

12. **Reason quantitatively.** What is the slope of the line? What does it tell you about the population growth of the town?

13. Use your calculator to find a quadratic equation that models the growth of the town.

14. Use your quadratic equation to predict the population of Williston in 2020.

15. Use your calculator to find an exponential equation that models the growth of the town.

16. Use your exponential equation to predict the population of Williston in 2020.

17. According to the exponential model, in what year will the town have a population greater than 40,000 for the first time? (*Hint*: Use the table feature of your calculator.) What assumptions do you make when you use the exponential model to answer this question?

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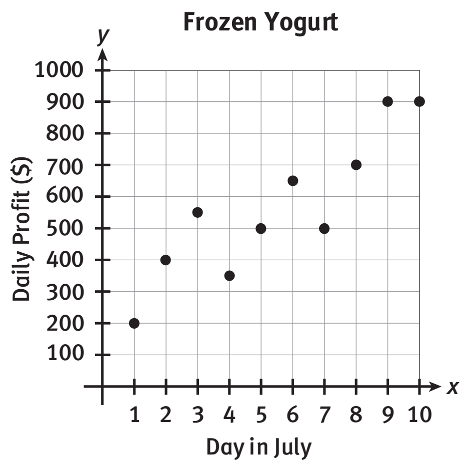
ACTIVITY 13 PRACTICE

Write your answers on notebook paper. Show your work.

Lesson 13-1

The scatter plot shows the relationship between the day of the month and a frozen yogurt stand's daily profit during the month of the July.



1. Are the data linear? Explain.
2. Draw a trend line on the scatter plot and name two points that your trend line passes through.
3. Write the equation of the trend line you drew in Item 2.
4. What do the variables in your equation represent?
5. What is the slope of the trend line? What does this tell you?
6. Use your trend line to predict the yogurt stand's daily profit on July 20.
7. The owner of a competing frozen yogurt stand finds that her daily profit each day in July is exactly $100 more than that of the stand in the scatter plot. Write the equation of a trend line for the competing stand.
8. The manager of a local history museum experiments with different prices for admission to the museum. For each price, the manager notes the number of visitors who enter the museum on that day. The table shows the data.

Price	$2.75	$3.50	$4.25	$5.75
Number of Daily Visitors	112	88	66	63

Which is a true statement about the data?

A. A trend line on the scatter plot has a positive slope.
B. The y-intercept of the trend line is above the x-axis.
C. The trend line predicts at least 70 visitors when the admission price is $6.25.
D. The trend line fits the data perfectly because the data is linear.

Lesson 13-2

Use your calculator to perform a linear regression for the following data. Use your linear regression for Items 9–12.

$(-6, -3), (-8, -4), (-2, 1), (1, 4), (3, 6), (5, 8), (7, 13)$

9. What is the equation of the line of best fit?
10. What is the value of the slope? What does this tell you about the relationship between x and y?
11. According to your model, what is the value of y when $x = -19$?
12. According to your model, for what value of x is $y = 100$?

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13. Look at the scatter plot on the previous page showing the daily profits of a frozen yogurt stand. What type of correlation, if any, does the scatter plot show?

14. Which of the following pairs of variables are likely to show a negative correlation?

A. the length of a shoe; the size of the shoe
B. the number of miles on a car's odometer; the age of the car
C. the weight of a watermelon; the price of the watermelon
D. the number of minutes you have waited for a bus; the number of minutes remaining until the bus arrives

15. At several times during the school year, Emilio collected data on the height of a plant in the classroom and the total number of quizzes he had taken so far in his science class. The data are shown below.

Height of Plant (cm)	16	19	22	26
Total Number of Quizzes	4	6	7	9

a. Is there a correlation between the variables? Explain.
b. Is there causation between the variables? Explain.

Lesson 13-3

The table shows the number of employees at a software company in various years.

Years Since 2000	4	6	8	10
Number of Employees	32	40	75	124

16. Make a scatter plot of the data.

17. Do you think a linear equation would be a good model for the data? Justify your answer.

18. Use your calculator to find a quadratic equation that models the growth of the company.

19. Use the quadratic model to predict the number of employees in the year 2015.

20. Use your calculator to find an exponential equation that models the growth of the company.

21. Use the exponential model to predict the number of employees in the year 2015.

22. How do the predictions given by the two models in Items 19 and 21 compare?

23. Use your calculator to compare the quadratic and exponential models. Enter the equation from Item 19 as Y_1 and the equation from Item 21 as Y_2. View the graphs in a window that allows you to compare their growth. What do you notice?

The table shows the total number of bacteria in a sample over five hours.

Hour	Number of Bacteria
1	12
2	144
3	1728
4	20,736
5	248,832

24. Use your calculator to find an exponential equation that models the bacteria data.

25. If this trend continues, how many bacteria will be growing in the sample after 9 hours?

MATHEMATICAL PRACTICES
Look for and Make Use of Structure

26. Is it possible to tell from the equation of a line of best fit whether there is a positive or negative correlation between two variables? If so, explain how. If not, explain why not.

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