Functions

2

Unit Overview

In this unit, you will build linear models and use them to study functions, domain, and range. Linear models are the foundation for studying slope as a rate of change, intercepts, and direct variation. You will learn to write linear equations given varied information and express these equations in different forms.

Key Terms

As you study this unit, add these and other terms to your math notebook. Include in your notes your prior knowledge of each word, as well as your experiences in using the word in different mathematical examples. If needed, ask for help in pronouncing new words and add information on pronunciation to your math notebook. It is important that you learn new terms and use them correctly in your class discussions and in your problem solutions.

Academic Vocabulary

- causation

Math Terms

- mapping
- relation
- vertical line test
- independent variable
- dependent variable
- continuous
- discrete
- *y*-intercept
- relative maximum
- relative minimum
- *x*-intercept
- parent function
- translation
- direct variation
- constant of variation
- indirect variation
- constant of variation
- indirect variation
- inverse function
- one-to-one
- arithmetic sequence
- explicit formula
- recursive formula
- slope-intercept form
- point-slope form
- standard form
- scatter plot
- trend line
- correlation
- line of best fit
- linear regression
- quadratic regression
- exponential regression

ESSENTIAL QUESTIONS

? How can you show mathematical relationships?

? Why are linear functions useful in real-world settings?

EMBEDDED ASSESSMENTS

This unit has three embedded assessments, following Activities 8, 11, and 13. They will give you an opportunity to demonstrate what you have learned.

UNIT 2

Getting Ready

Write your answers on notebook paper.
Show your work.

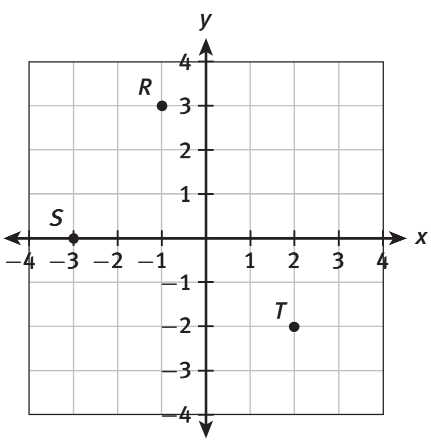
1. Copy and complete the table of values.

–1	–1
2	5
5	11
8	
11	23
	29

2. List the integers that make this statement true.
$-3 \le x < 4$

3. Evaluate for $a = 3$ and $b = -2$.
a. $2a - 5$ **b.** $3b + 4a$

4. Name the point for each ordered pair.
a. $(-3, 0)$ **b.** $(-1, 3)$ **c.** $(2, -2)$



5. Explain how you would plot $(3, -4)$ on a coordinate plane.

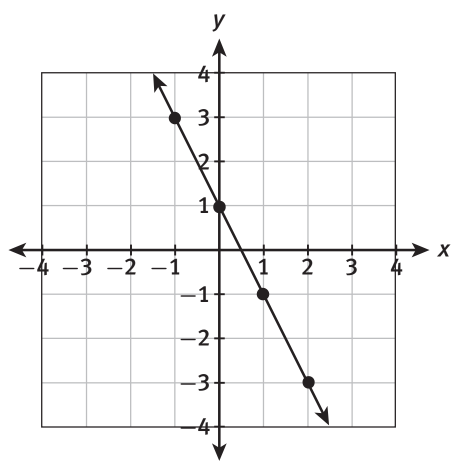
6. Which of the following equations represents the data in the table?

x	1	3	5	7
y	2	8	14	20

A. $y = 2x - 1$ **B.** $y = 3x - 1$
C. $y = x + 1$ **D.** $y = 2x + 1$

7. If $2x + 6 = 2$, what is the value of x?
A. 4 **B.** 2 **C.** 0 **D.** -2

8. Which of the following are the coordinates of a point on this line?



A. $(-1, 3)$ **B.** $(1, -3)$
C. $(-1, -3)$ **D.** $(1, 3)$

Functions and Function Notation

Vending Machines

Lesson 5-1 Relations and Functions

My Notes

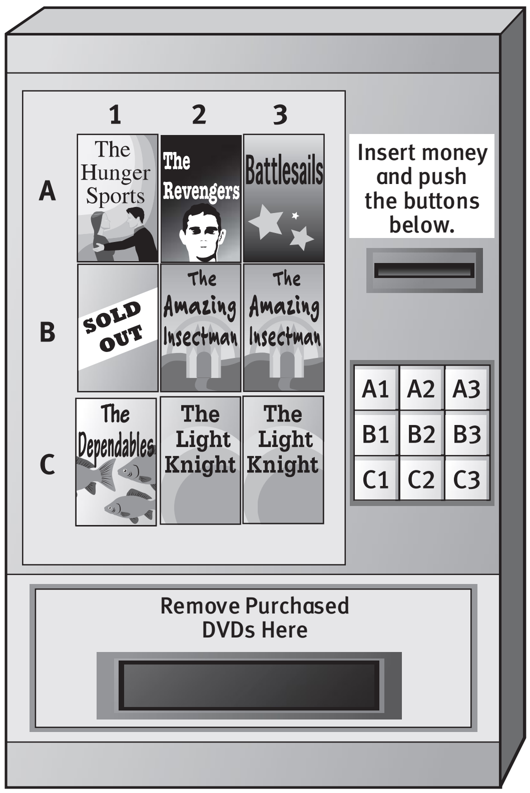
Learning Targets:

- Represent relations and functions using tables, diagrams, and graphs.
- Identify relations that are functions.

SUGGESTED LEARNING STRATEGIES: Visualization, Create Representations, Think-Pair-Share, Interactive Word Wall, Paraphrasing

Use this machine to answer the questions below.

DVD Vending Machine



1. What DVD would you receive if you inserted your money and pressed:

a. A1?

b. C2?

c. B3?

2. Assuming the machine were filled properly, describe what would happen if you pressed the same button twice.

My Notes

Each time you press a button, an **input**, you may receive a DVD, an **output**.

3. In the DVD vending machine situation, does every input have an output? Explain your response.

4. Each combination of input and output can be expressed as a **mapping** written *input* → *output*. For example, B2 → The Amazing Insectman.

a. Write as mappings each of the possible combinations of buttons pushed and DVDs received in the vending machine.

b. Create a table to illustrate how the inputs and outputs of the vending machine are related.

Mappings that relate values from one set of numbers to another set of numbers can be written as **ordered pairs**. A **relation** is a set of ordered pairs.

MATH TERMS

A **mapping** is a visual-representation of a relation in which an arrow associates each input with its output.

MATH TERMS

An **ordered pair** shows the relationship between two elements, written in a specific order using parentheses notation and a comma separating the two values.

MATH TERMS

relation

Lesson 5-1
Relations and Functions

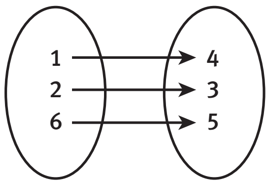
My Notes

Relations can have a variety of representations. Consider the relation {(1, 4), (2, 3), (6, 5)}, shown here as a set of ordered pairs. This relation can also be represented in these ways.

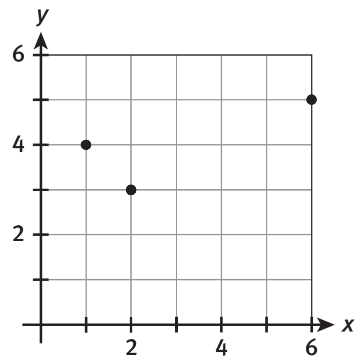
Table

x	y
1	4
2	3
6	5

Mapping



Graph

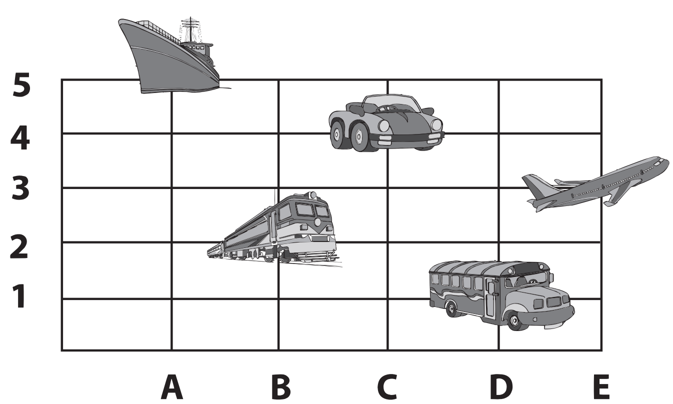


5. Write the following numerical mappings as ordered pairs.

Input		Output	Ordered Pairs
1	→	−2	(1, −2)
2	→	1	
3	→	4	
4	→	7	

Check Your Understanding

6. A vending machine at the Ocean, Road, and Air show creates souvenir coins. You select a letter and a number and the machine creates a souvenir coin with a particular vehicle imprinted on it. The graph shows the vending machine letter/number combinations for the different coins.



a. Make a table showing each coin's letter/number combination.
b. Write the letter/number combinations as a set of ordered pairs.
c. Write the letter/number combinations in a mapping diagram.

My Notes

MATH TERMS

function

A **function** is a relation in which each input is paired with exactly one output.

7. Compare and contrast the DVD Vending Machine with a function.

8. Suppose when pressing button C1 on the vending machine both "The Dependables" and "The Light Knight" come out. Describe how this vending machine resembles or does not resemble a function.

9. Imagine a machine where you input an age and the machine gives you the name of anyone who is that age. Compare and contrast this machine with a function. Explain by using examples and create a representation of the situation.

My Notes

10. Create an example of a situation (math or real-life) that behaves like a function and another that does not behave like a function. Explain why you chose each example to fit the category.

a. Behaves like a function:

b. Does not behave like a function:

11. Determine whether each list of ordered pairs represents a function. Explain your answers.

a. {(5, 4), (6, 3), (7, 2)}

b. {(4, 5), (4, 3), (5, 2)}

c. {(5, 4), (6, 4), (7, 4)}

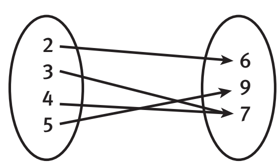
12. Attend to precision. Using positive integers, write two relations as lists of ordered pairs below, one that is a function and one that is not a function.

Function:

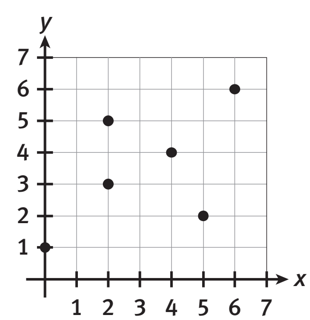
Not a function:

Check Your Understanding

13. Does the mapping shown represent a function? Explain.



14. Does the graph shown represent a function? Explain.



My Notes

LESSON 5-1 PRACTICE

For the Bingo card below, suppose that a combination of a column letter and a row number, such as B1, represents an input and the number at that location, such as 7, represents an output. Use this information for Items 15–17.

B	I	N	G	O
7	26	35	51	73
14	23	44	55	63
6	19	FREE	48	64
12	22	32	54	70
11	16	33	47	69

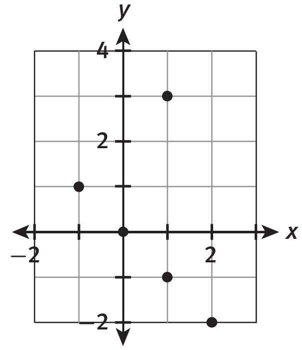
15. What output corresponds to I2?

16. What input corresponds to 54?

17. Does every input have a numerical output? Explain.

18. **Construct viable arguments.** Explain why each of the following is **not** a function.

a.



b.

x	*y*
12	−8
17	3
−4	9
17	−5

Learning Targets:

- Describe the domain and range of a function.
- Find input-output pairs for a function.

SUGGESTED LEARNING STRATEGIES: Quickwrite, Create Representations, Discussion Groups, Marking the Text, Sharing and Responding

The set of all inputs for a function is known as the **domain** of the function. The set of all outputs for a function is known as the **range** of the function.

1. Consider a vending machine where inserting 25 cents dispenses one pencil, inserting 50 cents dispenses 2 pencils, and so forth up to and including all 10 pencils in the vending machine.

a. Identify the domain in this situation.

b. Identify the range in this situation.

2. For each function below, identify the domain and range.

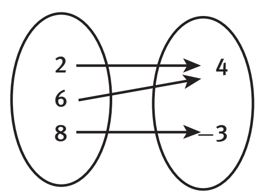
a.

input	output
7	6
3	–2
5	1

Domain:

Range:

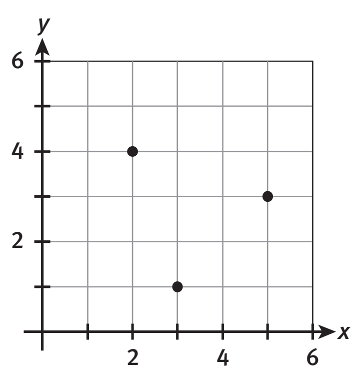
b.



Domain:

Range:

c.



Domain:

Range:

d. {(–7, 0), (9, –3), (–6, 2.5)}

Domain:

Range:

My Notes

MATH TERMS

domain
range

WRITING MATH

The **domain** and **range** of a function can be written using set notation.

For example, for the function {(1, 2), (3, 4), (5, 6)}, the domain is {1, 3, 5} and the range is {2, 4, 6}.

My Notes

3. Consider a machine that exchanges quarters for dollar bills. Inserting one dollar bill returns four quarters and you may insert up to five one-dollar bills at a time.

a. Is 7 a possible input for the relation this change machine represents? Justify your response.

b. Could 3.5 be included in the domain of this relation? Explain why or why not.

c. **Reason abstractly.** What values are **not** in the domain? Justify your reasoning.

d. Is 8 a possible output for the relation this change machine represents? Justify your response.

e. Could 3 be included in the range of this relation? Explain why or why not.

f. What values are **not** in the range? Justify your reasoning.

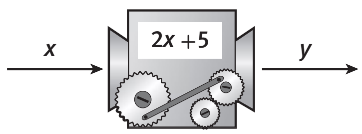
My Notes

MATH TERMS

A **finite** set has a fixed countable number of elements. An **infinite** set has an unlimited number of elements.

4. **Make sense of problems.** Each of the functions that you have seen has a **finite** number of ordered pairs. There are functions that have an **infinite** number of ordered pairs. Describe any difficulties that may exist trying to represent a function with an infinite number of ordered pairs using the four representations of functions that have been described thus far.

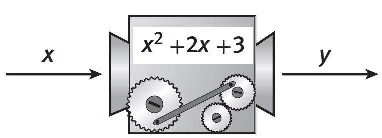
5. Sometimes, machine diagrams are used to represent functions. In the function machine below, the inputs are labeled x and the outputs are labeled y. The function is represented by the expression $2x + 5$.



   a. What is the output if the input is $x = 7$? $x = -2$? $x = \frac{1}{2}$?

   b. **Express regularity in repeated reasoning.** Is there any limit to the number of input values that can be used with this expression? Explain.

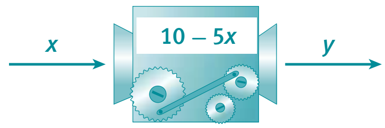
Consider the function machine below.



6. Use the diagram to find the (input, output) ordered pairs for the following values.
   a. $x = -5$ b. $x = \frac{3}{5}$ c. $x = -10$

My Notes

7. Make a function machine for the expression $10 - 5x$. Use it to find ordered pairs for $x = 3$, $x = -6$, $x = 0.25$, and $x = \frac{3}{4}$.



Creating a function machine can be time consuming and awkward. The function represented by the diagram in Item 5 can also be written algebraically as the equation $y = 2x + 5$.

8. For each function, find ordered pairs for $x = -2$, $x = 5$, $x = \frac{2}{3}$, and $x = 0.75$. Create tables of values.

a. $y = 9 - 4x$

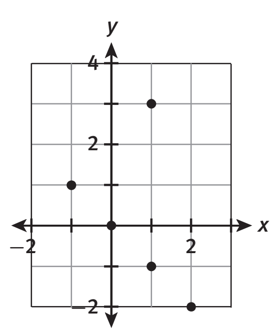
b. $y = \frac{1}{x}$

Check Your Understanding

9. The set $\{(3, 5), (-1, 2), (2, 2), (0, -1)\}$ represents a function. Identify the domain and range of the function.

10. Identify the domain and range for each function.

a.



b.

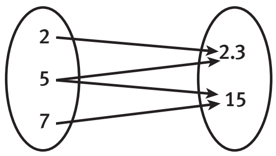
x	y
12	−8
17	3
−4	9

My Notes

LESSON 5-2 PRACTICE

Identify the domain and range.

11.



12.

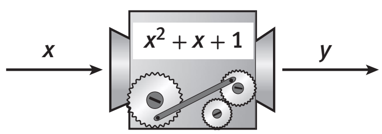
x	y
1.5	4
−0.3	8
$\frac{1}{6}$	3

13. **Model with mathematics.** At an arcade, there is a machine that accepts game tokens and returns tickets that can be redeemed for prizes. Inserting 5 tokens returns 3 tickets and inserting 10 tokens returns 8 tickets. You must insert tokens in multiples of 5 or 10, and you have a total of 20 tokens.

a. Identify the domain in this situation.

b. Identify the range in this situation.

14. For the function machine shown, copy and complete the table of values.



x	y
−1	
0	
$\frac{1}{2}$	
1.2	

15. For each function below, find ordered pairs for $x = -1$, $x = 3$, $x = \frac{1}{2}$, and $x = 0.4$. Write your results as a set of ordered pairs.

a. $y = 4x$ **b.** $y = 2 - x^2$

My Notes

MATH TIP

It is important to recognize that $f(x)$ **does not** mean f multiplied by x.

MATH TIP

Notice that $f(x) = y$. For a **domain** value x, the associated **range** value is $f(x)$.

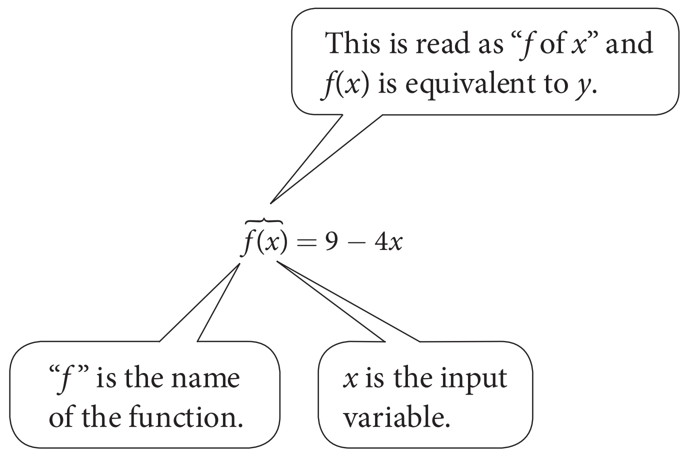
Learning Targets:

- Use and interpret function notation.
- Evaluate a function for specific values of the domain.

SUGGESTED LEARNING STRATEGIES: Create Representations, Discussion Groups

When referring to the functions in Item 8 in Lesson 5-2, it can be confusing to distinguish among them since each begins with "$y =$." Function notation can be used to help distinguish among different functions.

For instance, the function $y = 9 - 4x$ in Item 8a can be written:



1. To distinguish among different functions, it is possible to use different names. Use the name h to write the function from Item 8b using function notation.

Function notation is useful for evaluating functions for multiple input values. To evaluate $f(x) = 9 - 4x$ for $x = 2$, you substitute 2 for the variable x and write $f(2) = 9 - 4(2)$. Simplifying the expression yields $f(2) = 1$.

2. Use function notation to evaluate $f(x) = 9 - 4x$ for $x = 5$, $x = -3$, and $x = 0.5$.

My Notes

3. Use the values for x and $f(x)$ from Item 2. Display the values using each representation.

a. list of ordered pairs

b. table of values

c. mapping

d. graph

4. Given the function $f(x) = 9 - 4x$ as shown above, what value of x results in $f(x) = 1$?

5. Evaluate each function for $x = -5$ and $x = \frac{4}{3}$.

a. $f(x) = 2x - 7$

b. $g(x) = 6x - x^2$

c. $h(x) = \frac{2}{x^2}$

6. **Reason quantitatively.** Recall the money-changing machine from Item 3 in Lesson 5-2, in which customers can insert up to five one-dollar bills at a time and receive an equivalent amount of quarters. The function $f(x) = 4x$ represents this situation. What does x represent? What does $f(x)$ represent?

My Notes

A function whose domain is the set of positive consecutive integers forms a sequence. The terms of the sequence are the range values of the function. For the sequence 4, 7, 10, 13, … $f(1) = 4$, $f(2) = 7$, $f(3) = 10$, and $f(4) = 13$.

7. Consider the sequence −4, −2, 0, 2, 4, 6, 8, ….

a. What is $f(3)$?

b. What is $f(7)$?

Check Your Understanding

8. Evaluate the functions for the domain values indicated.

a. $p(x) = 3x + 14$ for $x = -5, 0, 4$

b. $h(t) = t^2 - 5t$ for $t = -2, 0, 5, 7$

9. Consider the sequence −7, −3, 1, 5, 9, ….

a. What is $f(2)$?

b. What is $f(5)$?

LESSON 5-3 PRACTICE

Use the function $y = x^2 - 3x - 4$ for Items 10–12.

10. Write the function in function notation.

11. Evaluate the function for $x = -2$. Express your answer in function notation.

12. **Make use of structure.** For what value of x does $f(x) = -4$?

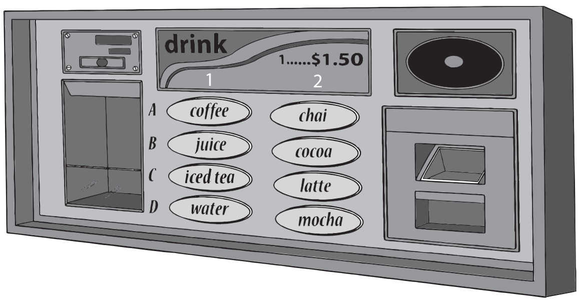
13. Consider the sequence $\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3, \ldots$. What is $f(4)$?

ACTIVITY 5 PRACTICE

Write your answers on notebook paper. Show your work.

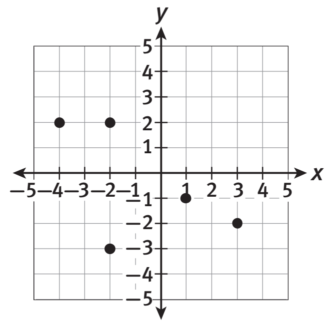
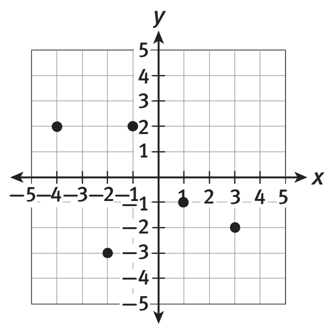
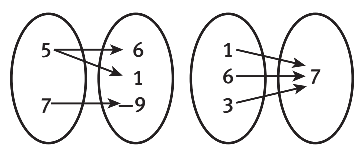
Lesson 5-1

Use the Beverage Vending Machine to answer Items 1–6.



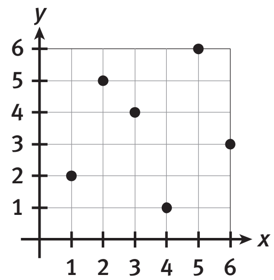
1. List all of the possible inputs.
2. List all of the possible outputs.
3. Which output results from an input of 2C?
   A. Juice
   B. Iced tea
   C. Latte
   D. Cocoa
4. Which number/letter combination would you input if you wanted the machine to output juice?
   A. 2A
   B. 1B
   C. 2B
   D. 1D
5. In a mapping of the relation shown by the vending machine, what drink would 1D map to?
6. In a table of the relation shown by the vending machine, what number/letter combination would correspond to cocoa?

For Items 7–9, two relations are given. One relation is a function and one is not. Identify each and explain.

7. $\{(5, -2), (-2, 5), (2, -5), (-5, 2)\}$
   $\{(5, -2), (-2, 5), (5, 2), (-5, 2)\}$
8. 
   
9. 
10. What value(s) of x in the relation below would create a set of ordered pairs that is not a function? Justify your answer.

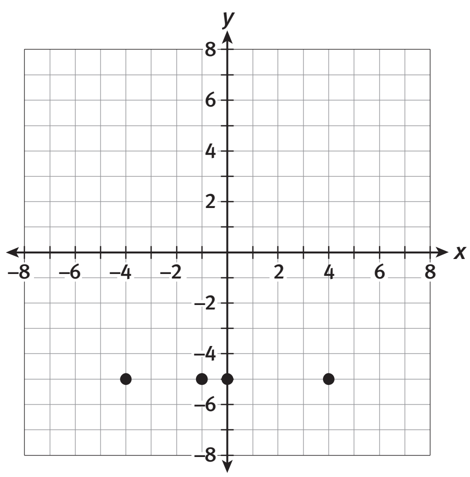
$$\{(0, 5)\ (1, 5)\ (2, 6)\ (x, 7)\}$$

11. Does the graph shown represent a function? Explain.



Lesson 5-2

Use the graph for Items 12–14.



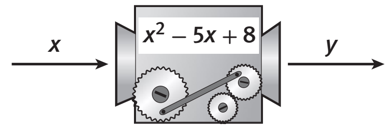
12. Identify the domain of the relation represented in the graph.

13. Identify the range of the relation represented in the graph.

14. Does the relation shown in the graph represent a function? Explain.

Lesson 5-3

Use the function machine for Items 15–17.



15. How would you write the function shown in the function machine in function notation?

16. What is the value of $f(-2)$?

17. What value(s) of x results in $f(x) = 8$?

18. Given the function $f(x) = -2x - 5$, determine the value of $f(-3)$.

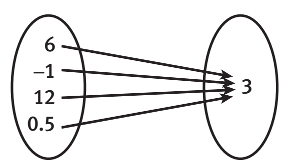
The first seven numbers in the Fibonacci sequence are: 0, 1, 1, 2, 3, 5, 8. Use this information for Items 19 and 20.

19. What is $f(2)$?

20. What is $f(6)$?

MATHEMATICAL PRACTICES
Construct Viable Arguments and Critique the Reasoning of Others

21. Dora said that the mapping diagram below does not represent a function because each value in the domain is paired with the same value in the range. Explain the error in Dora's reasoning.



Interpreting Graphs of Functions
Shake, Rattle, and Roll
Lesson 6-1 Key Features of Graphs

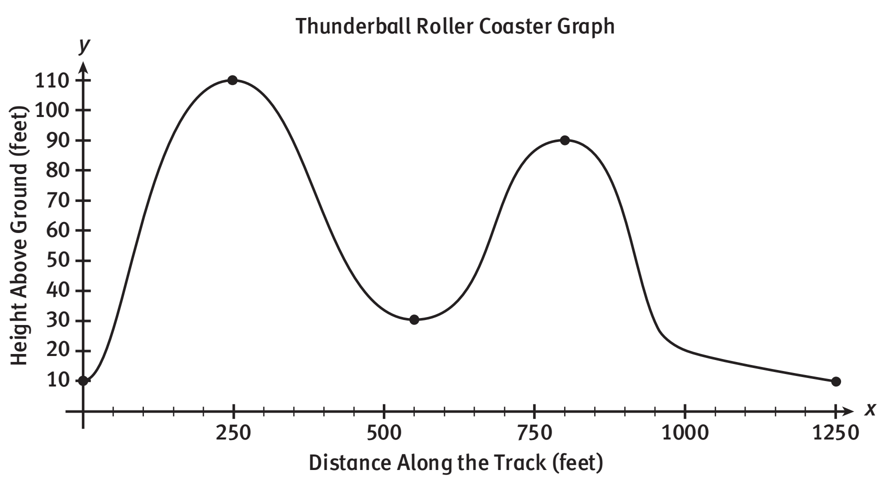
ACTIVITY 6

Learning Targets:

- Relate the domain and range of a function to its graph.
- Identify and interpret key features of graphs.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Visualization, Interactive Word Wall, Discussion Groups

Roller coasters can be scary but fun to ride. Below is the graph of the heights reached by the cars of the Thunderball Roller Coaster over its first 1250 feet of track. The graph displays a function because each input value has one and only one output value. You can see this visually using the **vertical line test**. Study this graph to determine the domain and range.



The domain gives all values of the **independent variable:** in this case, the distance along the track in feet. The domain values are graphed along the horizontal or x-axis. The domain of the function above can be written in set notation as:

$$\{\text{all real values of } x\text{: } 0 \le x \le 1250\}$$

Read this notation as: *the set of all real values of x, between 0 and 1250, inclusive*.

The range gives the values of the **dependent variable**: in this case, the height of the roller coaster above the ground in feet. The range values are graphed on the vertical or y-axis. The range of the function above can be written in set notation as:

$$\{\text{all real values of } y\text{: } 10 \le y \le 110\}$$

Read this notation as: *the set of all real values of y, between 10 and 110, inclusive.*

My Notes

MATH TERMS

The **vertical line test** is a visual check to see if a graph represents a function. For a function, every vertical line drawn in the coordinate plane will intersect the graph in at most one point. This is equivalent to having each domain element associated with one and only one range element.

MATH TERMS

An **independent variable** is the variable for which input values are substituted in a function.
A **dependent variable** is the variable whose value is determined by the input or value of the independent variable.

My Notes

CONNECT TO AP

The **absolute maximum** of a function $f(x)$ is the greatest value of $f(x)$ for all values in the domain. The **absolute minimum** of a function $f(x)$ is the least value of $f(x)$ for all values in the domain. Unlike relative maximums and relative minimums, absolute maximums and absolute minimums may correspond to the endpoints of graphs.

MATH TIP

An open interval is an interval whose endpoints are not included. For example, $0 < x < 5$ is an open interval, but $0 \leq x \leq 5$ is not.

The graph above shows data that are **continuous**. The points in the graph are connected, indicating that domain and range are sets of real numbers with no breaks in between. A graph of **discrete** data consists of individual points that are not connected by a line or curve.

Many other useful pieces of information about a function can be determined by looking at its graph.

- The ***y*-intercept** of a function is the point at which the graph of the function intersects the y-axis. The y-intercept is the point at which $x = 0$.
- A **relative maximum** of a function $f(x)$ is the greatest value of $f(x)$ for values in a limited open domain interval.
- A **relative minimum** of a function $f(x)$ is the least value of $f(x)$ for values in a limited open domain interval.

Because they must occur within open intervals of the domain, relative maximums and relative minimums cannot correspond to the endpoints of graphs.

Use the Thunderball Roller Coaster Graph on the previous page for Items 1–5.

1. **Reason abstractly.** What is the y-intercept of the function shown in the graph, and what does it represent?

2. Identify a relative maximum of the function represented by the graph.

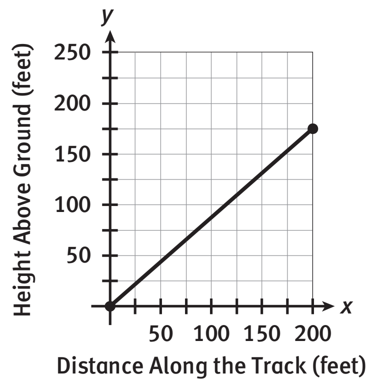
3. Identify the absolute maximum of the function represented by the graph. Interpret its meaning in the context of the situation.

4. Identify a relative minimum of the function represented by the graph.

5. Identify the absolute minimum of the function represented by the graph. Interpret its meaning in the context of the situation.

My Notes

Suppose you got on a roller coaster called Cougar Mountain that immediately started climbing the track in a linear fashion, as shown in the graph.



6. Identify the domain and range of the function.

7. Identify the y-intercept of the function.

8. Identify the absolute maximum and minimum of the function.

9. Does the function have any relative maximum or minimum values? Explain.

10. How are the **extrema** different on this linear graph versus the nonlinear graph for the Thunderball Roller Coaster?

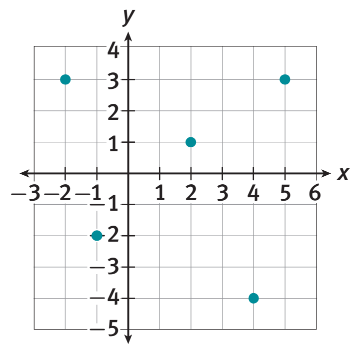
MATH TERMS

Extrema refers to all maximum and minimum values.

My Notes

Check Your Understanding

11. The graph below shows five points that make up the function h. Is the function h continuous? Explain.



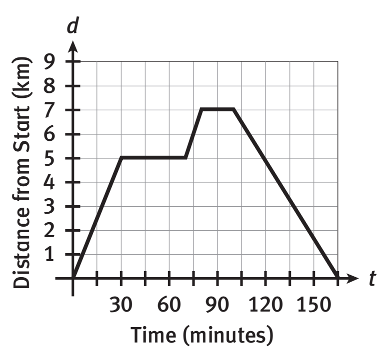
12. A function has three relative maximums: −2, 10.3, and 28. One of the relative maximums is also the absolute maximum. What is the absolute maximum?

Tell whether each statement is sometimes, always, or never true. Explain your answers.

13. A relative minimum is also an absolute minimum.

14. An absolute minimum is also a relative minimum.

Tom hiked along a circular trail known as the Juniper Loop. The graph shows his distance d from the starting point after t minutes.



15. Identify the domain and range of the function shown in the graph.

16. Identify the absolute minimum of the function. What does it represent?

My Notes

17. In this function, the absolute minimum corresponds to two points on the graph. What are the two points? What do they represent in this context?

18. Identify the absolute maximum of the function. What does it represent?

19. What points on the graph correspond to the absolute maximum? What does this mean in the context of Tom's hike?

20. Identify any relative minimums for the function shown in the graph.

21. Identify any relative maximums for the function shown in the graph.

Check Your Understanding

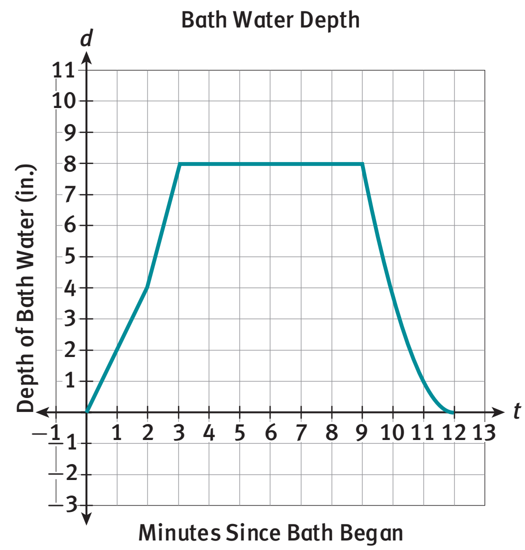
22. What are the independent and dependent variables for the function representing Tom's hike?

23. Explain how to determine the maximum and minimum values of a function by examining its graph.

24. Is it possible for a function to have more than one absolute maximum or absolute minimum value? Explain.

My Notes

Model with mathematics. Use the graph below for Items 25–30.



25. What are the independent and dependent variables? Explain.

26. Use set notation to write the domain and range of the function.

27. Is the function discrete or continuous? Explain.

28. What is the y-intercept? Interpret the meaning of the y-intercept in this context.

29. Identify any relative maximums or minimums of the function.

30. Identify the absolute maximum and absolute minimum values. Interpret their meanings in this context.

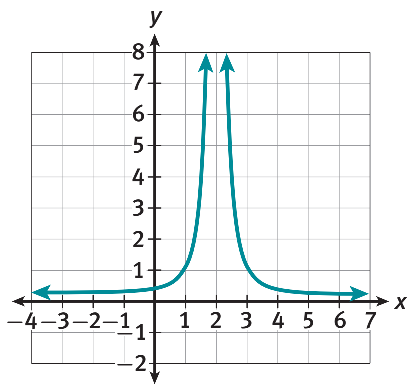
My Notes

Learning Targets:

- Relate the domain and range of a function to its graph and to its function rule.
- Identify and interpret key features of graphs.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Levels of Questions, Think Aloud, Create Representations, Summarizing

Examine the graph of the function $f(x) = \frac{1}{(x-2)^2}$, graphed below.



1. Describe how this graph is different from the graphs in Lesson 6-1.

Example A

Give the domain and range of the function $f(x) = \frac{1}{(x-2)^2}$.

Then find the y-intercept, the absolute maximum, and the absolute minimum.

To find the domain and range:

Step 1: Study the graph.
The sketch of this graph is a portion of the function represented by the equation $f(x) = \frac{1}{(x-2)^2}$.

Step 2: Look for values for which the domain causes the function to be undefined. Look how the graph behaves near $x = 2$.

Solution: The domain and range of $f(x) = \frac{1}{(x-2)^2}$ can be written:

Domain: $\{\textit{all real values of } x : x \neq 2\}$

Range: $\{\textit{all real values of } y : y > 0\}$

MATH TIP

Notice the result when $x = 2$ is substituted into $f(x)$.

$$f(2) = \frac{1}{(2-2)^2} = \frac{1}{0}$$

Division by zero is undefined in mathematics.

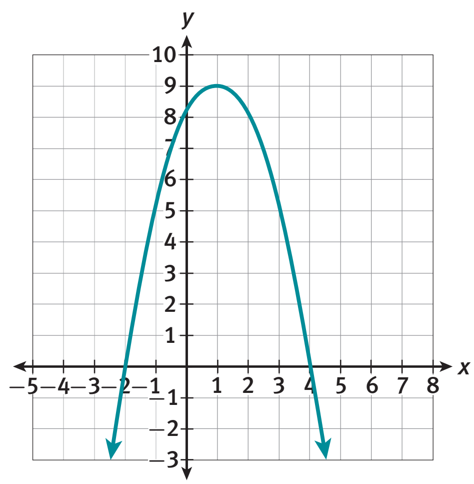
My Notes

To determine the y-intercept and identify any maximums or minimums:

Study the graph. We can see that the function intersects the y-axis at (0, 0.25). The value of $f(x)$ keeps getting larger as x approaches 2 from both sides. The value of $f(x)$ approaches, but never reaches, 0 as x gets further from 2 on both sides.

Solution: The y-intercept is (0, 0.25). The function does not have an absolute maximum or minimum.

The function $f(x) = 8 + 2x - x^2$ is graphed below.



a. Identify the domain and range of the function.
Domain:

Range:

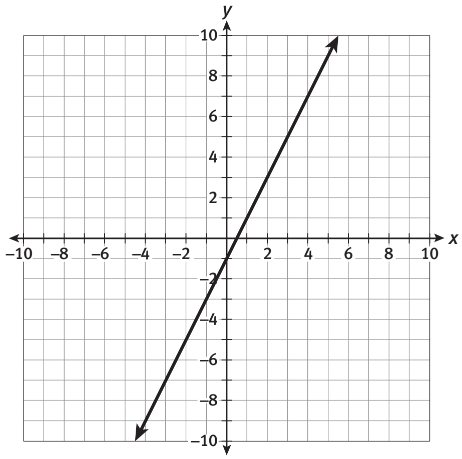
b. Identify is the y-intercept.

c. Identify any relative or absolute minimums of the function.

d. Identify any relative or absolute maximums of the function.

My Notes

2. The equation $y = 2x - 1$ is graphed below.



a. Identify the domain and range.
Domain:
Range:

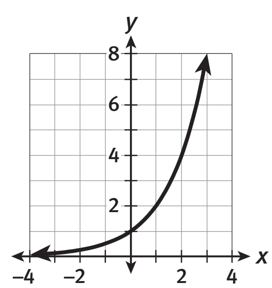
b. What is the y-intercept of $y = 2x - 1$?

c. Identify any relative or absolute minimums of $y = 2x - 1$.

d. Identify any relative or absolute maximums of $y = 2x - 1$.

e. **Construct viable arguments.** Explain whether this equation represents a function and how you determined this.

3. The function $y = 2^x$ is graphed below.



a. Identify the domain and range.
Domain:
Range:

b. What is the y-intercept of the function $y = 2^x$?

My Notes

c. Identify any relative or absolute minimums of $y = 2^x$.

d. Identify any relative or absolute maximums of $y = 2^x$.

MATH TIP

The domain is restricted to avoid situations where division by zero or taking the square root of a negative number would occur.

4. If you have access to a graphing calculator, work with a partner to graph the equations listed in the table below. Each equation is a function.

a. Using the graphs you create, determine the domain and range for each function from the possibilities listed below the chart.

b. Select the appropriate domain from choices 1–6 and record your answer in the Domain column. Then select the appropriate range from choices a–f and record the appropriate range in the Range column.

c. When the chart is complete, compare your answers with those from another group.

Function	Domain	Range
$y = -3x + 4$		
$y = x^2 - 6x + 5$		
$y = 9x - x^2$		
$y = \|x + 1\|$		
$y = 3 + \sqrt{x}$		
$y = \frac{4}{x}$		

Possible Domains

1) all real numbers
2) all real x, such that $x \neq -2$
3) all real x, such that $x \neq 0$
4) all real x, such that $x \neq 2$
5) all real x, such that $x \geq 0$
6) all real x, such that $x \leq 0$

Possible Ranges

a) all real numbers
b) all real y, such that $y \neq 0$
c) all real y, such that $y \geq -4$
d) all real y, such that $y \geq 0$
e) all real y, such that $y \leq 20.25$
f) all real y, such that $y \geq 3$

My Notes

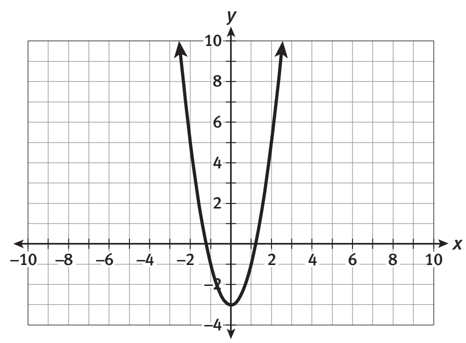
Check Your Understanding

5. How can you determine from a function's graph whether the function has any maximum or minimum values?

6. How can you determine the domain of a function by examining its graph? By examining its function rule?

7. Give an example of a function that has a restricted domain. Justify your answer.

The function $f(x) = 2x^2 - 3$ is graphed below.

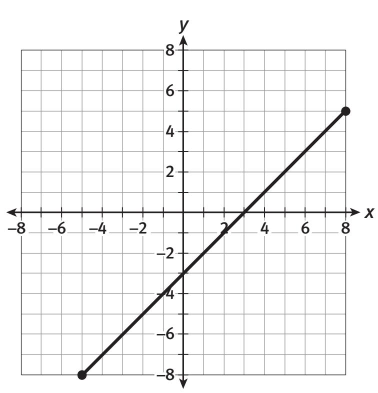


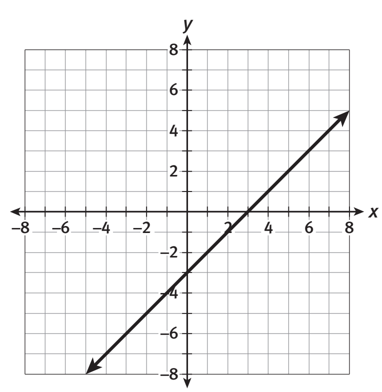
8. Give the domain, range, and y-intercept.

9. Identify any relative or absolute minimums.

10. Identify any relative or absolute maximums.

11. Attend to precision. Examine the graphs below. Explain why one function has an absolute minimum and an absolute maximum and the other function does not. Identify the absolute minimum and maximum values of the function for which they exist.





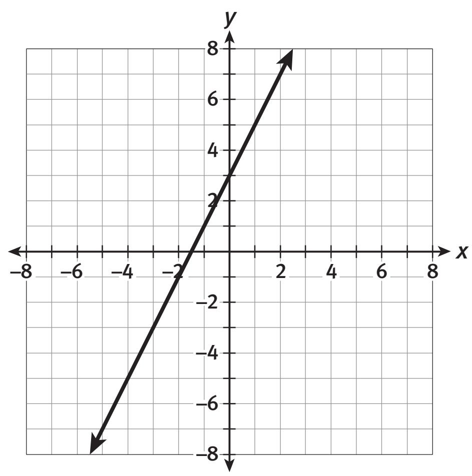
My Notes

Learning Targets:

- Identify and interpret key features of graphs.
- Determine the reasonable domain and range for a real-world situation.

SUGGESTED LEARNING STRATEGIES: Visualization, Discussion Groups, Look for a Pattern

The function $f(x) = 3 + 2x$ is graphed below.



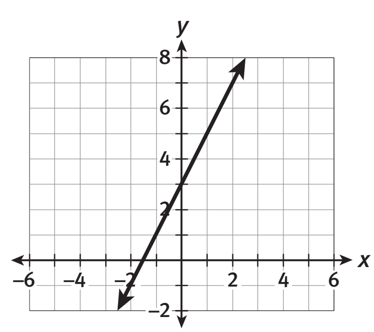
1. What are the domain and range of the function?
Domain:
Range:

In many real-world situations, not all values make sense for the domain and/or range. For example, distance cannot be negative; number of people cannot be a decimal or a fraction. In such situations, the values that make sense for the domain and range are called the *reasonable* domain and range.

Example A

A taxi ride costs an initial rate of \$3.00, which is charged as soon as you get in the cab, plus \$2 for each mile traveled. The cost of traveling x miles is given by the function $f(x) = 3 + 2x$. What are the reasonable domain and range?

Step 1: Sketch a graph of the function.



MATH TIP

Graph a function by substituting several values for x and generating ordered pairs. You can organize the ordered pairs in a table.

x	$f(x) = 3 + 2x$	(x, y)
0	3	(0, 3)
1	5	(1, 5)
2	7	(2, 7)

My Notes

Step 2: Determine the reasonable domain. Think about what the variable x represents. What values make sense?

The variable x represents the number of miles, so it does not make sense for x to be negative.

The reasonable domain is $\{x: x \geq 0\}$.

Step 3: Use the reasonable domain and the graph to determine the reasonable range.

From the graph, all y-values corresponding to the reasonable domain values are greater than or equal to 3. The reasonable range is $\{y: y \geq 3\}$.

Solution: The reasonable domain is $\{x: x \geq 0\}$. The reasonable range is $\{y: y \geq 3\}$.

Try These A

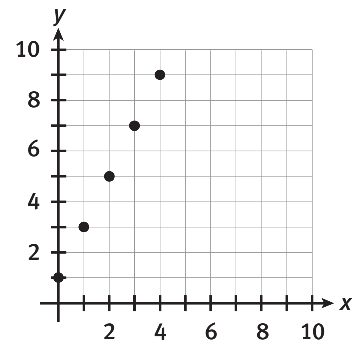
a. A banquet hall charges \$15 per person plus a \$100 setup fee. The cost for x people is given by the function $f(x) = 100 + 15x$. What are the reasonable domain and range?

b. Eight Ball Billiards charges \$5 to rent a table plus \$10 per hour of game play, rounded to the nearest whole hour. The cost of playing billiards for x hours is given by the function $f(x) = 5 + 10x$. What are the reasonable domain and range?

2. Reason quantitatively. Are the domain and range of $f(x) = 3 + 2x$ that you found in Item 1 the same as the reasonable domain and range of $f(x) = 3 + 2x$ found in Example A? Explain.

My Notes

3. The graph below represents a real-world situation.



a. Identify the domain and range.

b. Describe a real-world situation that matches the graph. Your answers to Part (a) should be the reasonable domain and range for your situation.

c. Identify the independent and dependent variables in your real-world situation.

Check Your Understanding

4. For a function that models a real-world situation, the dependent variable y represents a person's height. What is a reasonable range? Explain.

5. A tour company charges \$25 to hire a tour director plus \$75 per tour member. The total cost for a group of x people is given by $f(x) = 25 + 75x$. What is the reasonable domain? Explain.

Talk the Talk Cellular charges a base rate of \$20 per month for unlimited texts plus \$0.15/minute of talk time. The monthly cost for x minutes is given by $f(x) = 20 + 0.15x$.

6. **Make sense of problems.** What is the independent variable and what is the dependent variable? Explain how you know.

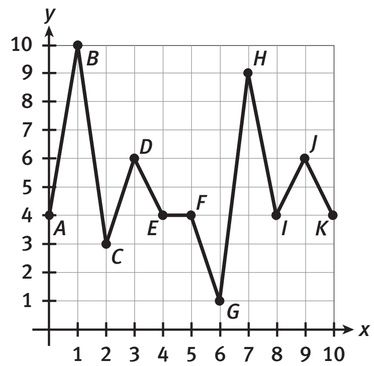
7. What are the reasonable domain and range? Explain.

ACTIVITY 6 PRACTICE

Write your answers on notebook paper.
Show your work.

Lesson 6-1

Use the graph below for Items 1–5.



1. Which point corresponds to the absolute maximum of the function?
   A. B
   B. D
   C. G
   D. H

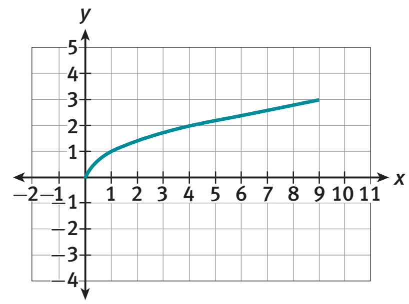
2. Which represents the range of the function shown in the graph?
   A. $\{0 \le x \le 10\}$
   B. $\{1 \le x \le 10\}$
   C. $\{0 \le y \le 10\}$
   D. $\{1 \le y \le 10\}$

3. Which point does **not** correspond to a relative minimum?
   A. B
   B. C
   C. E
   D. I

4. Is the function represented by the graph discrete or continuous? Explain.

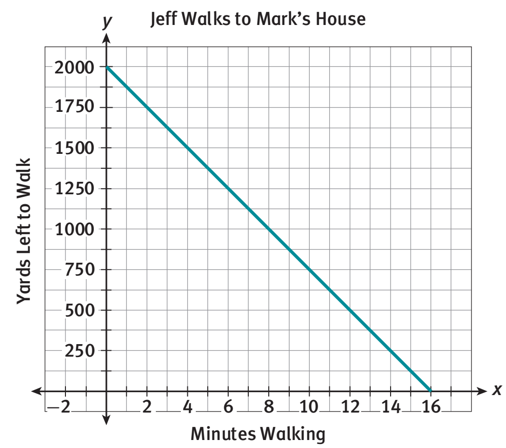
5. What is the y-intercept of the function shown in the graph?

6. **a.** Give the domain and range for the function graphed below. Explain why this graph represents a function.



   b. What is the y-intercept of the function shown in the graph?
   c. Identify any extrema of the function shown in the graph.

Jeff walks at an average rate of 125 yards per minute. Mark's house is located 2000 yards from Jeff's house. The graph below shows how far Jeff still needs to walk to reach Mark's house. Use the graph for Items 7–10.



7. Identify the independent and dependent variables.

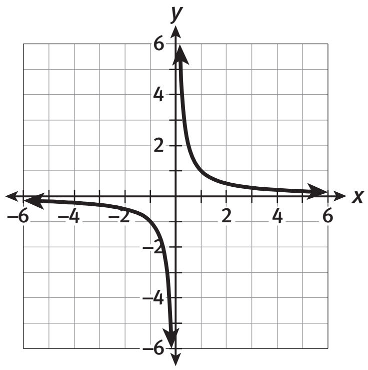
8. Identify the absolute minimum and absolute maximum values. What do these values represent?

9. Identify any relative maximums or minimums.

10. What is the y-intercept? What does it represent?

Lesson 6-2

Use the graph for Items 11–13.

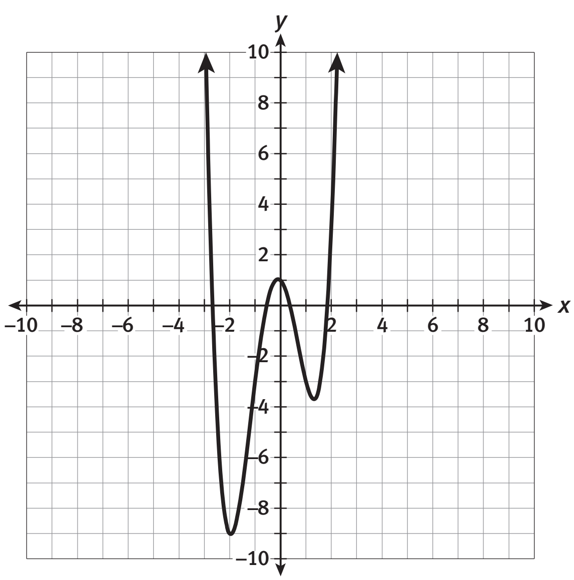


11. What is the domain of the function shown in the graph?

12. What is the range of the function shown in the graph?

13. What is the y-intercept of the function shown in the graph?

Use the graph below for Items 14–16.



14. What is the y-intercept of the function shown in the graph?

15. Identify any relative maximums.

16. Identify any relative minimums.

Lesson 6-3

A fundraising organization will donate $250 plus half of the money it raises from a charity event. Use this information for Items 17–20.

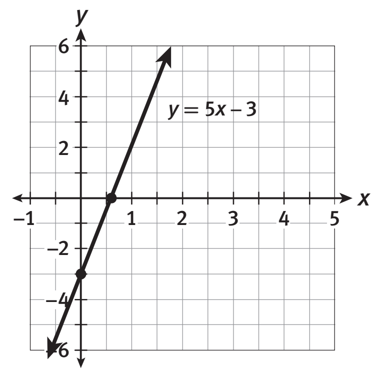
17. What is the independent variable?

18. What is the dependent variable?

19. What is the reasonable domain? Explain.

20. What is the reasonable range? Explain.

21. Describe a real-world situation that matches the graph shown.



MATHEMATICAL PRACTICES
Look For and Make Use of Structure

22. The graph of a function is a horizontal line. What is true about the absolute maximum and absolute minimum values of this function? Explain.

Graphs of Functions

Experiment Experiences

Lesson 7-1 The Spring Experiment

My Notes

Learning Targets:

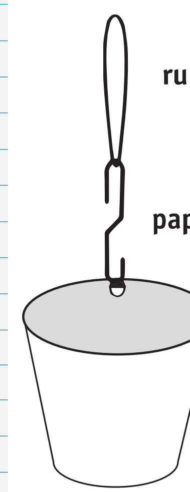
- Graph a function given a table.
- Write an equation for a function given a table or graph.

SUGGESTED LEARNING STRATEGIES: Discussion Groups, Look for a Pattern, Sharing and Responding, Think-Pair-Share, Create Representations, Construct an Argument

For the following experiment, you will need a paper cup, a rubber band, a paper clip, a measuring tape, and several washers.

A. Punch a small hole in the side of the paper cup, near the top rim.

B. Use the bent paper clip to attach the paper cup to the rubber band as shown in the diagram in the *My Notes* section.



1. What is the length of the rubber band?

Drop washers one at a time into the cup. Each time you add a washer, measure the length of the rubber band. Subtract the original length you recorded in Item 1 to find the distance that the rubber band has stretched.

2. Make a table of your data.

Number of Washers x	Length of Stretch from Original Length y
1	
2	
3	
4	
5	

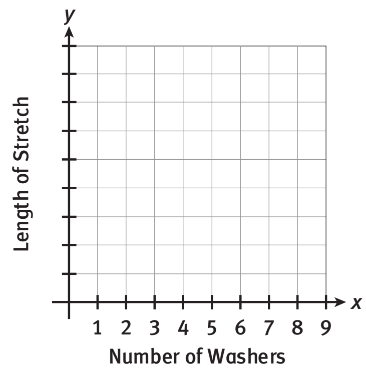
3. What patterns do you notice that might help you determine the relationship between the number of washers in the cup and the length of the rubber band stretch?

My Notes

CONNECT TO SCIENCE

What you have revealed with your experiment is an example of Hooke's Law. Hooke's Law states that the distance d that a spring (in this case the rubber band) is stretched by a hanging object varies directly with the object's weight w.

4. Use your table to make a graph. Be sure to label an appropriate scale and the units on the y-axis.



5. Describe your graph.

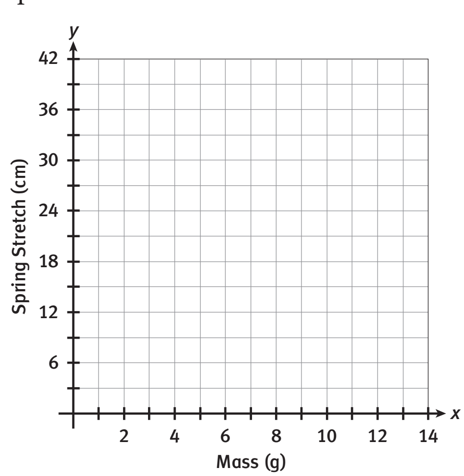
6. Model with mathematics. Use your graph and any patterns you described in Item 3 to write an equation that describes the relationship between the number of washers and the length of the stretch.

7. Use your graph or your equation to predict the length of the stretch for 8 washers and for 10 washers.

A group of students performed a similar experiment with a spring and various masses. The data they collected is shown in the table below.

Mass (g)	Spring Stretch (cm)
2	6
4	12
6	18
8	24
10	30
12	36

8. Make a graph of the data in the table.



My Notes

9. Reason quantitatively. How much does the spring stretch for each additional gram of mass added? Explain how you found your answer.

10. Reason abstractly. Use the students' data to write an equation that gives the distance d that the spring will stretch in terms of the mass m. Explain your equation.

11. Use the equation or the graph to determine the length of the stretch for a mass of 1 gram. Graph the outcome on your graph.

12. Use the equation or the graph to determine the length of the stretch for a mass of 7 grams. Graph the outcome on your graph.

13. Use the equation or the graph to determine the length of the stretch for a mass of 13 grams. Graph the outcome on your graph.

14. a. What do you notice about the points you graphed in Items 11–13?

b. How could you represent the set of all possible masses and corresponding stretches?

15. What is the y-intercept of the graph? What does it represent?

16. What is the reasonable domain? Explain.

My Notes

Mr. Hardiff's class conducts an experiment with a spring and a set of weights. They record their data, but some of the information is missing.

Weight (oz)	Spring Stretch (in.)
5	12.5
8	20
10	25
12	
15	
16	

17. How much does the spring stretch for each additional ounce of weight?

18. Describe how to use your answer to Item 17 to write an equation for the data in the table.

19. Use your equation from Item 18 to complete the table.

Check Your Understanding

20. A 4.5-pound weight stretches a spring 18 inches and a 7.5-pound weight stretches the same spring 30 inches. How much does the spring stretch for each additional pound of weight? Explain how you found your answer.

LESSON 7-1 PRACTICE

Jeremy and his classmates conduct an experiment with a set of weights and a spring. They record their results in the table. Use the table to answer Items 21–24.

Student	Mass (lb)	Spring Stretch (in.)
Jeremy	5	7.5
Adele	8	12
Roberto	14	21
Shanice	21	36
Guillaume	28	42

21. Make a graph of the data.

22. **Critique the reasoning of others.** Which student made a mistake when taking their turn at the experiment? Explain how you know.

23. If the mistake in Item 22 were corrected, what would the correct data point be?

24. Write an equation to describe the students' data, using the corrected data point you identified in Item 23.

My Notes

Learning Target:

- Graph a function describing a real-world situation and identify and interpret key features of the graph.

SUGGESTED LEARNING STRATEGIES: Discussion Groups, Look for a Pattern, Construct an Argument, Think-Pair-Share, Summarizing, Sharing and Responding

1. The Empire State Building in New York City is 1454 feet tall. How long do you think it will take a penny dropped from the top of the Empire State Building to hit the ground?

In 1589, the mathematician and scientist Galileo conducted an experiment to answer a question much like the one in Item 1. Galileo dropped balls from the top of the Leaning Tower of Pisa in Italy and determined the time it took them to reach the ground. Galileo used several balls identical in shape but differing in mass. Because the balls all reached the ground in the same amount of time, he developed the theory that all objects fall at the same rate.

Galileo's findings can be represented with the equation $h(t) = 1600 - 16t^2$, where $h(t)$ represents the height in feet of an object t seconds after it has been dropped from a height of 1600 feet.

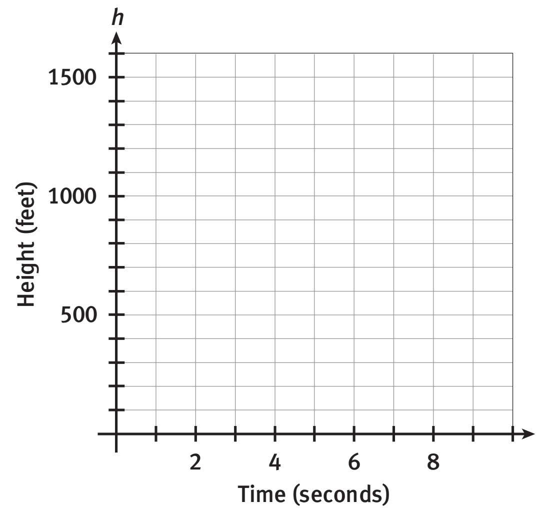
2. Make a table of values for Galileo's function $h(t) = 1600 - 16t^2$.

t (seconds)	$h(t)$ (feet)
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

My Notes

3. **Construct viable arguments.** Why would negative domain values not be appropriate in this context?

4. Using your table of values, graph Galileo's function.



5. What is the reasonable domain of the function represented in your graph? What is the reasonable range?

6. What is the y-intercept?

7. What does the y-intercept represent?

8. What is the ***x*-intercept**? What does the x-intercept represent?

9. Identify any extrema of the function shown in the graph. What do the extrema represent?

MATH TERMS

The ***x*-intercept** is the point where a graph crosses the x-axis. The y-coordinate of the x-intercept is 0.

"Your homework assignment is to graph this function," your math teacher says. She then points to the following function on the board:

$$f(x) = x^2 - 2x$$

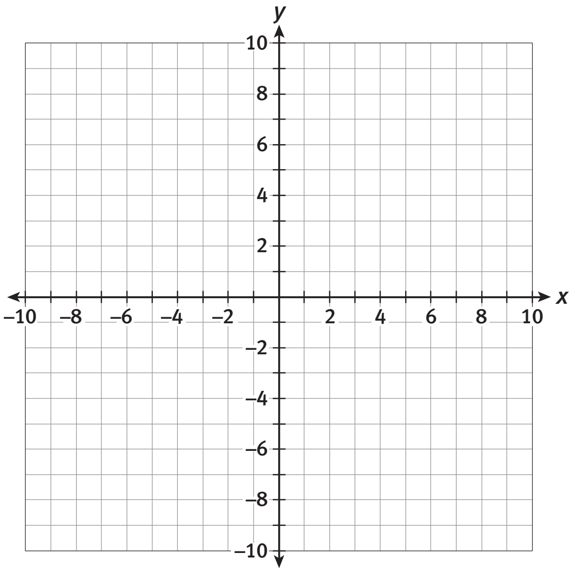
In this case, the function is not limited by a real-world situation. Therefore, it is important to use different types of domain values as you prepare to graph.

My Notes

10. Using various values for x, make a table of values for $f(x) = x^2 - 2x$.

x	$f(x)$

11. Using your table of values, graph the function.



12. Describe the differences between the domain of $f(x) = x^2 - 2x$ and the domain of Galileo's function.

13. State the range of $f(x) = x^2 - 2x$.

14. Identify the y-intercept of $f(x) = x^2 - 2x$.

15. What is the absolute maximum of $f(x) = x^2 - 2x$? What is the absolute minimum?

My Notes

Check Your Understanding

16. Revisit your answer to Item 1 and revise it if necessary. About how long do you think it will take a penny dropped from the top of the Empire State Building to hit the ground? How can you use Galileo's equation to help you answer this question?

LESSON 7-2 PRACTICE

The area of a rectangle with a perimeter of 20 units is given by $f(w) = 10w - w^2$, where w is the width of the rectangle. Assume that w is a whole number. Use this function to answer Items 17–20.

17. Make a table of values and a graph of the function.

18. **Attend to precision.** Give a reasonable domain for the function in this context. Explain your answers.

19. Identify the y-intercept of the function. What does the y-intercept represent within this context?

20. What is the absolute maximum of the function? What is the absolute minimum?

For Items 21–23, use the function $f(x) = x^2 - 9$.

21. Make a table of values and a graph of the function.

22. What are the domain and range?

23. Identify the y-intercept, the absolute maximum, and the absolute minimum.

My Notes

Learning Targets:

- Given a verbal description of a function, make a table and a graph of the function.
- Graph a function and identify and interpret key features of the graph.

SUGGESTED LEARNING STRATEGIES: Discussion Groups, Look for a Pattern, Construct an Argument, Paraphrasing, Marking the Text, Think-Pair-Share

In the late nineteenth century, the scientist Marie Curie performed experiments that led to the discovery of radioactive substances.

A radioactive substance is a substance that gives off radiation as it decays. Scientists describe the rate at which a radioactive substance decays as its *half-life*. The half-life of a substance is the amount of time it takes for one-half of the substance to decay.

CONNECT TO SCIENCE

How much is half a life?
The half-life of a radioactive substance can be as little as 0.0018 seconds for Polonium-215 and as much as 4.5 billion years for Uranium-238.

1. Radium has a half-life of 1600 years. How much radium will be left from a 1000-gram sample after 1600 years?

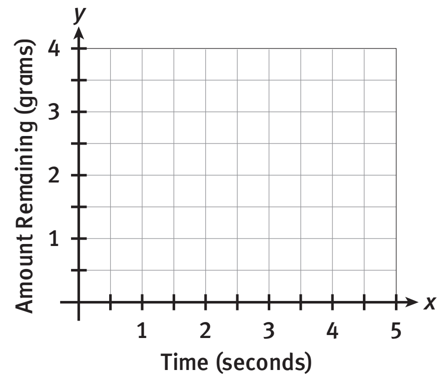
2. How much radium will be left after another 1600 years?

3. Suppose a radioactive substance has a half-life of 1 second and you begin with a sample of 4 grams. Complete the table of values.

Time (seconds)	Amount Remaining (grams)
0	4
1	
2	
3	
4	
5	

My Notes

4. Graph the data from the table on the grid below.



5. **Make use of structure.** Will the amount of the substance that remains ever reach 0? Explain.

6. What are the reasonable domain and range of the function represented in the graph? Explain.

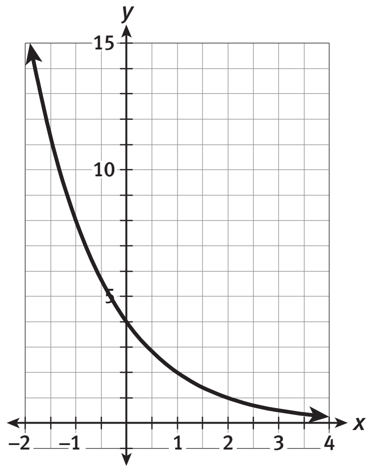
7. What is the y-intercept and what does it represent?

8. Identify the absolute maximum and minimum of the function represented in the graph, and tell what they represent in the context.

Lesson 7-3
The Radioactive Decay Experiment

My Notes

The function that describes the substance's decay is $f(x) = 4\left(\frac{1}{2}\right)^x$. The graph of this function when it does not model a real-world situation is shown below.



9. What are the domain and range of the function?

10. How is this graph different from your graph in Item 4?

11. How do the values of y change as the values of x increase?

12. How do the values of y change as the values of x decrease?

13. Identify the absolute maximum and absolute minimum of the function.

My Notes

Check Your Understanding

14. A scientist has g grams of a radioactive substance. Write an expression that shows the amount of the substance that remains after one half-life.

15. Critique the reasoning of others. Dylan looked at the function $f(x) = 4\left(\frac{1}{2}\right)^x$ and said, "This function is always greater than 0, so 0 is the absolute minimum." Explain why Dylan is incorrect.

LESSON 7-3 PRACTICE

Suppose the value of your new car is reduced by half every year that you own it. You paid $20,000 for your new car.

16. Describe how this situation is similar to the half-life of a radioactive substance.

17. Copy and complete the table below.

Time (years)	Value ($)
0	20,000
1	
2	
3	
4	
5	

18. Make sense of problems. For insurance purposes, a vehicle is considered scrap when its value falls below $500. After how many years will your new car be considered scrap?

ACTIVITY 7 PRACTICE

Write your answers on notebook paper.
Show your work.

Lesson 7-1

A weight of 15 ounces stretches a spring 10 inches. A weight of 24 ounces stretches the same spring 16 inches. Use this information to answer Items 1–4.

1. How many inches does the spring stretch per ounce of additional weight?
A. $\frac{2}{3}$ inch
B. $\frac{3}{2}$ inches
C. 25 inches
D. 150 inches

2. Write an equation to describe the relationship between the distance d that the spring stretches and the weight w that is attached to it.

3. How much will the spring stretch for a weight of 9 ounces?

4. The spring is stretched 14 inches. How many ounces is the weight that is attached to it?

A spring stretches 2.5 inches for each ounce of weight. Use this information for Items 5–7.

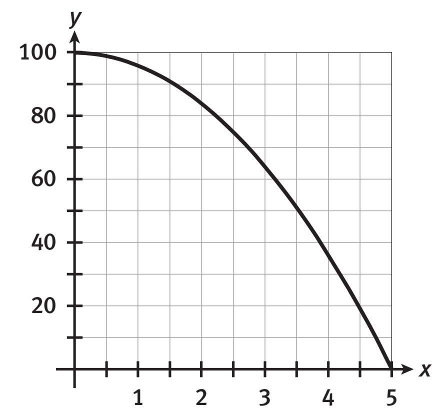
5. Determine a function that represents this situation.

6. If you were to graph the function represented by this situation, what would be the reasonable domain? Explain.

7. Which of the following data points would **not** lie on the graph representing this function?
A. (0, 0)
B. (1, 2.5)
C. (2.5, 1)
D. (10, 25)

Lesson 7-2

Suppose that the height of an object after x seconds is given by $f(x) = 100 - 4x^2$, as shown in the graph below.



Use the function or the graph for Items 8–14.

8. What is the reasonable domain of the function?

9. What is the reasonable range of the function?

10. Identify the y-intercept of the function.

11. What does the y-intercept represent?

12. Identify the x-intercept of the function.

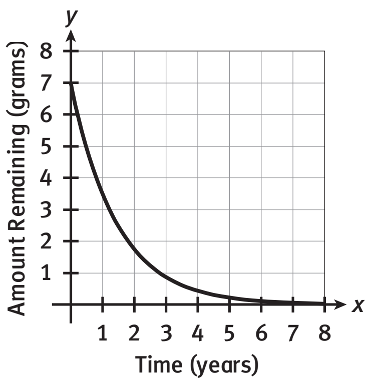
13. What does the x-intercept represent?

14. Loni says that because of the negative sign in front of $4x^2$, the reasonable domain for this function is only negative values. Is her reasoning correct? Explain.

Lesson 7-3

15. The half-life of a radioactive substance is 1 hour. If you begin with 100 ounces of the substance, how many hours does it take for 12.25 ounces to remain?

The graph below represents a radioactive decay situation. Use this graph for Items 16–18.



16. What is the original amount of the radioactive substance? Explain how you know.

17. What are the reasonable domain and range?

18. Identify the absolute maximum and absolute minimum values of the function. What do these values represent?

Barry has a piece of paper whose area is 150 square inches. He cuts the paper in half and discards one of the pieces. He repeats this procedure several times. Use this information for Items 19–24.

19. Copy and complete the table below to show the area of the remaining piece of paper after x cuts.

Number of Cuts, x	Area of Remaining Piece, y
0	150
1	
2	
3	
4	

20. Describe how this situation is similar to the half-life of a radioactive substance.

21. If you were to graph the points from the table, would you connect the points? Explain.

22. Describe how the reasonable domain in this situation is different from the reasonable domain in a radioactive decay situation.

23. Identify the y-intercept. What does it represent?

24. Identify the absolute maximum value. What does it represent?

MATHEMATICAL PRACTICES
Construct Viable Arguments and Critique the Reasoning of Others

25. Maude receives $100 for her birthday. "I am going to spend half of my birthday money each day until none is left," she decides. Is it reasonable for her to believe that she will eventually spend all of the money? Justify your answer.

Transformations of Functions

Transformers

Lesson 8-1 Exploring $f(x) + k$

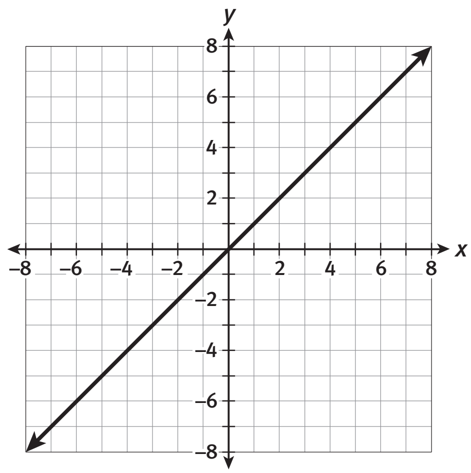
My Notes

Learning Targets:

- Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$.
- Identify the transformation used to produce one graph from another.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Interactive Word Wall, Think-Pair-Share, Create Representations, Discussion Groups

The equation and the graph of $y = x$ or $f(x) = x$ are referred to as the linear **parent function**. The graph of $f(x) = x$ is shown below.



MATH TERMS

A **parent function** is the most basic function of a particular category or type.

1. Complete the table for $g(x) = x + 5$.

x	$f(x) = x$	$g(x) = x + 5$
-3	-3	2
-2	-2	
-1	-1	
0	0	
1	1	
2	2	
3	3	

2. **Make use of structure.** How do the y-values for $g(x)$ compare to the y-values for $f(x)$? Make a conjecture about the graph of $g(x)$.

My Notes

3. Test your conjecture by using a graphing calculator to graph $g(x) = x + 5$. Graph this on the grid in Item 1.

a. What is the y-intercept of the parent function?

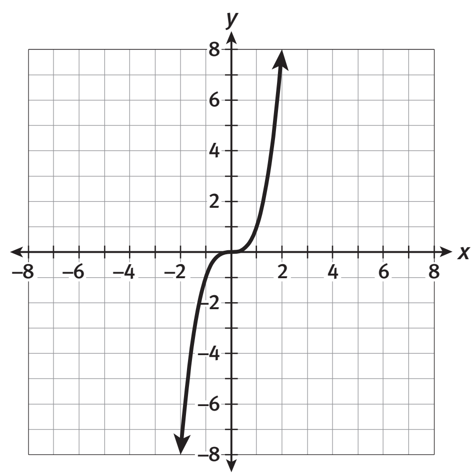
b. What is the y-intercept of $g(x)$?

c. What is the x-intercept of the parent function?

d. What is the x-intercept of $g(x)$?

e. Revisit your original conjecture in Item 2 and revise it if necessary. How does the graph of $g(x)$ differ from the graph of the parent function, $f(x) = x$?

The graph of $f(x) = x^3$ is shown below.



4. Make a conjecture about the graph of $g(x) = x^3 - 4$.

5. Graph both $f(x)$ and $g(x)$ on a graphing calculator. Sketch the graph of $g(x)$ on the grid above. Label a few points on each graph.

6. Revisit your original conjecture in Item 4 about the graph of $g(x)$ and revise it if necessary. How does the graph of $g(x)$ differ from the graph of $f(x)$?

7. **Express regularity in repeated reasoning.** How does the value of k in the equation $g(x) = f(x) + k$ change the graph of $f(x)$?

My Notes

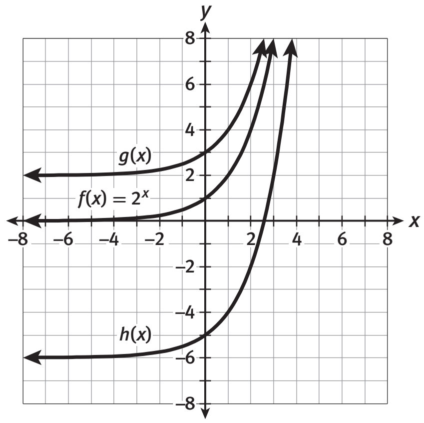
A change in the position, size, or shape of a graph is a **transformation**. The changes to the graphs in Items 1–6 are examples of a transformation called a **vertical translation**.

MATH TERMS

A **vertical translation** of a graph shifts the graph up or down. A vertical translation preserves the shape of the graph.

8. In the figure, the graphs of $g(x)$ and $h(x)$ are vertical translations of the graph of $f(x) = 2^x$.

a. Write the equation for $g(x)$.



b. Write the equation for $h(x)$.

Check Your Understanding

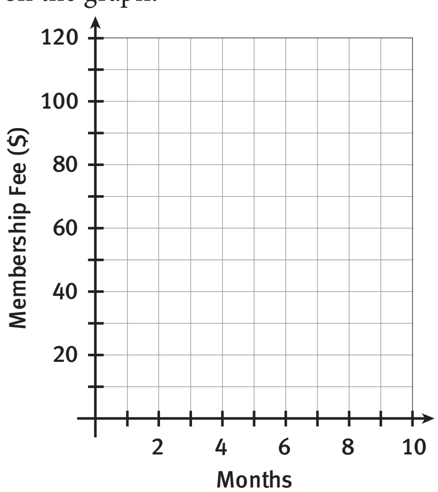
9. Without graphing, describe the transformation from the graph of $f(x) = x^2$ to the graph of $g(x) = x^2 + 7$.

10. Suppose $f(x) = x - 2$. Describe the transformation from the graph of $f(x)$ to the graph of $g(x) = x + 3$. Use a graphing calculator to check your answer.

Ray's Gym charges an initial sign-up fee of \$25.00 and a monthly fee of \$15.00.

11. Reason abstractly. Write a function that describes the gym's total membership fee for x months.

12. Graph the function you wrote in Item 11 on the grid below. Label several points on the graph.



13. Identify the y-intercept. What does the y-intercept represent?

My Notes

14. How would the function change if the initial sign-up fee were increased by $5.00? How would the graph change?

Check Your Understanding

15. The membership fee at Gina's Gym is given by the function $g(x) = 15x + 20$, where x is the number of months.

a. How do the fees at Gina's Gym compare to those at Ray's Gym?

b. Without graphing, describe how the graph of $g(x)$ compares to the graph of $f(x)$.

16. The y-intercept of a function $f(x)$ is $(0, b)$. What is the y-intercept of $f(x) + k$?

LESSON 8-1 PRACTICE

Identify the transformation from the graph of $f(x) = x^2$ to the graph of $g(x)$. Then graph $f(x)$ and $g(x)$ on the same coordinate plane.

17. $g(x) = x^2 - 7$

18. $g(x) = x^2 + 10$

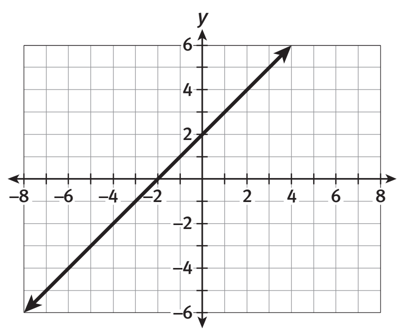
Write the equation of the function described by each of the following transformations of the graph of $f(x) = x^3$.

19. Translated up 9 units

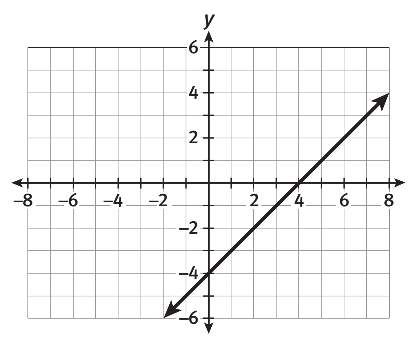
20. Translated down 5 units

Each graph shows a vertical translation of the graph of $f(x) = x$. Write an equation to describe each graph.

21.



22.



23. Model with mathematics. Orange Taxi charges $2.75 as soon as you step into the taxi and $2.50 per mile. Magenta Taxi charges $3.25 as soon as you step into the taxi and $2.50 per mile.

a. Write a function $f(x)$ that describes the total cost of a ride of x miles with Orange Taxi. Write a function $g(x)$ that describes the total cost of a ride of x miles with Magenta Taxi.

b. Without graphing, explain how the graph of $g(x)$ compares to the graph of $f(x)$.

c. Check your answer to Part (b) by graphing the functions.

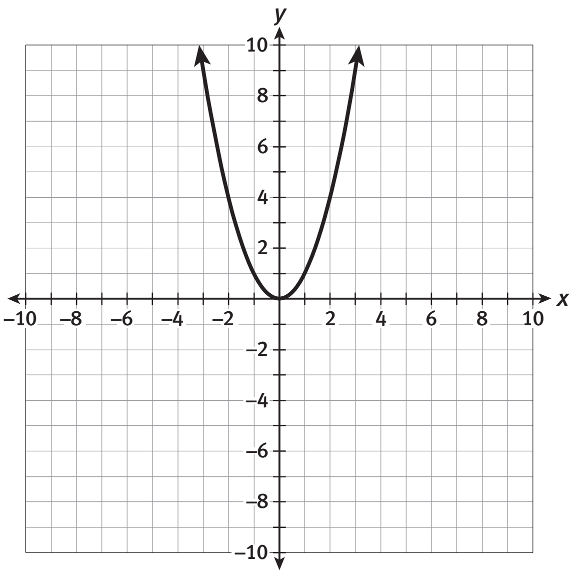
My Notes

Learning Targets:

- Identify the effect on the graph of replacing $f(x)$ by $f(x + k)$.
- Identify the transformation used to produce one graph from another.

SUGGESTED LEARNING STRATEGIES: Predict and Confirm, Look for a Pattern, Create Representations, Think-Pair-Share, Discussion Groups

The function $f(x) = x^2$ is graphed below.



1. Write a new function, $g(x)$, by replacing x with $x + 7$.

2. Graph both $f(x) = x^2$ and $g(x)$ on a graphing calculator. Sketch the graph of $g(x)$ on the grid above, labeling at least a few points on each graph.

3. What is the x-intercept of $f(x) = x^2$?

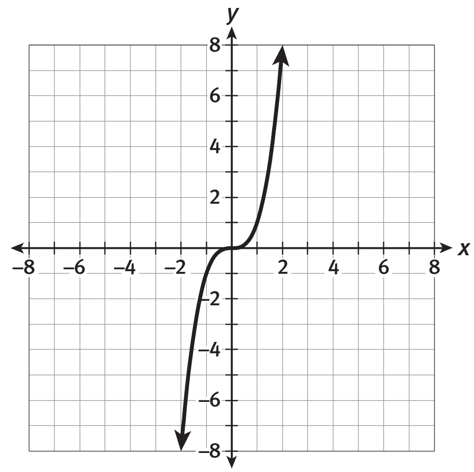
4. What is the x-intercept of $g(x)$?

5. Describe the transformation from the graph of $f(x) = x^2$ to the graph of $g(x)$.

Note that the function $g(x)$ can be written as $f(x + 7)$. This means that x is replaced with $x + 7$ in the function $f(x)$.

My Notes

The graph of $f(x) = x^3$ is shown below.



6. Make a conjecture about the graph of $g(x) = (x - 3)^3$.

7. Graph both $f(x)$ and $g(x)$ on a graphing calculator. Sketch the graph of $g(x)$ on the grid above, labeling at least a few points on each graph.

8. Revisit your original conjecture in Item 6 about the graph of $g(x)$ and revise it if necessary. How does the graph of $g(x)$ differ from the graph of $f(x)$?

9. How does the value of k in the equation $g(x) = f(x + k)$ change the graph of the function $f(x)$?

The changes to the graphs in Items 1–8 are examples of a transformation called a **horizontal translation**.

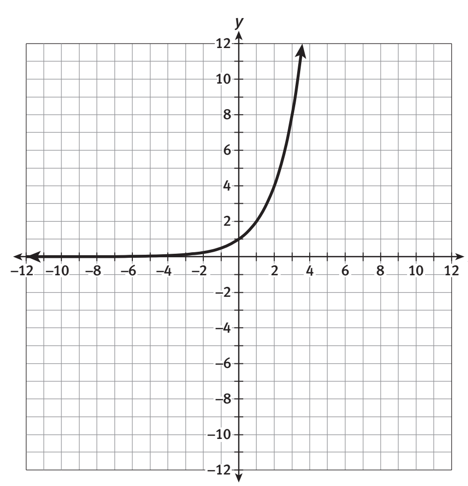
MATH TERMS

A **horizontal translation** of a graph shifts the graph left or right. Like a vertical translation, a horizontal translation preserves the shape of the graph.

10. The figure shows the graph of the function $f(x) = 2^x$.

a. Without using a graphing calculator, sketch the graph of $g(x) = f(x + 8) = 2^{x+8}$ on the grid.

b. Use a graphing calculator to check your graph in Part (a). Revise your graph if necessary.

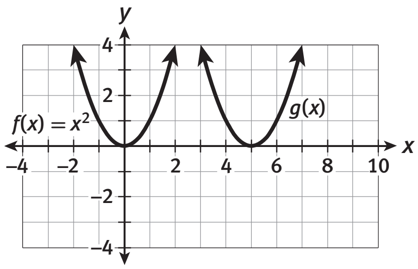


Check Your Understanding

11. Without graphing, describe the transformation from the graph of $f(x) = x^2$ to the graph of $g(x)$.

a. $g(x) = (x + 4)^2$
b. $g(x) = f(x - 7)$
c. $g(x) = (x - 2)^2 + 5$
d. $g(x) = (x + 9)^2 - 1$

12. The function $f(x) = x^2$ and another function, $g(x)$, are graphed below. Write the equation for $g(x)$. Explain how you found your answer.



13. Make sense of problems. Julio went to a theme park in July. He paid $15 to enter the park and $3.00 for each ride. He went on x rides.

a. Write a function that describes the total cost of Julio's trip to the theme park.

b. Julio went back to the theme park in September. The entrance fee was the same and each ride still cost $3.00. However, this time Julio went on 5 more rides. Use your function from Part (a) to describe Julio's second trip.

c. How does the equation for Julio's second trip to the park change the graph of the first trip?

d. What kind of transformation describes the change from the first graph to the second graph?

e. Julio went to the park again in October and went on 8 fewer rides than he did in July. Use your function from Part (a) to describe Julio's third trip. How does this change the initial graph?

My Notes

f. Julio goes to the park again in November. Now it is the off-season and the entrance fee is $10 less than it was in July. He goes on the same number of rides as he did in July. Write a function to describe Julio's fourth trip. How does the graph of the initial trip change with this new situation?

Check Your Understanding

14. The x-intercept of the function $f(x)$ is $(a, 0)$. What is the x-intercept of the function $f(x + k)$?

15. Without graphing, explain how the graph of $y = (x - 4)^3$ is related to the graph of $y = (x + 4)^3$.

LESSON 8-2 PRACTICE

Identify the transformation from the graph of $f(x) = x^2$ to the graph of $g(x)$. Then graph $f(x)$ and $g(x)$ on the same coordinate plane.

16. $g(x) = (x - 1)^2$

17. $g(x) = (x + 3)^2$

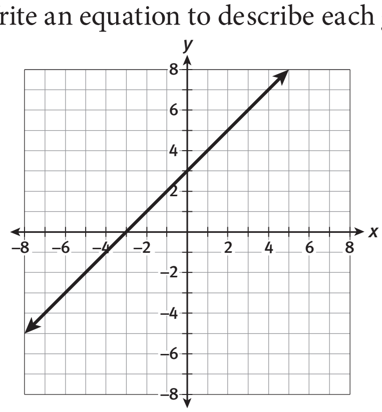
Write the equation of the function described by each of the following transformations of the graph of $f(x) = x^3$.

18. Translated 7 units to the left

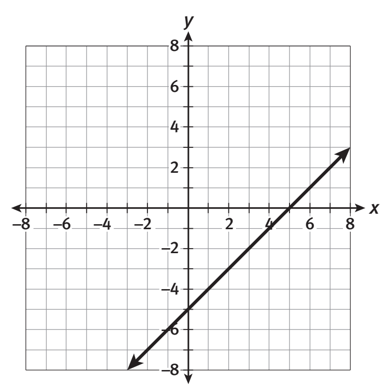
19. Translated 8 units to the right

20. Each graph shows a horizontal translation of the graph of $f(x) = x$. Write an equation to describe each graph.

a.



b.



c. Critique the reasoning of others. Molly said that the graphs above are also vertical translations of the graph of $f(x) = x$. Is Molly correct? Explain.

ACTIVITY 8 PRACTICE

Write your answers on notebook paper.
Show your work.

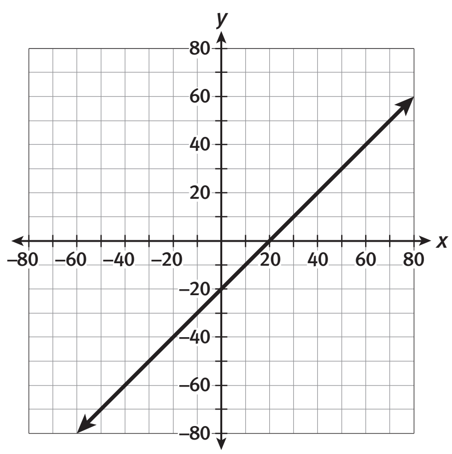
Lesson 8-1

In Items 1–4, identify the transformation from the graph of $f(x) = x^3$ to the graph of $g(x)$.

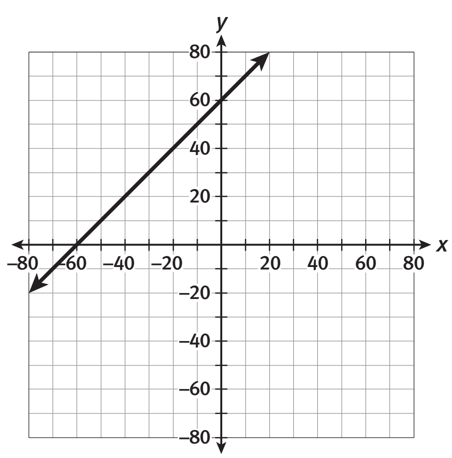
1. $g(x) = x^3 + 11$
2. $g(x) = x^3 - 4$
3. $g(x) = x^3 + 0.1$
4. $g(x) = -2 + x^3$
5. The graph of $f(x) = x^2$ is translated 9 units down to create the graph of $g(x)$. Which of the following is the equation for $g(x)$?
   A. $g(x) = x^2 + 9$
   B. $g(x) = x^2 - 9$
   C. $g(x) = (x + 9)^2$
   D. $g(x) = (x - 9)^2$

In Items 6 and 7, each graph shows a vertical translation of the graph of $f(x) = x$. Write an equation to describe the graph.

6.



7.



For Items 8 and 9, determine the equation of the function described by each of the following transformations of the graph of $f(x) = 3^x$.

8. Translated 15 units down
9. Translated 2.1 units up
10. An air conditioner costs \$450 plus \$40 per month to operate.
    a. Write a function that describes the total cost of buying and operating the air conditioner for x months.
    b. Use your calculator to graph the function.
    c. What is the y-intercept? What does it represent?
    d. How would the function change if the price of the air conditioner were reduced to \$425? How would the graph change?

Given that $g(x) = f(x) + k$, with $k \neq 0$, determine whether each statement is always, sometimes, or never true.

11. The graph of $g(x)$ is a vertical translation of the graph of $f(x)$.
12. The graphs of $f(x)$ and $g(x)$ are both lines.
13. The graph of $f(x)$ has the same y-intercept as the graph of $g(x)$.
14. Caitlin drew the graph of $f(x) = x^2$. Then she translated the graph 6 units up to get the graph of $g(x)$. Next, she translated the graph of $g(x)$ 8 units down to get the graph of $h(x)$. Which of these is an equation for $h(x)$?
    A. $h(x) = x^2 + 14$
    B. $h(x) = x^2 + 2$
    C. $h(x) = x^2 - 2$
    D. $h(x) = x^2 - 14$

Lesson 8-2

In Items 15–18, identify the transformation from the graph of $f(x) = 2^x$ to the graph of $g(x)$.

15. $g(x) = 2^x - 3$

16. $g(x) = 2^{(x-3)}$

17. $g(x) = 2^x + 4$

18. $g(x) = 2^{(x+4)}$

19. The graph of which function is a translation of the graph of $f(x) = x^2$ five units to the right?

A. $g(x) = x^2 - 5$
B. $g(x) = (x + 5)^2$
C. $g(x) = (x - 5)^2$
D. $g(x) = x^2 + 5$

Write the equation of the function described by each of the following transformations of the graph of $f(x) = x^3$.

20. Translated 7 units up

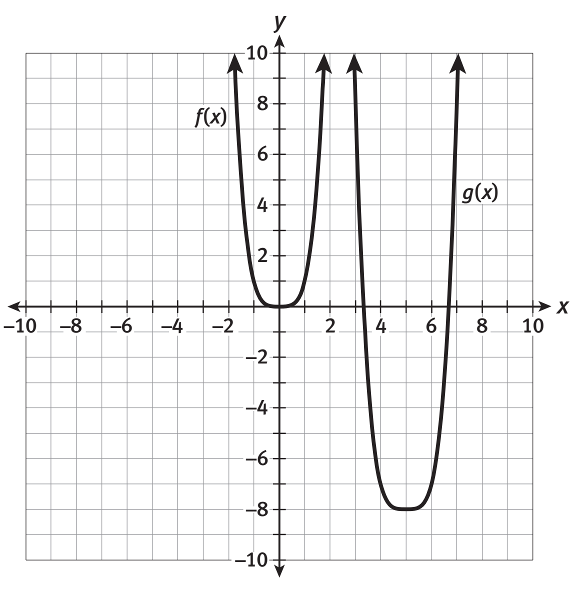
21. Translated 4 units down

22. Translated 2 units right

23. Translated 5 units down

24. Translated 3 units left

25. The figure shows the graph of $f(x) = x^4$ and the graph of $g(x)$. Write an equation for the graph of $g(x)$.



Without graphing, describe the transformation from the graph of $f(x) = x^2$ to the graph of $g(x)$.

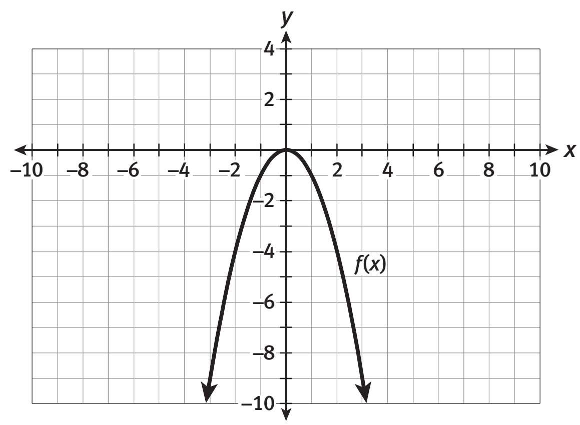
26. $g(x) = (x - 7)^2 + 1$

27. $g(x) = f(x + 4)$

28. $g(x) = (x + 9)^2 - 0.2$

29. $g(x) = f(x - 2) - 3$

30. The graph of $f(x)$ is shown below. Which of the following is a true statement about the graph of $g(x) = f(x + 3)$?



A. The x-intercept of $g(x)$ is (3, 0).
B. The x-intercept of $g(x)$ is (−3, 0).
C. The y-intercept of $g(x)$ is (0, 3).
D. The y-intercept of $g(x)$ is (0, −3).

MATHEMATICAL PRACTICES
Model with Mathematics

31. In 2011, the ticket price for entrance to a state fair was \$12. Each ride had an additional \$4.00 fee. In 2012, the entrance ticket cost \$15 and the rides remained \$4.00 each.

a. Write a function $f(x)$ for the cost of visiting the fair and riding x rides in 2011.
b. Write a function $g(x)$ for the cost of visiting the fair and riding x rides in 2012.
c. What transformation could you use to obtain the graph of $g(x)$ from the graph of $f(x)$?
d. What transformation could you use to obtain the graph of $f(x)$ from the graph of $g(x)$?

Rates of Change

Ramp it Up

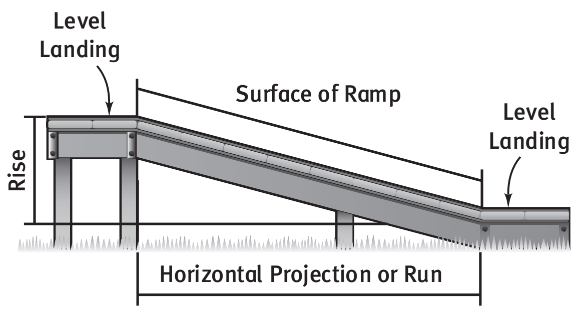
Lesson 9-1 Slope

Learning Targets:

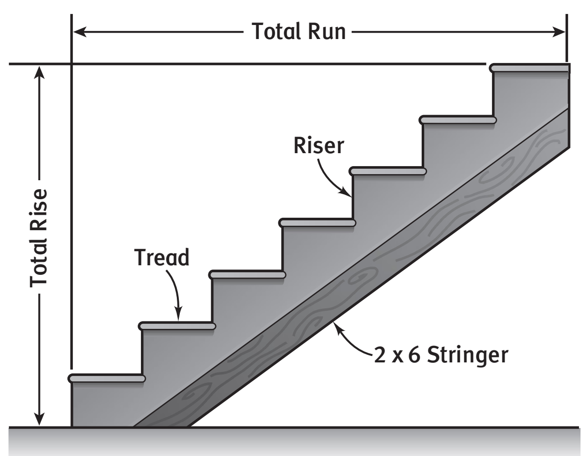
- Determine the slope of a line from a graph.
- Develop and use the formula for slope.

SUGGESTED LEARNING STRATEGIES: Close Reading, Summarizing, Sharing and Responding, Discussion Groups, Construct an Argument, Identify a Subtask

Margo's grandparents are moving in with her family. The family needs to make it easier for her grandparents to get in and out of the house. Margo has researched the specifications for building stairs and wheelchair ramps. She found the government website that gives the Americans with Disabilities Act (ADA) accessibility guidelines for wheelchair ramps and discovered the following diagram:



Then, Margo decided to look for the requirements for building stairs and found the following diagram:



1. What do you think is meant by the terms *rise* and *run* in this context?

My Notes

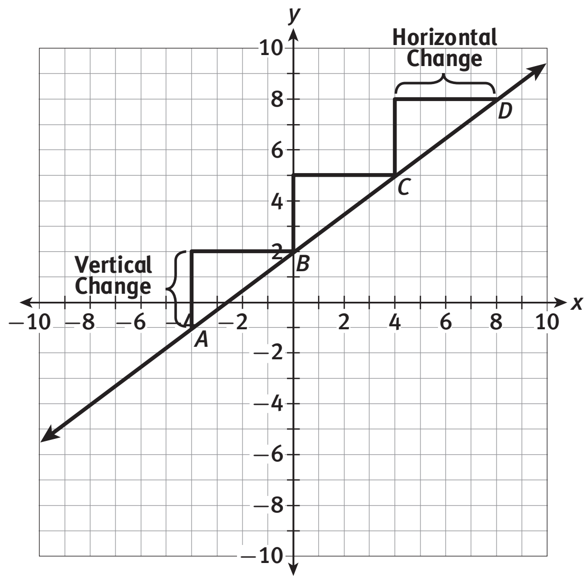
CONNECT TO SOCIAL SCIENCE

The table gives information from the ADA website about the slope of wheelchair ramps.

Slope	Maximum Rise		Maximum Run	
	in.	mm	ft	m
$\frac{1}{16} < m \leq \frac{1}{12}$	30	760	30	9
$\frac{1}{20} \leq m < \frac{1}{16}$	30	760	40	12

My Notes

Consider the line in the graph below:



Vertical change can be represented as a *change in y*, and horizontal change can be represented by a *change in x*.

2. What is the vertical change between:
a. points *A* and *B*? **b.** points *A* and *C*? **c.** points *C* and *D*?

3. What is the horizontal change between:
a. points *A* and *B*? **b.** points *A* and *C*? **c.** points *C* and *D*?

MATH TERMS

Slope is a measure of the amount of decline or incline of a line. The variable *m* is often used to represent slope.

The ratio of the vertical change to the horizontal change determines the **slope** of the line.

$$\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x}$$

4. Find the slope of the segment of the line connecting:
a. points *A* and *B* **b.** points *A* and *C* **c.** points *C* and *D*

5. What do you notice about the slope of the line in Items 4a, 4b, and 4c?

6. What does your answer to Item 5 indicate about points on a line?

My Notes

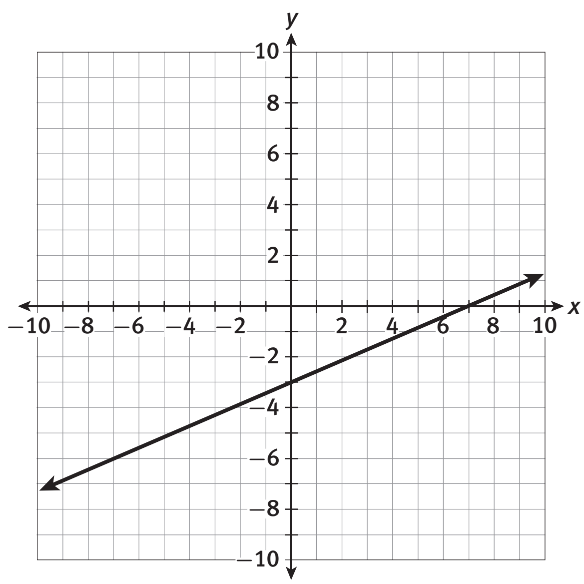
7. Slope is sometimes referred to as $\frac{\text{rise}}{\text{run}}$. Explain how the ratio $\frac{\text{rise}}{\text{run}}$ relates to the ratios for finding slope mentioned above.

8. **Reason quantitatively.** Would the slope change if you counted the run (horizontal change) before you counted the rise (vertical change)? Explain your reasoning.

WRITING MATH

In mathematics the Greek letter Δ (delta) represents a change or difference between mathematical values.

9. Determine the slope of the line graphed below.



MATH TIP

Select two points on the line and use them to compute the slope.

My Notes

Although the slope of a line can be calculated by looking at a graph and counting the vertical and horizontal change, it can also be calculated numerically.

10. Recall that the slope of a line is the ratio $\frac{\text{change in } y}{\text{change in } x}$.

a. Identify two points on the graph above and record the coordinates of the two points that you selected.

	x-coordinate	*y*-coordinate
1st point		
2nd point		

b. Which coordinates relate to the vertical change on the graph?

c. Which coordinates relate to the horizontal change on a graph?

d. Determine the vertical change.

e. Determine the horizontal change.

f. Calculate the slope of the line. How does this slope compare to the slope that you found in Item 10?

g. If other students in your class selected different points for this problem, should they have found different values for the slope of this line? Explain.

11. It is customary to label the coordinates of the first point (x_1, y_1) and the coordinates of the second point (x_2, y_2).

a. Write an expression to calculate the vertical change, $\triangle y$, of the line through these two points.

b. Write an expression to calculate the horizontal change, $\triangle x$, of the line through these two points.

c. Write an expression to calculate the slope of the line through these two points.

My Notes

Check Your Understanding

12. Use the slope formula to determine the slope of a line that passes through the points (4, 9) and (−8, −6).

13. Use the slope formula to determine the slope of the line that passes through the points (−5, −3) and (9, −10).

14. Explain how to find the slope of a line from a graph.

15. Explain how to find the slope of a line when given two points on the line.

LESSON 9-1 PRACTICE

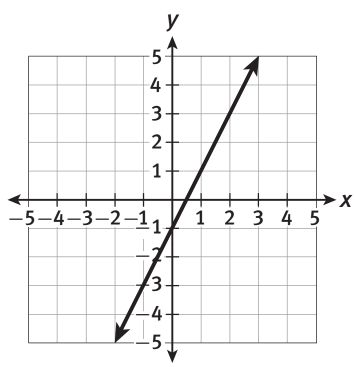
16. Find $\triangle x$ and $\triangle y$ for the points (7, −2) and (9, −7).

17. **Critique the reasoning of others.** Connor determines the slope between (−2, 4) and (3, −3) by calculating $\frac{4-(-3)}{-2-3}$. April determines the slope by calculating $\frac{3-(-2)}{-3-4}$. Explain whose reasoning is correct.

18. When given a table of ordered pairs, you can find the slope by choosing any two ordered pairs from the table. Determine the slope represented in the table below.

x	5	7	9	11
y	5	3	1	−1

19. Determine the slope of the given line.



My Notes

Learning Targets:

- Calculate and interpret the rate of change for a function.
- Understand the connection between rate of change and slope.

SUGGESTED LEARNING STRATEGIES: Discussion Groups, Create Representations, Look for a Pattern, Think-Pair-Share

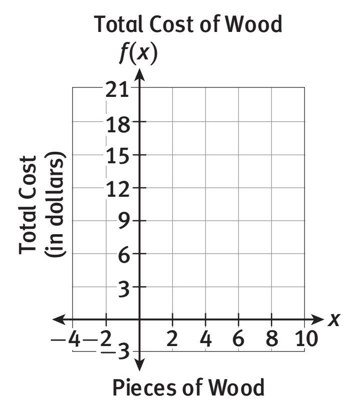
The **rate of change** for a function is the ratio of the change in y, the dependent variable, to the change in x, the independent variable.

1. Margo went to the lumberyard to buy supplies to build the wheelchair ramp. She knows that she will need several pieces of wood. Each piece of wood costs \$3.

a. **Model with mathematics.** Write a function $f(x)$ for the total cost of the wood pieces if Margo buys x pieces of wood.

b. Make an input/output table of ordered pairs and then graph the function.

Pieces of Wood, x	Total Cost, $f(x)$



c. What is the slope of the line that you graphed?

d. By how much does the cost increase for each additional piece of wood purchased?

e. How does the slope of this line relate to the situation with the pieces of wood?

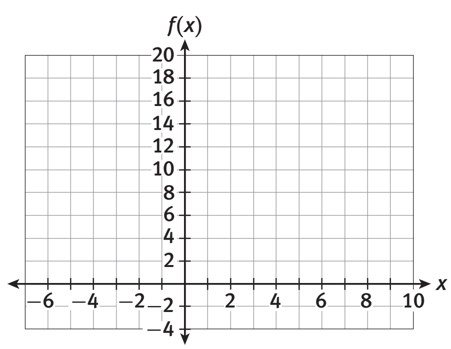
f. Is there a relationship between the slope of the line and the equation of the line? If so, describe that relationship.

2. Margo is going to work with a local carpenter during the summer. Each week she will earn \$10.00 plus \$2.00 per hour.

a. Write a function $f(x)$ for Margo's total earnings if she works x hours in one week.

b. Make an input/output table of ordered pairs and then graph the function. Label your axes.

Hours, x	Earnings, $f(x)$ (dollars)



My Notes

c. How much will Margo's earnings change if she works 6 hours instead of 2? If she works 4 hours instead of 3? How much do Margo's earnings change for each additional hour worked?

d. Does the function have a constant rate of change? If so, what is it?

e. What is the slope of the line that you graphed?

f. Describe the meaning of the slope within the context of Margo's job.

g. Describe the relationship between the slope of the line, the rate of change, and the equation of the line.

h. How much will Margo earn if she works for 8 hours in one week?

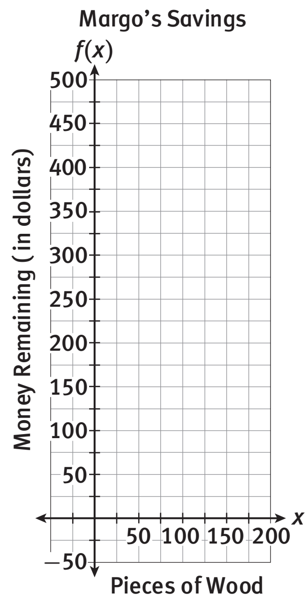
3. By the end of the summer, Margo has saved $375. Recall that each of the small pieces of wood costs $3.

a. Write a function $f(x)$ for the amount of money that Margo still has if she buys x pieces of wood.

My Notes

b. Make an input/output table of ordered pairs and then graph the function.

Pieces of Wood, x	Money Remaining, $f(x)$ (dollars)



c. How much will the amount Margo has saved change if she buys 100 instead of 25 pieces of wood? If she buys 50 instead of 0 pieces of wood? For each additional piece of wood? Explain.

d. Does the function have a constant rate of change? If so, what is it?

e. What is the slope of the line that you graphed?

f. How are the rate of change of the function and the slope related?

g. Describe the meaning of the slope within the context of Margo's savings.

h. How does this slope differ from the other slopes that you have seen in this activity?

My Notes

Check Your Understanding

4. The constant rate of change of a function is −5. Describe the graph of the function as you look at it from left to right.

5. Does the table represent data with a constant rate of change? Justify your answer.

x	y
2	−5
4	5
7	20
11	40

LESSON 9-2 PRACTICE

6. The art museum charges an initial membership fee of $50.00. For each visit the museum charges $15.00.

a. Write a function $f(x)$ for the total amount charged for x trips to the museum.

b. Make a table of ordered pairs and then graph the function.

c. What is the rate of change? What is the slope of the line?

d. How does the slope of this line relate to the number of museum visits?

7. **Critique the reasoning of others.** Simone claims that the slope of the line through (−2, 7) and (3, 0) is the same as the slope of the line through (2, 1) and (12, −13). Prove or disprove Simone's claim.

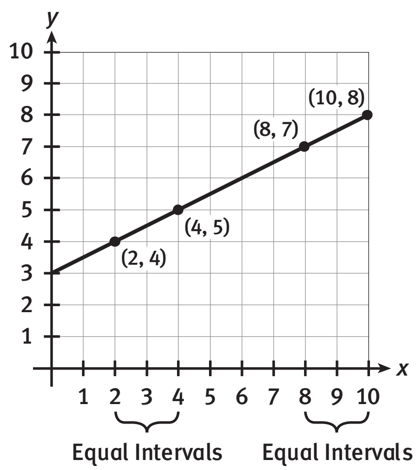
My Notes

Learning Targets:

- Show that a linear function has a constant rate of change.
- Understand when the slope of a line is positive, negative, zero, or undefined.
- Identify functions that do not have a constant rate of change and understand that these functions are not linear.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Think-Pair-Share, Construct an Argument, Sharing and Responding, Summarizing

You have seen that for a linear function, the rate of change is constant and equal to the slope of the line. This is because linear functions increase or decrease by equal differences over equal intervals. Look at the graph below.



1. Over the interval 2 to 4, by how much does the function increase? Explain.

2. Over the equal interval 8 to 10, by how much does the function increase? Explain.

"Equal differences over equal intervals" is an equivalent way of referring to constant slope. "Differences" refers to $\triangle y$, and "intervals" refers to $\triangle x$. "Equal differences over equal intervals" means $\frac{\triangle y}{\triangle x}$, which represents the slope, will always be the same.

My Notes

3. The table below represents a function.

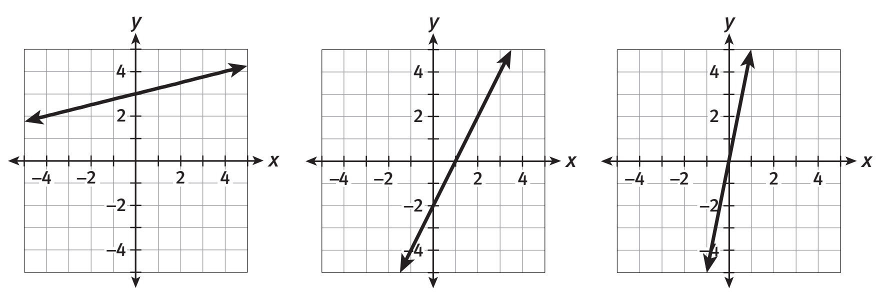
x	*y*
−8	62
−6	34
−1	−1
1	−1
5	23
7	47

a. Determine the rate of change between the points (−8, 62) and (−6, 34).

b. Determine the rate of change between the points (−1, −1) and (1, −1).

c. **Construct viable arguments.** Is this a linear function? Justify your answer?

4. a. Determine the slopes of the lines shown.

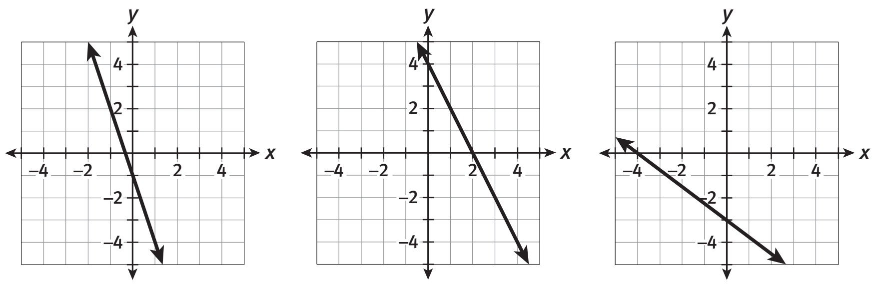


b. **Express regularity in repeated reasoning.** Describe the slope of any line that rises as you view it from left to right.

Lesson 9-3
More About Slopes

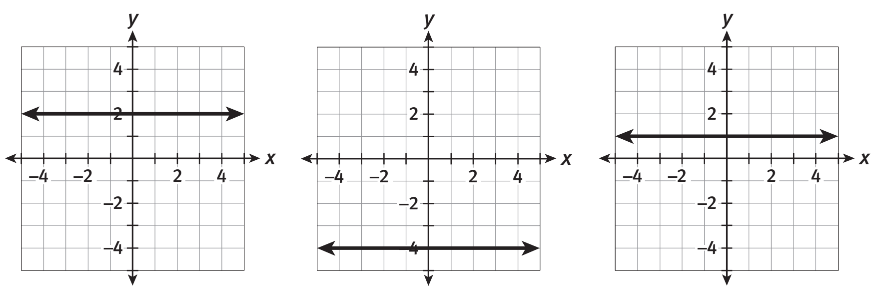
My Notes

5. a. Determine the slopes of the lines shown.



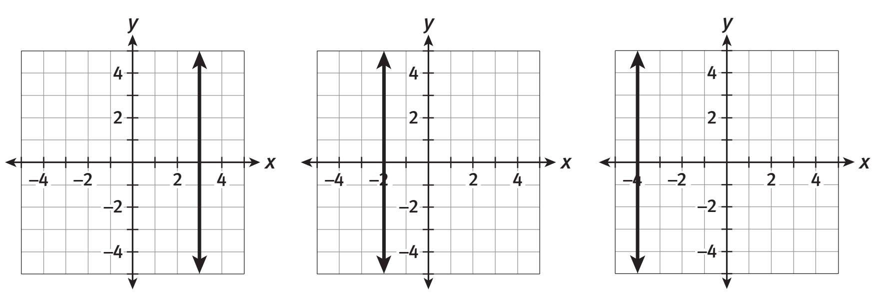
b. Express regularity in repeated reasoning. Describe the slope of any line that falls as you view it from left to right.

6. a. Determine the slopes of the lines below.



b. What is the slope of a horizontal line?

7. a. Determine the slopes of the lines shown.



b. What is the slope of a vertical line?

My Notes

8. Summarize your findings in Items 4–7. Tell whether the slopes of the lines described in the table below are positive, negative, 0, or undefined.

Up from left to right	Down from left to right	Horizontal	Vertical

Check Your Understanding

9. Suppose you are given several points on the graph of a function. Without graphing, how could you determine whether the function is linear?

10. How can you tell from a graph if the slope of a line is positive or negative?

11. Describe a line having an undefined slope. Why is the slope undefined?

LESSON 9-3 PRACTICE

12. **Make use of structure.** Sketch a line for each description.
a. The line has a positive slope.
b. The line has a negative slope.
c. The line has a slope of 0.

13. Does the table represent a linear function? Justify your answer.

x	*y*
1	−1
4	9
7	19
11	29

14. Are the points (12, 11), (2, 7), (5, 9), and (1, 5) part of the same linear function? Explain.

ACTIVITY 9 PRACTICE

Write your answers on notebook paper. Show your work.

Lesson 9-1

1. Find $\triangle x$ and $\triangle y$ for each of the following pairs of points.
  a. (2, 6), (−6, −8)
  b. (0, 9), (4, −8)
  c. (−3, −3), (7, 10)

For Items 2 and 3, use the table to calculate the slope.

2.

x	*y*
−5	−1
0	2
5	5
10	8

3.

x	*y*
−4	20
−3	14
0	−4
2	−16

4. Two points on a line are (−10, 1) and (5, −5). If the *y*-coordinate of another point on the line is −3, what is the *x*-coordinate?

For Items 5–7, determine the slope of the line that passes through each pair of points.

5. (−4, 11) and (1, −9)

6. (−10, −3) and (−5, 1)

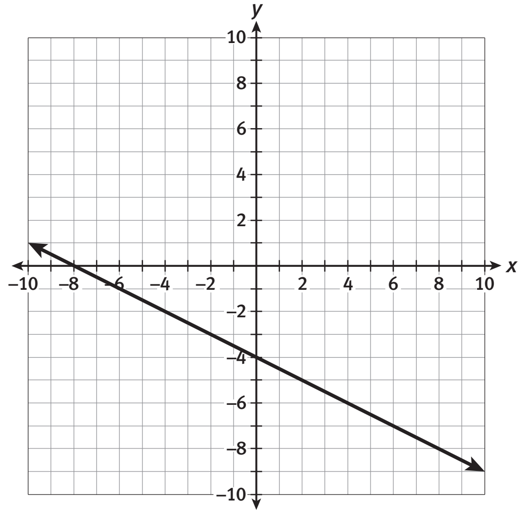
7. (−2, −7) and (−8, −4)

8. Are the three points (2, 3), (5, 6), and (0, −2) on the same line? Explain.

9. Which of the following pairs of points lies on a line with a slope of $-\frac{3}{5}$?
  A. (4, 0), (−2, 10)
  B. (4, 2), (10, 4)
  C. (−4, −10), (0, −2)
  D. (10, −2), (0, 4)

For Item 10, determine the slope of the line that is graphed.

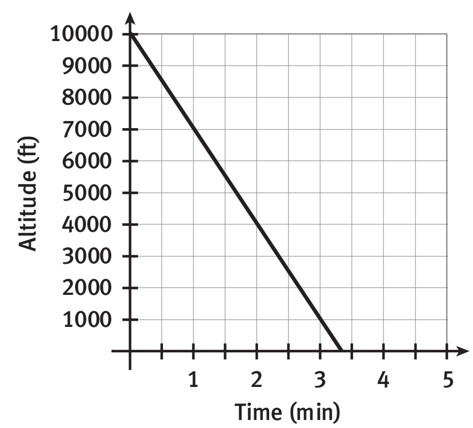
10.



Lesson 9-2

11. Juan earns $7 per hour plus $20 per week making picture frames.
  a. Write a function *g*(*x*) for Juan's total earnings if he works *x* hours in one week.
  b. Without graphing the function, determine the slope.
  c. Describe the meaning of the slope within the context of Juan's job.

12. The graph shows the height of an airplane as it descends to land.



  a. Does the function have a constant rate of change? If so, what is it?
  b. What is the slope of the line?
  c. How are the rate of change and the slope of the line related?
  d. Describe the meaning of the slope within the context of the situation.

Lesson 9-3

For Items 13–15, tell whether the function is linear. Justify your response.

13.

x	y
−3	44
−1	4
0	−1
1	4

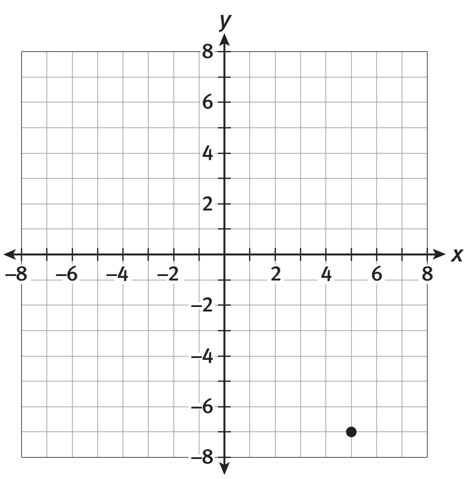
14.

x	y
−5	−7
0	−8
5	−9
10	−10

15.

x	y
4	−30
6	−46
8	−62
9	−70

16. One point on the line described by $y = -2x + 3$ is shown below. Use your knowledge of slope to give the coordinates of three more points on the line.

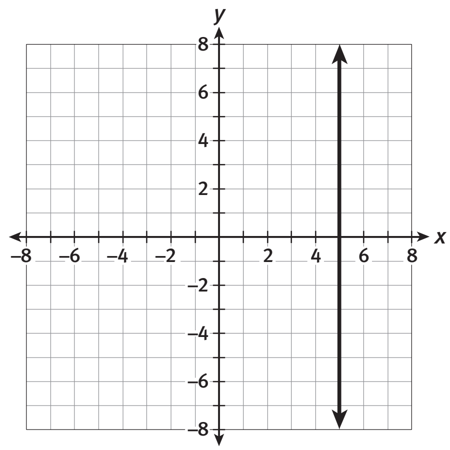


17. Which of the following is **not** a linear function?

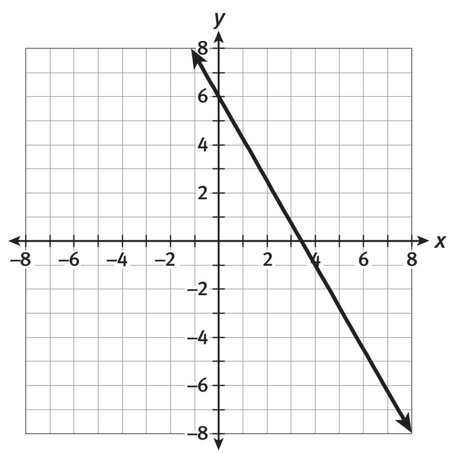
A. (4, −6), (7, −12), (8 −14), (10, −18), (2, −2)
B. (−2, −6), (1, 0), (4, −30), (0, 2), (7, −96)
C. (−4, 9), (0, 7), (2, 6), (6, 4), (8, 3)
D. (2, 18), (6, 50), (−3, −22), (0, 2), (3, 26)

For Items 18 and 19, identify the slope of the line in each graph as positive, negative, 0, or undefined.

18.



19.



20. The slope of a line is 0. It passes through the point (−3, 4). Identify two other points on the line. Justify your answers.

MATHEMATICAL PRACTICES
Look For and Make Use of Structure

21. Describe three different ways to determine the slope of a line and the similarities and differences between the methods.