

Extensions of Linear Concepts

3

Unit Overview

In this unit, you will extend your study of linear concepts to the study of piecewise-defined functions and systems of linear equations and inequalities. You will learn to solve systems of equations and inequalities in a variety of ways.

Key Terms

As you study this unit, add these and other terms to your math notebook. Include in your notes your prior knowledge of each word, as well as your experiences in using the word in different mathematical examples. If needed, ask for help in pronouncing new words and add information on pronunciation to your math notebook. It is important that you learn new terms and use them correctly in your class discussions and in your problem solutions.

Math Terms

- piecewise-defined function
- linear inequality
- solutions of a linear inequality
- boundary line
- half-plane
- closed half-plane
- open half-plane
- system of linear equations
- substitution method
- elimination method
- parallel
- coincident
- independent
- dependent
- inconsistent
- consistent
- system of linear inequalities
- solution region

ESSENTIAL QUESTIONS



Why would you use multiple representations of linear equations and inequalities?



How are systems of linear equations and systems of linear inequalities useful in analyzing real-world situations?

EMBEDDED ASSESSMENTS

These assessments, following Activities 16 and 18, will give you an opportunity to demonstrate what you have learned about piecewise-defined functions, inequalities, and systems of equations and inequalities.

Embedded Assessment 1:

Graphing Inequalities and Piecewise-Defined Functions p. 249

Embedded Assessment 2:

Systems of Equations and Inequalities p. 283

Getting Ready

Write your answers on notebook paper.
Show your work.

1. Which of the following tables of values represents linear data?

A.

x	y
2	3
4	6
6	6
8	3

B.

x	y
1	1
2	4
3	7
4	10

C.

x	y
2	3
3	4
4	3
6	4

D.

x	y
1	3
2	6
3	12
4	24

2. Give an algebraic representation of these data.

x	1	4	7	10
y	1	7	13	19

3. You open a savings account with a deposit of \$100. In each month after your initial deposit, you add \$25 to your account. Provide a table to display the total amount of money in the account after 1, 2, 3, and 4 months. Give a graphical and an algebraic representation that would allow you to determine the amount of money A in dollars you have deposited after m months.

4. Graph $2x + 3y = 4$.

5. Describe the graph of $y = 3$.

6. Which ordered pair is a solution of $y > x + 5$?

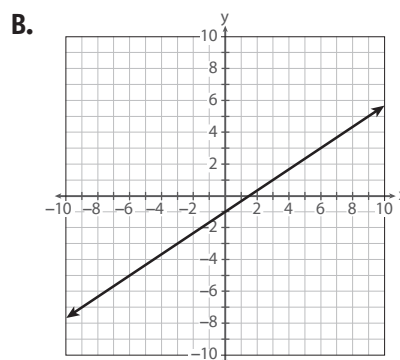
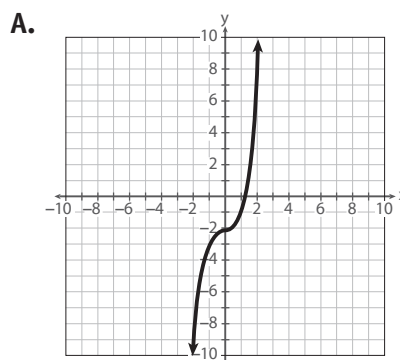
- A. (2, 8) B. (-5, 0)
C. (1, 6) D. (0, -5)

7. Compare and contrast the graphs of these two compound statements:

$$-1 < x \leq 3$$

$$x < -1 \text{ or } x \geq 3$$

8. Which of the following represents a function with a constant rate of change?



C. $y = \frac{4}{x}$

D.

x	1	4	16	64
y	5	10	15	20

My Notes

3. **Model with mathematics.** Let w be a dog's weight in pounds and let $A(w)$ be the amount of food in ounces that the dog should be fed each day. Use the table in Item 1 and function notation to write a function that expresses $A(w)$ in terms of w .

After several hours, all of the dogs had finished their food except for Hercules. The vet asked, "How much food did you give Hercules?"

After hearing Miriam's answer, the vet told Miriam that she had overfed Hercules and suggested that Miriam check the complete feeding chart posted on the office wall.

Barko Dog Food Feeding Chart

Weight of Dog (pounds)	Daily Amount of Barko (ounces)
3	3
12	12
20	20
50	35
100	60
over 100 pounds	60 ounces plus 1 ounce for each additional 10 pounds of weight

4. Miriam had assumed that each dog should be fed as many ounces of dog food as the dog weighed in pounds. Explain why the data in the new chart indicate that her original assumption was not correct.

Adult dogs that weigh less than 20 pounds are classified as *small* dogs. Dogs that weigh 20 to 100 pounds are classified as *mid-size* dogs. Finally, dogs that weigh more than 100 pounds are classified as *large* dogs. Miriam had not known that the formulas for feeding small, mid-size, and large dogs are all different.

Lesson 14-1

Function Notation and Rate of Change

ACTIVITY 14

continued

- The vet tells Miriam that the data for feeding mid-size dogs are linear. Verify the vet's statement using the feeding chart.
- Determine a rate of change that describes the number of additional ounces of food a mid-size dog should be fed for each pound of weight greater than 20. Explain how you found your answer.
- Using the rate of change you found in Item 6 and the feeding chart, use function notation to write a function that expresses $A(w)$, the ounces of dog food, in terms of w , the dog's weight in pounds, for mid-size dogs.
- How many ounces was Hercules overfed? Justify your response.

Check Your Understanding

Use the table for Items 9 and 10.

Grapes (pounds)	Cost (dollars)
0.5	\$1.20
1.5	\$3.60
2.0	\$4.80

- Determine the rate of change of the data. Express the rate as cost per pound.
- Do the data in the table represent a linear relationship? Explain.
- Tom has read the first 40 pages of a book. He plans to read another 12 pages each day. Use function notation to write a function that expresses the number of pages read p after d days.

My Notes

My Notes

LESSON 14-1 PRACTICE

12. Todd had five gallons of gasoline in his motorbike. After driving 100 miles, he has three gallons left. What is Todd's rate of change in miles per gallon?
13. This table shows how the balance in John's savings account has changed over the course of a year.

Month	Balance (dollars)
1	\$ 400
3	\$ 600
7	\$1000
10	\$1300
12	\$1500

How much did John save per month during the year?

14. Linda purchased a house for \$144,000. Thinking of possibly refinancing after 11 years, she had her house appraised and found that it is now worth \$245,000. Find the rate of change of the value of the house in dollars per year.
15. **Make sense of problems.** The cost in dollars of producing x vehicles for a company is given by $C(x) = 1200x + 5500$. Interpret the rate of change of this linear function.
16. A 500-liter tank full of oil is being drained at the constant rate of 20 liters per minute. Use function notation to write a linear function expressing the number of liters in the tank V after t minutes.

Learning Targets:

- Write linear equations in two variables given a table of values, a graph, or a verbal description.
- Determine the domain and range of a linear function, determine their reasonableness, and represent them using inequalities.

SUGGESTED LEARNING STRATEGIES: Identify a Subtask, Think-Pair-Share, Create Representations, Construct an Argument, Interactive Word Wall

1. Rewrite the function that you wrote in Item 7 in Lesson 14-1.
2. a. Remember that this function is true only for mid-size dogs. Describe the appropriate input values for the function.

- b. Use your description of the appropriate input values to express the domain of the function using set notation.

domain = { w : _____ }

Miriam knows that she also has several large dogs to feed. When she looks at the chart, she reads the instruction “60 ounces plus 1 ounce for each additional 10 pounds of weight.”

3. How much additional dog food should large dogs be fed for each pound of weight greater than 100 pounds?
4. **Critique the reasoning of others.** A 140-pound Great Dane has arrived for a short stay at the kennel. Miriam says that the dog should be fed 64 ounces of food per day. Her friend Chase says that the dog requires 100 ounces of food per day. Who is correct? Justify your response.

My Notes

My Notes																			

My Notes

- Write a function that expresses the amount of food, $A(w)$, in ounces that a large dog should be fed, as a function of w , the weight of the dog in pounds. Determine the domain and range of the function.

Miriam realizes that there are three different algebraic feeding rules to follow because dogs are different sizes. She organizes her feeding rules in a list so that she can quickly refer to them whenever she has to decide how much food to feed a dog.

- Complete Miriam's list by writing the appropriate function. Indicate the domain by writing the appropriate inequality symbols.

$A(w) = \underline{\hspace{2cm}}$, when $0 \underline{\hspace{1cm}}$ $w \underline{\hspace{1cm}}$ 20

$A(w) = \underline{\hspace{2cm}}$, when $20 \underline{\hspace{1cm}}$ $w \underline{\hspace{1cm}}$ 100

$A(w) = \underline{\hspace{2cm}}$, when $w \underline{\hspace{1cm}}$ 100

- The vet has a German Shepard named Max, and Miriam knows that the vet feeds Max 63 ounces of food each day. Miriam also knows that the vet feeds her cocker spaniel Min 32 ounces of food each day. If the two dogs are being correctly fed, what is each dog's weight? Explain your reasoning.

Lesson 14-2

Writing Functions and Finding Domain and Range

ACTIVITY 14

continued

Miriam decides to make a table that lists the weight of the dogs she will be feeding, in the order that she will feed them. A portion of Miriam's table is shown below.

8. Complete the table using the rules you wrote in Item 6.

Weight of Dog (pounds)	Amount of Dog Food (ounces)
21	
70	
16	
120	
15	
56	
27	
107	
7	
25	

The functions in Item 6 can be written as **piecewise-defined functions**. Every possible weight w has exactly one feeding amount A assigned to it, but the rule for determining that feeding amount changes for dogs of different sizes. The different feeding rules, along with their domains, are considered to be the pieces of a single piecewise-defined function.

9. Miriam decides to write a piecewise-defined function to represent the functions she wrote in Item 6.
- a. Explain how you know that each equation in Item 6 represents a function.

- b. Complete the piecewise-defined function for the feeding rules.

$$A(w) = \begin{cases} w, & \text{_____} \\ \text{_____}, & \text{when } 20 \leq w \leq 100 \\ 0.1w + 50, & \text{when } \text{_____} \end{cases}$$

- c. Explain why the equation from Part (b) is also a function. Use the domain of each rule to help justify your answer.

My Notes

MATH TERMS

A **piecewise-defined function** is a function that is defined differently for different intervals in its domain.

My Notes

Check Your Understanding

A wholesale grocery store has the following sale on mixed nuts:

<p>SALE! Mixed nuts Less than 4 lbs: \$2/lb 4–10 lbs: \$6 + \$0.50/lb</p>

10. Write a function to represent the cost $C(x)$ of buying less than four pounds of mixed nuts. Identify the domain and range.
11. Write a function to represent the cost $C(x)$ of buying 4 to 10 pounds of mixed nuts. Identify the domain and range.
12. Make a table to show the cost of 4, 5, and 8 pounds of mixed nuts.
13. Use your answers to Items 10 and 11 to write a piecewise-defined function to represent the cost $C(x)$ of x pounds of mixed nuts.

LESSON 14-2 PRACTICE

14. Speed Cell Wireless offers a plan of \$40 for the first 400 minutes, and an additional \$0.50 for every minute over 400. Let t represent the total talk time in minutes. Write a piecewise-defined function to represent the cost $C(t)$.
15. If Pam works more than 40 hours per week, her hourly wage for every hour over 40 is 1.5 times her normal hourly wage of \$7. Write a piecewise-defined function that gives Pam's weekly pay $P(h)$ in terms of the number h of hours she works.
16. A man walks for 45 minutes at a rate of 3 mi/h, then jogs for 75 minutes at a rate of 5 mi/h, then rests for 30 minutes, and finally walks for 90 minutes at a rate of 3 mi/h. Write a piecewise-defined function expressing the distance $D(t)$ he traveled as a function of time t .
17. **Model with mathematics.** A state income tax law reads as follows:

Annual Income (dollars)	Tax
$< \$2000$	\$0 tax
\$2000 – \$6000	2% of income over \$2000
$> \$6000$	5% of income over \$6000

Write a piecewise-defined function to represent the income tax law.

Learning Targets:

- Evaluate a function at specific inputs within the function’s domain.
- Graph piecewise-defined functions.

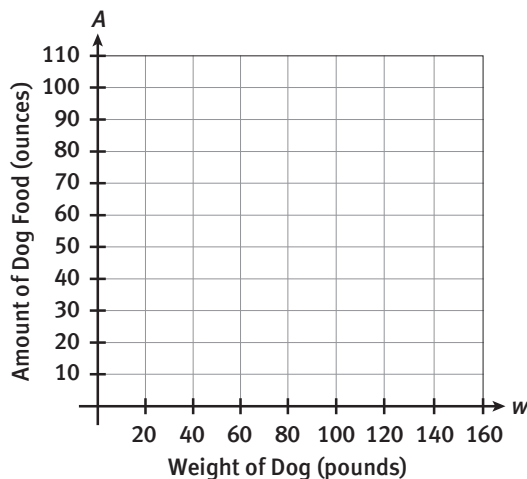
SUGGESTED LEARNING STRATEGIES: Identify a Subtask, Think-Pair-Share, Construct an Argument, Create Representations, Discussion Groups

1. Rewrite the piecewise-defined feeding function that you wrote in Item 9 in Lesson 14-2.

2. Miriam must feed a dog that weighs 57 pounds.
 - a. Which piece of the feeding function should Miriam use? Explain your answer.

 - b. Use your answer to Part (a) to evaluate $A(57)$.

3. **Make use of structure.** When graphing a piecewise-defined function, it is necessary to graph each piece of the function only for its appropriate interval of the domain. Graph the feeding function on the axes below. When finished, your graph should consist of three line segments.



My Notes

CONNECT TO AP

This is a connection to the concept of continuity in AP Calculus.

TECHNOLOGY TIP

To graph a piecewise-defined function in your calculator, enter the function into $Y=$, in dot mode, using parentheses to indicate the domain intervals. For example

$$f(x) = \begin{cases} -x^2 - 1, & \text{when } x < 1 \\ x + 2, & \text{when } x \geq 1 \end{cases}$$

would be entered as

$$Y_1 = (-x^2 - 1)(x < 1) + (x + 2)(x \geq 1).$$

My Notes

4. Why is the graph of a piecewise-defined function more useful than three separate graphs?
5. How can you conclude that the graph represents a function?

Check Your Understanding

Use the piecewise-defined function for Items 6 and 7.

$$f(x) = \begin{cases} x + 1, & \text{when } -3 < x \leq 1 \\ 2x, & \text{when } 1 < x \leq 4 \\ x - 1, & \text{when } 4 < x < 10 \end{cases}$$

6. Find $f(1)$ and $f(4)$. Explain which piece of the function you used to find each value.
7. Sketch a graph of the function.

LESSON 14-3 PRACTICE

8. **Model with mathematics.** Graph the piecewise-defined function that you wrote in Item 15 in Lesson 14-2 to describe Pam's pay.

For Items 9 and 10, graph each piecewise-defined function.

9. $f(x) = \begin{cases} 2x, & \text{when } x > 0 \\ x, & \text{when } x \leq 0 \end{cases}$

10. $f(x) = \begin{cases} \frac{1}{2}x + \frac{3}{2}, & \text{when } x < 1 \\ -x + 3, & \text{when } x \geq 1 \end{cases}$

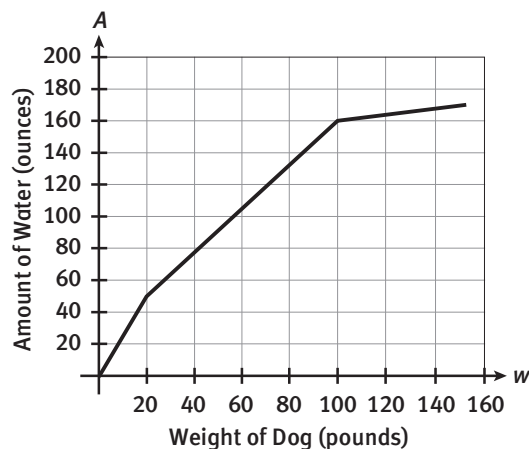
For Items 11 and 12, consider the piecewise-defined function

$$f(x) = \begin{cases} \frac{1}{2}x - 10, & \text{when } x \leq 3 \\ -x - 1, & \text{when } x > 3 \end{cases}$$

11. Find $f(-4)$.
12. Find $f(6)$.

My Notes

The graph below shows how many ounces of water a dog should drink daily, based on the dog's weight.



4. How much water should a 20-pound dog consume each day?

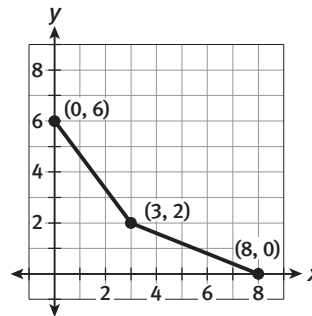
5. How much water should a 100-pound dog consume each day?

6. Compare the water function with the feeding function. Describe the similarities and differences.

7. Write a piecewise-defined function for the graph.

LESSON 14-4 PRACTICE

11. Write a piecewise-defined function for the graph shown.



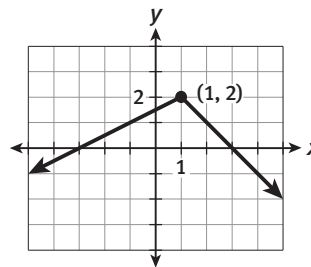
12. Reason abstractly. Compare the function in Item 11 to the function

$$f(x) = \begin{cases} 2x + 1, & \text{when } x < 1 \\ -x + 4, & \text{when } x \geq 1 \end{cases}$$

Describe the similarities and differences.

13. Graph the function $f(x) = \begin{cases} 2x + 8, & \text{when } x < -2 \\ 4, & \text{when } -2 \leq x < 2 \\ -2x + 8, & \text{when } x \geq 2 \end{cases}$. State the domain and range.

14. How does the function in Item 13 compare to the function shown in the graph below?



ACTIVITY 14 PRACTICE

Write your answers on notebook paper.
Show your work.

Lesson 14-1

Use the tables for Items 1–5.

x	y
0	2
1	3
2	4
3	5
4	6
5	7

x	y
6	-5
7	-6
8	-7
9	-8
10	-9
11	-10

- Find the rate of change for each table.
- At what x -value does the rate of change switch patterns?
- Find a function that represents the first table of data, and write the domain for the function.
- Find a function that represents the second table of data, and write the domain for the function.

Lesson 14-2

- Write the piecewise-defined function that represents the data in the tables.
- A store is having a special sale on designer soaps. For every three bars of soap purchased, one is given for free. The bars of soap cost \$2 each, and there is a limit of eight bars of soap per customer (including free ones).
 - Write a piecewise-defined function $C(b)$ that gives the total cost C for b bars of soap.
 - Write the domain for this function in terms of the context.
- A museum has the following prices for admission:

Children under 12: free

Kids age 12–17: \$3

Adults age 18–64: \$8

Senior citizens 65 and over: \$4

Write a piecewise-defined function that gives the cost $C(n)$ for a museum visitor who is n years old. What is the range of the function?

Lesson 14-3

- A piecewise-defined function is shown.

$$h(x) = \begin{cases} x + 3, & \text{when } x < 3 \\ bx, & \text{when } x \geq 3 \end{cases}$$

What value of b is needed so that $h(3) = 6$?

- A. 1 B. 2
C. 3 D. 6

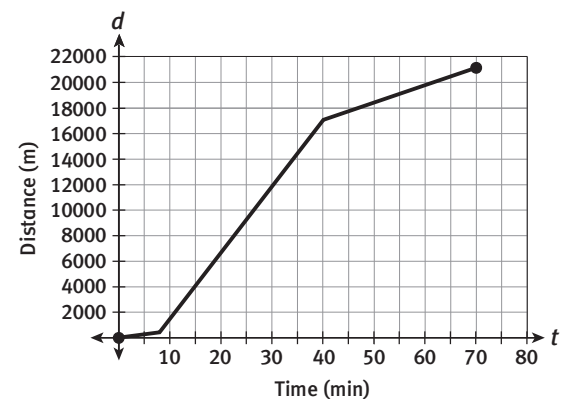
- Sketch a graph of the function.

$$f(x) = \begin{cases} x, & \text{when } x < 2 \\ -x + 4, & \text{when } x \geq 2 \end{cases}$$

- Ashley participated in a triathlon.
 - She swam for 10 minutes at a rate of 40 meters/min.
 - Then she biked for 40 minutes at a rate of 400 meters/min.
 - Finally, she ran for 25 minutes at a rate of 200 meters/min.
 - Write a piecewise-defined function expressing the distance $d(t)$ in meters that Ashley traveled as a function of time t in minutes.
 - Graph your function from Part (a).

Lesson 14-4

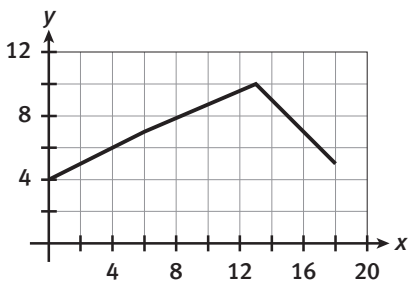
- Refer back to Item 10. Wanda competed in the same triathlon as Ashley. A graph of the distance that Wanda covered as a function of time is shown below.



- Tell who completed each leg of the triathlon (swimming, biking, and running) faster, Ashley or Wanda. Justify your answers.
- Who completed the triathlon first, Ashley or Wanda? Justify your answer.

ACTIVITY 14*continued***Piecewise-Defined Linear Functions**
Breakfast for Bowser

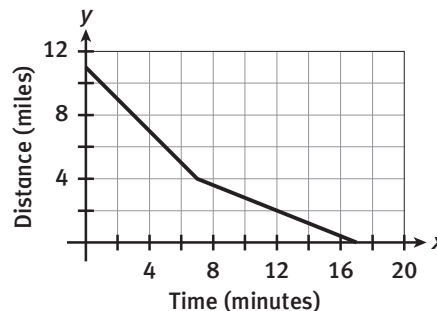
Use the graph for Items 12–15.



12. What is the domain of the piecewise-defined function?
- A. $0 \leq x \leq 18$ B. $0 \leq x < 10$
 C. $4 \leq x \leq 10$ D. $x \neq 6$
13. What is the value of the function for $x = 6$?
- A. 5 B. 6
 C. 7 D. 10
14. Which of the following defines the function for the domain $0 \leq x < 6$?
- A. $y = 0.5x$ B. $y = 0.5x + 4$
 C. $y = 2$ D. $y = -x + 2$
15. What is the range of the piecewise-defined function?
- A. $0 \leq y \leq 18$ B. $0 \leq y \leq 10$
 C. $4 \leq y \leq 10$ D. $y \neq 7$

A function gives Tanya's distance y (in miles) from home x minutes after she leaves her friend's house.

Use the graph of the function for Items 16–18.



16. What are the domain and range of the function?
- A. $0 \leq x \leq 11, 0 \leq y \leq 17$
 B. $0 \leq x \leq 17, 0 \leq y \leq 11$
 C. $0 \leq x \leq 7, 0 \leq y \leq 4$
 D. $0 \leq x \leq 4, 0 \leq y \leq 7$
17. Which statement is true?
- A. After 7 minutes, Tanya's average speed increased.
 B. After 7 minutes, Tanya's average speed decreased.
 C. After 4 minutes, Tanya's average speed increased.
 D. After 4 minutes, Tanya's average speed decreased.
18. How far does Tanya live from her friend's house?
- A. 4 miles B. 7 miles
 C. 11 miles D. 17 miles

MATHEMATICAL PRACTICES**Attend to Precision**

19. Explain why a restricted domain and a restricted range are sometimes appropriate for a piecewise-defined function. Describe instances when the domain and range need not be restricted.

Comparing Equations

A Tale of a Trucker

Lesson 15-1 Writing Equations from Graphs and Tables

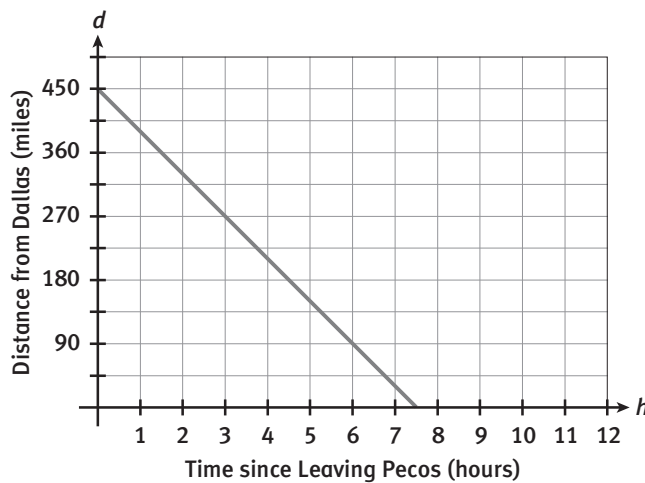
Learning Targets:

- Write a linear equation given a graph or a table.
- Analyze key features of a function given its graph.

SUGGESTED LEARNING STRATEGIES: Summarizing, Visualization, Look for a Pattern, Create Representations, Discussion Groups, RAFT

Travis Smith and his brother, Roy, are co-owners of a trucking company. One of their regular weekly jobs is to transport fruit grown in Pecos, Texas, to a Dallas, Texas, distributing plant. Travis knows that his customers are concerned about the speed with which the brothers can deliver the produce, because fruit will spoil after a certain length of time.

1. Travis wants to address his customers' concerns with facts and figures. He knows that a typical trip between Pecos and Dallas takes 7.5 hours. He makes the following graph.



- a. What information does the graph provide?
- b. Locate the point with coordinates of (3, 270) on the graph. Label it point A. Describe the information these coordinates provide.

My Notes

CONNECT TO BUSINESS

Companies that ship fruit from distribution plants to stores around the country use refrigerated trucks to keep the fruit fresher longer. For example, the optimal temperature range for shipping cantaloupes is 36–41°F. What would the temperature range look like on a number line graph?

My Notes

- c. According to the graph, how many hours will it take Travis to reach Dallas from Pecos? Explain how you determined your answer.

- d. Interstate 20 is the direct route between Pecos and Dallas. Based upon his graph, at what average speed does Travis expect to travel? Explain how you determined your answer.

2. Complete the table to show Travis's distance from Dallas at each hour.

Hours Since Leaving Pecos, h	Distance from Dallas, d
0	
1	
2	
3	
4	
5	
6	
7	

WRITING MATH

The graph falls from left to right, and the numbers in the second column of the table decrease. When you write the equation, the coefficient of h should be negative.

- 3. **Model with mathematics.** Use your table and the graph to write an equation that expresses Travis's distance d from Dallas as a function of the number of hours h since he left Pecos.

- 4. The graph of an equation in two variables is the set of all solutions of the equation plotted in the coordinate plane.
 - a. The graph appears to pass through the point (6, 90). Use your equation to verify this.

Lesson 15-1

Writing Equations from Graphs and Tables

ACTIVITY 15

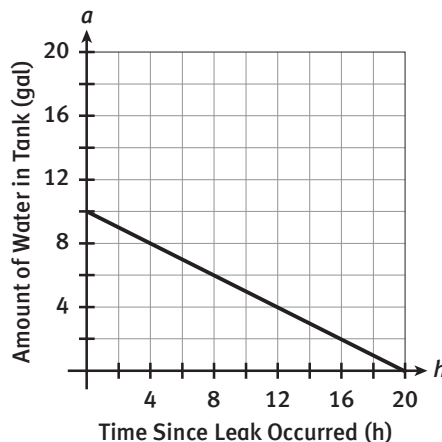
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- b. Generate four more ordered-pair solutions to your equation by substituting 1, 5, 8, and 10 for h . Does each point appear on the graph showing Travis's distance from Dallas? Explain.
- c. Which of the solution points you generated in Part (b) make sense in this context?
- d. Explain why only some solution points make sense in this context.
- e. Use what you know about the solution points and the graph to state a reasonable domain and range for the function in the given context.

My Notes

Check Your Understanding

The graph shows the amount of water remaining in a tank after a leak occurs. Use the graph for Items 5–7.



5. Make a table to show the amount of water in the tank at five different times.
6. **Make use of structure.** Use your table and the graph to write an equation that expresses the amount of water in the tank a as a function of the number of hours h since the leak occurred.
7. State a reasonable domain and range for the function in the given context.

My Notes

LESSON 15-1 PRACTICE

A farm in Plainville, Texas, will ship fruit to the distribution plant in Dallas. The farm is 350 miles from the plant.

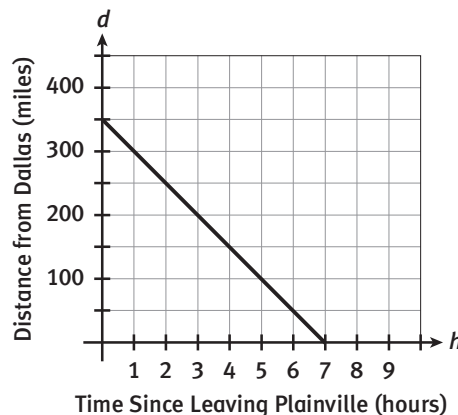
8. The table shows part of a trip that Travis and Roy made from the farm to the distribution plant.

Distance from Dallas After Leaving Plainville

Time h (hours)	Distance d (miles)
2	230
2.5	200
3	170
3.5	140

Determine and interpret the rate of change of the data in the table.

9. A competing shipping company guarantees the fastest delivery times. The graph shows a recent trip from the farm to the distribution plant.



Determine and interpret the rate of change of the data in the graph.

10. Suppose both companies leave the farm at the same time and head to Dallas. Predict who will arrive first.
11. Write equations to represent the trip for each company. Determine how long it takes each company to drive from the farm to the plant in Dallas.
12. **Construct viable arguments.** Suppose you work at the farm. Write a memo to your supervisor about which trucking company to use and why.

MATH TIP

When writing your answer to Item 12, you can use a RAFT.

- Role—farm worker
- Audience—your supervisor
- Format—a memo
- Topic—which trucking company to use and why

My Notes

3. Travis can average anywhere from 45 mi/h to 60 mi/h, as shown on the two previous graphs.
- On the graph in Item 1, sketch the vertical line $h = 3$. Highlight the segment of that line that gives all the possible distances from Dallas three hours after Travis leaves Pecos.
 - How far might Travis be from Dallas after 3 hours of travel? Explain in your own words and then write your answer as an inequality.

The distances you found in Item 3b are the possible distances after three hours of travel. The line segment can be described as the set of ordered pairs (h, d) where $h = 3$ and the value of d is between the values indicated in the inequality you wrote in Item 3b.

4. Travis can average anywhere from 45 mi/h to 60 mi/h, as shown on the graph in Item 1.
- On the graph, sketch the vertical line $h = 5$. Highlight the segment of that line that gives all the possible distances from Dallas five hours after Travis leaves Pecos.
 - Reason quantitatively.** How far might Travis be from Dallas after 5 hours of travel? Explain in your own words, and then write your answer as an inequality.
 - Describe what the inequality and the line segment tell Travis about his trip.
5. Use the graph in Item 1.
- Draw the line segment for $h = 7.5$. How far might Travis be from Dallas after 7.5 hours of travel? Write your answer as an inequality.
 - Describe what the inequality and the line segment tell Travis about his trip.

WRITING MATH

If an inequality includes the greatest and least numbers, write the inequality using \leq or \geq . If the greatest and least numbers are not included, write the inequality using $<$ or $>$.

Lesson 15-2
Comparing Functions with Inequalities

ACTIVITY 15
continued

My Notes

6. Use the graph in Item 1.
 - a. Draw the line segment for $h = 9$. How far might Travis be from Dallas after 9 hours of travel? Write your answer as an inequality.

 - b. Describe what the inequality and the line segment tell Travis about his trip.

7. There is a region that could be filled by similar vertical line segments for all values of h that Travis could be on his trip. Shade this region on the graph in Item 1.

8. Travis likes to stop for a break after driving half the distance to Dallas.
 - a. On the graph in Item 1, sketch the horizontal line that represents half the distance to Dallas. What is the equation of this line? Highlight the segment of the line that gives all possible times that he may stop.

 - b. How much time might remain in the trip when Travis stops for a break? Explain how you know.

9. After Travis drives for 4 hours and 20 minutes at 45 mi/h, he wonders how much farther he has to drive.
 - a. Would you prefer to use the graph or the equation to determine the answer? Explain your choice.

 - b. How much farther does Travis have to drive? Justify your response.

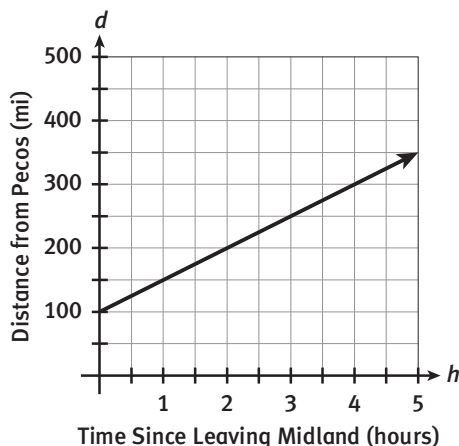
My Notes

Check Your Understanding

10. Explain how you can use the graph in Item 1 to determine how far Travis might be from Dallas after 6 hours of travel.
11. Assuming Travis can average anywhere from 45 mi/h to 60 mi/h, is it possible for him to be 300 miles from Dallas after 2 hours of travel? Justify your answer.
12. Suppose Travis maintains an average speed of 50 mi/h throughout his trip. Describe how the graph of his distance from Dallas would compare to the graph in Item 1.

LESSON 15-2 PRACTICE

Travis's sister, Amy, lives in Midland, which is 100 miles from Pecos. After visiting his sister, Travis plans to drive his truck to Dallas. The graph shows his planned trip.



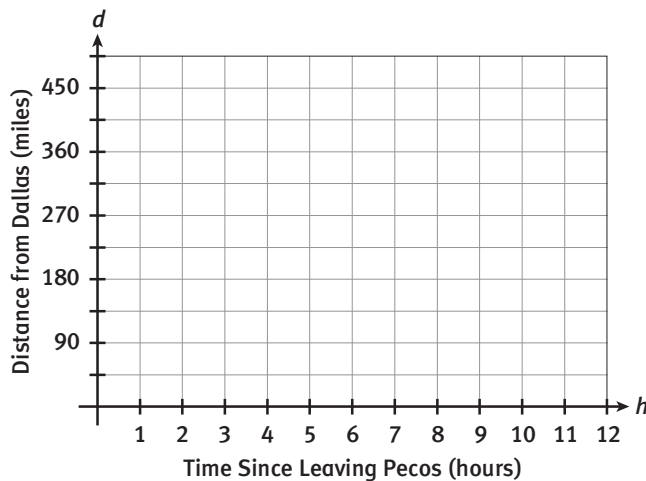
13. At what average speed does Travis expect to travel?
14. Travis hopes he will be able to maintain an average speed of 60 mi/h once he leaves his sister's house. Copy the graph above and draw the graph of Travis's distance from Pecos, based on an average speed of 60 mi/h.
15. **Make sense of problems.** Write an equation for the line you graphed in Item 14. Interpret the meanings of the constant and the coefficient of h in terms of the context.
16. How far might Travis be from Pecos after 3 hours of travel? Write your answer as an inequality.

Learning Targets:

- Write a linear equation given a verbal description.
- Graph and analyze functions on the same coordinate plane.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Create Representations, Identify a Subtask

1. Suppose that the speed limit on all parts of Interstate 20 has been changed to 70 mi/h. Travis finds that he can now average between 50 mi/h and 70 mi/h on the trip between Pecos and Dallas.
 - a. Write an equation that expresses Travis’s distance d from Dallas as a function of the hours h since he left Pecos if his average speed is 50 mi/h.
 - b. Write an equation that expresses Travis’s distance d from Dallas as a function of the hours h since he left Pecos if his average speed is 70 mi/h.
 - c. **Reason abstractly.** On the grid below, graph the two equations that you found in Parts (a) and (b). Describe how changing the coefficient of h in the equation affects the graph. Explain why this makes sense.



- d. Shade the region of the graph above for the ordered pairs (h, d) such that h represents all the possible times and d represents all the possible distances from Dallas after h hours of travel.

My Notes

My Notes

- e. Describe Travis's possible distances from Dallas after 4 hours of travel.
- f. Write an inequality to represent Travis's possible distances from Dallas after 4 hours.

Check Your Understanding

Sumi and Jerome start riding their bikes at the same time, traveling in the same direction on the same bike path. Sumi rides at a constant speed of 10 ft/s. Jerome rides at a constant speed of 15 ft/s.

2. Write an equation that expresses the distance d that Sumi has traveled as a function of the time s in seconds since she started riding her bike.
3. Write an equation that expresses the distance d that Jerome has traveled as a function of the time s in seconds since he started riding his bike.
4. Graph the equations from Items 2 and 3 on the same coordinate plane.
5. Explain how to use your graph to find the distance between Sumi and Jerome after they have ridden their bikes for 5 seconds.

LESSON 15-3 PRACTICE

Graph the equations for Items 6 and 7 in the first quadrant of the same graph. The equations represent the height y in centimeters of Ellie's and James's model gliders after x seconds.

6. Ellie: $y = -20x + 220$
7. James: $y = -15x + 220$
8. Draw a vertical line segment that connects the graphs at $x = 5$. Describe what the segment represents in words and with an inequality.
9. Interpret the meanings of the coefficients of x and the constants in terms of the context.
10. **Construct viable arguments.** Whose glider was in the air longer? Justify your response.
11. Julian's model glider begins at an initial height of 300 centimeters and descends at a rate of 18 cm per second. Write an equation that expresses the height y of Julian's glider after x seconds.

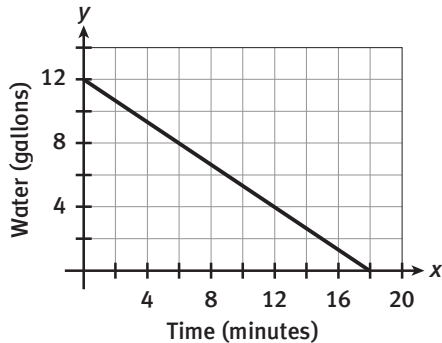
ACTIVITY 15 PRACTICE

Write your answers on notebook paper.

Show your work.

Lesson 15-1

- The graph shows the number of gallons of water y remaining in a tub x minutes after the tub began draining.



Which describes something that is being drained at nine times the rate of the tub?

- A pool is drained at a rate of nine gallons per minute.
 - A rain barrel is drained at a rate of six gallons per minute.
 - A cooler is drained at a rate of 12 gallons per minute.
 - A pond is drained at a rate of 18 gallons per minute.
- The equation $d = 30 - 60h$ gives the distance d in miles that a bus is from the station h hours after leaving an arena. How far is the station from the arena?

A. 0.5 miles	B. 30 miles
C. 60 miles	D. 120 miles
 - Claire walks home from a friend's house two miles away. She sketches a graph of her walk, showing her distance from home as a function of time in minutes. Is the slope of the graph positive or negative? Explain.

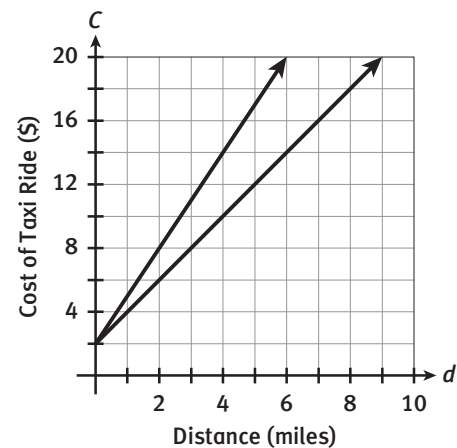
Use the table for Items 4–9.

Minutes Since Leaving Store, m	Distance in Miles from Home, d
10	1.5
15	1.25
20	1

- The table represents Holly's walk home from the store. What is Holly's rate in miles per minute?
- How far is the store from Holly's home? Explain how you know.
- Write an equation that relates distance d to the number of minutes m .

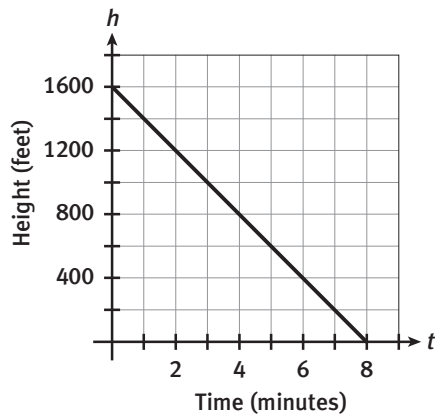
Lesson 15-2

- Tim left school on his bike at the same time Holly left the store. The equation $d = 4 - \frac{m}{5}$ gives Tim's distance from Holly's house after m minutes. Sketch a graph of Holly's and Tim's trips on the same coordinate plane.
- Compare the total time for Tim's trip to the total time for Holly's trip.
- Part of Tim's trip includes the way Holly will walk. Use your graph to estimate when Tim will run into Holly.
- Kane researched the cost of a taxi ride in a nearby city. He found conflicting information about the per-mile cost of a ride. The graph below shows his findings.



- What can you conclude about the cost per mile of a taxi ride?
- How much should Kane expect to pay for a five-mile taxi ride? Explain.

The graph shows the planned descent of a hot-air balloon. Use the graph for Items 11–16.



11. Write an equation that expresses the height of the balloon h as a function of the time t since the beginning of the descent.
12. State a reasonable domain and range for the function in Item 11 in this context.
13. The pilot thinks it may be possible to increase the rate of descent to a maximum of 400 ft/min. Copy the above graph and then draw a graph of the descent at 400 ft/min on the same coordinate plane.
14. On your graph from Item 13, draw the vertical line $t = 3$. Use the line to determine all the possible heights of the balloon after 3 minutes. Write your answer as an inequality.
15. The actual descent of the hot-air balloon is given by the equation $h = 1600 - 250t$. Compare the graph of the actual descent to the two lines graphed in Item 13.
16. How long does it take the hot-air balloon to make the descent? Explain your reasoning.

Lesson 15-3

17. Wendy leaves Sacramento and heads to San Jose. She averages 50 mi/h on the 120-mile trip. Which equation describes Wendy's distance d from San Jose h hours after she leaves Sacramento?
A. $d = 50h - 120$ **B.** $d = 120 - 50h$
C. $d = 120 + 50h$ **D.** $d = 50h$
18. Fresno is 150 miles from Bakersfield. A driver traveling to Bakersfield leaves Fresno and averages 62 miles per hour. Write an equation for the distance d from Fresno, given the time t , in hours since the driver left.
19. A driver traveling from Fresno to Bakersfield averaged 57 miles per hour. Write an equation for the distance d from Fresno, given the time t , in hours since the driver left.
20. Sketch a graph of the equations you created in Items 18 and 19 on the same coordinate axis.
21. If a driver traveling between Fresno and Bakersfield knew he could average between 57 and 62 miles per hour, sketch a graph of the region that represents all possible distances from Fresno that the driver could be at time t .

MATHEMATICAL PRACTICES

Make Sense of Problems and Persevere in Solving Them

22. The amount of water w in aquarium A, in gallons, is given by $w = 50 - 3.5m$, where m is the number of minutes since the aquarium started draining. The amount of water w in aquarium B, in gallons, m minutes after the aquarium started draining, is given by the table below. Which aquarium drains more quickly, and how long does it take until it is empty?

Minutes, m	Gallons of Water, w
0	50
5	40

Inequalities in Two Variables

ACTIVITY 16

Shared Storage

Lesson 16-1 Writing and Graphing Inequalities in Two Variables

Learning Targets:

- Write linear inequalities in two variables.
- Read and interpret the graph of the solutions of a linear inequality in two variables.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Look for a Pattern, Think-Pair-Share, Create Representations, Sharing and Responding

Axl and Aneesa pay for and share memory storage at a remote Internet site. Their plan allows them upload 250 terabytes (TB) or less of data each month. Let x represent the number of terabytes Axl uploads in a month, and let y represent the number of terabytes Aneesa uploads in a month.

1. Look at the data for four months below. Determine if Axl and Aneesa stayed within the limits of their monthly plan.

Axl's Uploads (x)	Aneesa's Uploads (y)	Within Plan? (Yes/No)
50 TB	120 TB	
135 TB	100 TB	
100 TB	150 TB	
162 TB	128 TB	

2. Write an inequality that models the plan's restriction on uploading.
3. **Reason quantitatively.** If Axl uploads 100 terabytes, write an inequality that shows how many terabytes Aneesa can upload.
4. If Aneesa uploads 75 terabytes, write an inequality that shows how many terabytes Axl can upload.

My Notes

ACTIVITY 16*continued***Lesson 16-1****Writing and Graphing Inequalities in Two Variables****My Notes****MATH TIP**

Some equations and inequalities are graphed on the coordinate plane, but contain only one variable.

$$x = 1$$

$$y < 3$$

Other equations and inequalities that are graphed on the coordinate plane contain two variables.

$$x + y = 10$$

$$y \geq 7 - x$$

MATH TERMS

A **linear inequality** is an inequality in the form

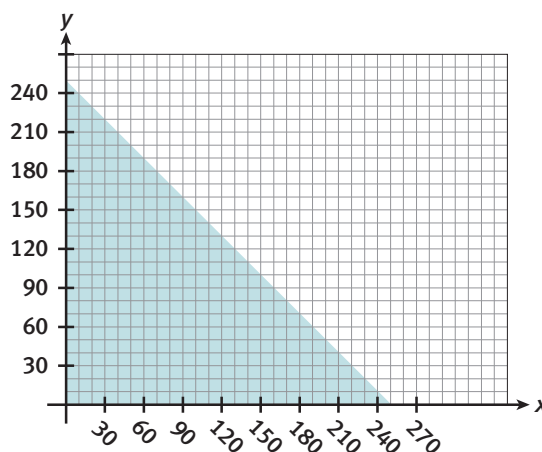
$$Ax + By > C, Ax + By \geq C,$$

$$Ax + By < C, \text{ or } Ax + By \leq C,$$

where A , B , and C are constants and A and B are not both zero.

A **solution of a linear inequality** is an ordered pair (x, y) that makes the inequality true.

The graph below represents the possible values for the number of terabytes that Axl and Aneeza can upload. An ordered pair (x, y) represents the number of terabytes that Axl and Aneeza upload. For example, the coordinates $(100, 75)$ mean that Axl uploaded 100 TB and Aneeza uploaded 75 TB of data.



5. Graph each ordered pair on the graph above. Determine if it will allow Axl and Aneeza to remain within their plan, and explain your answers.
- $(200, 50)$
 - $(150, 150)$
 - $(225, 25)$
 - $(20, 120)$
 - $(120, 200)$

The graph of Axl and Aneeza's storage plan above is an example of a graph of a **linear inequality** in two variables. All the points in the shaded region are **solutions of the linear inequality**.

Lesson 16-1

Writing and Graphing Inequalities in Two Variables

ACTIVITY 16

continued

Check Your Understanding

Axl and Aneezha decide to change to a 350-terabyte plan. Use this information for Items 6–8.

6. How does the graph in Item 5 change?
7. Graph the inequality that represents Axl and Aneezha's new plan. Let x represent the number of terabytes Axl can upload, and let y represent the number of terabytes Aneezha can upload.
8. Which ordered pairs will work with the new plan? Justify your response.
 - a. (200, 50)
 - b. (150, 150)
 - c. (240, 180)
 - d. (0, 300)
 - e. (210, 180)
 - f. (360, -10)
9. Explain how to decide whether an ordered pair is a solution to a linear inequality.

LESSON 16-1 PRACTICE

You are on a treasure-diving ship that is hunting for gold and silver coins. You reel in a wire basket that contains gold and silver coins, among other things. The basket holds no more than 50 pounds of material. Each gold coin weighs about 0.5 ounce, and each silver coin weighs about 0.25 ounce. You want to know the different numbers of each type of coin that could be in the basket.

10. Write an inequality that models the weight in the basket.
11. Give three solutions of the inequality you wrote in Item 10.
12. **Construct viable arguments.** Explain why there cannot be 400 gold coins and 2800 silver coins in the basket.

My Notes

My Notes

Learning Targets:

- Graph on a coordinate plane the solutions of a linear inequality in two variables.
- Interpret the graph of the solutions of a linear inequality in two variables.

SUGGESTED LEARNING STRATEGIES: Vocabulary Organizer, Interactive Word Wall, Create Representations, Identify a Subtask, Construct an Argument

The solutions of a linear inequality in two variables can be represented in the coordinate plane.

MATH TERMS

A line that separates the coordinate plane into two regions is a **boundary line**. The two regions are **half-planes**.

MATH TIP

The origin $(0, 0)$ is usually an easy point to test if it is not on the boundary line.

MATH TERMS

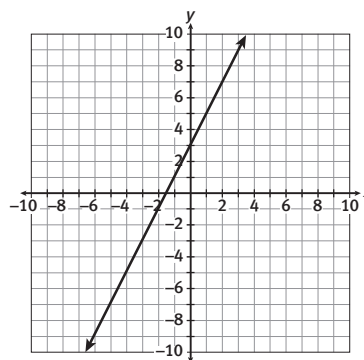
A **closed half-plane** includes the points on the boundary line.

An **open half-plane** does not include the points on the boundary line.

Example A

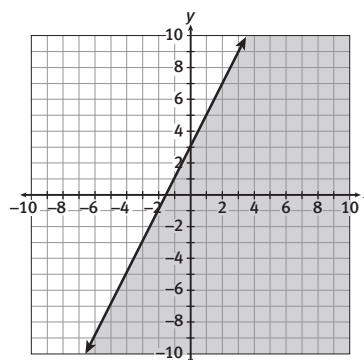
Graph the linear inequality $y \leq 2x + 3$.

Step 1: Graph the corresponding linear equation $y = 2x + 3$. The line you graphed is the **boundary line**.



Step 2: Test a point in one of the **half-planes** to see if it is a solution of the inequality. Using $(0, 0)$, $0 \leq 2(0) + 3$ is a true statement. So $(0, 0)$ is a solution.

Step 3: If the point you tested is a solution, shade the half-plane in which it lies. If it is not, shade the other half-plane. $(0, 0)$ is a solution, so shade the half-plane containing the solution point $(0, 0)$.



In Example A, the solution set includes the points on the boundary line. The solution set is a **closed half-plane**. In an inequality containing $<$ or $>$ the solution set does not include the points on the boundary line, so the boundary line is dashed, and the solution set is an **open half-plane**.

Lesson 16-2

Graphing Inequalities in Two Variables

ACTIVITY 16

continued

Example B

Graph $x + 2y > 8$.

Step 1: Solve the inequality for y .

$$x - x + 2y > 8 - x \quad \text{Subtract } x \text{ from each side.}$$

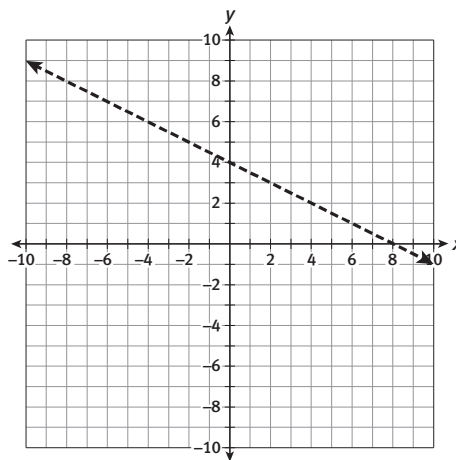
$$2y > -x + 8$$

$$\frac{2y}{2} > \frac{-x + 8}{2} \quad \text{Divide each side by 2.}$$

$$y > -\frac{1}{2}x + 4 \quad \text{Simplify.}$$

Step 2: Graph the boundary line,

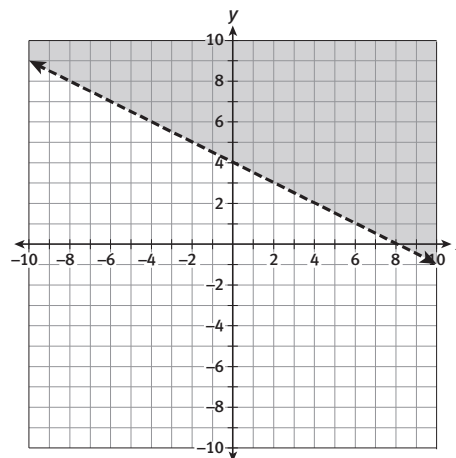
$y = -\frac{1}{2}x + 4$. Since the inequality uses the $<$ symbol, the solution set is an open half-plane. Draw a dashed line.



Step 3: Check the test point $(0, 0)$:

$$0 > -\frac{1}{2}(0) + 4 \text{ is false.}$$

Shade the half-plane that does **not** contain the point $(0, 0)$.



Try These A–B

Graph each inequality.

a. $3x - 4y \geq -12$

c. $-2y > 6 + x$

b. $y < 3x - 2$

d. $y \leq -4x + 5$

- When graphing a linear inequality, is it possible for a test point located on the boundary line to determine which half-plane should be shaded? Explain.

My Notes

MATH TIP

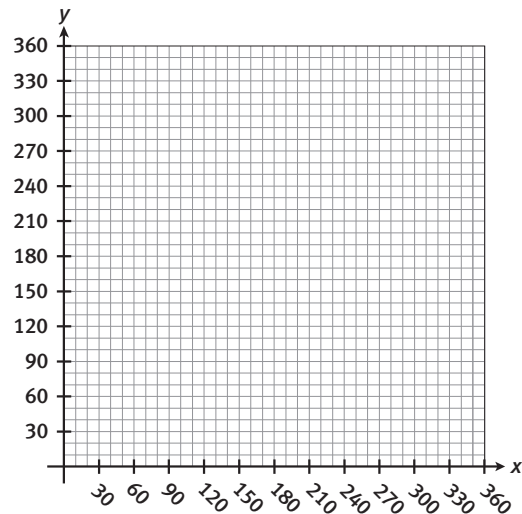
When multiplying or dividing each side of an inequality by a negative number, you must reverse the inequality symbol.

My Notes

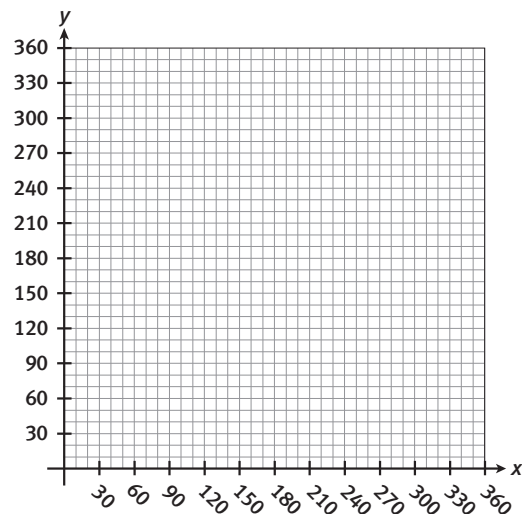
2. Critique the reasoning of others. Axl tried to graph the inequality $2x + 4y < 8$. He first graphed the linear equation $2x + 4y = 8$. He then chose the test point $(2, 1)$ and used his result to shade above the line. Explain Axl's mistake and how he should correct it.

3. Axl and Aneeza try a new plan in which they can upload no more than 350 terabytes per month. On the grids below, the x -axis represents the number of terabytes that Axl can upload and the y -axis represents the number of terabytes that Aneeza can upload.

a. Suppose Aneeza does not upload anything for a month. Write an inequality that represents the amount of data that Axl can upload during that month. Graph the inequality on the grid.



b. Suppose Axl does not upload anything for a month. Write an inequality that represents the amount of data that Aneeza can upload during that month. Graph the inequality on the grid.



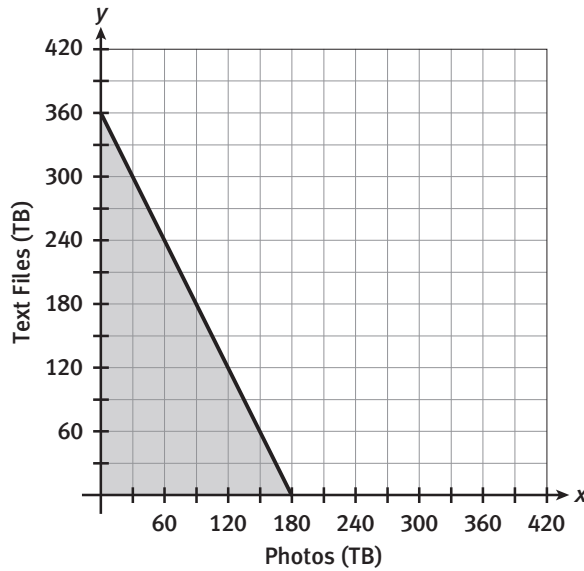
Lesson 16-2

Graphing Inequalities in Two Variables

ACTIVITY 16

continued

4. The graph below represents another plan that Axl and Aneeza considered. The x -axis represents the number of terabytes of data from photos that can be uploaded each month. The y -axis represents the number of terabytes of data from text files.



- a. Identify the x -intercept and y -intercept. What do they represent in this context?
- b. Determine the equation for the boundary line of the graph. Justify your response.
- c. **Model with mathematics.** Write a linear inequality to represent this situation. Then describe the plan in your own words.

My Notes

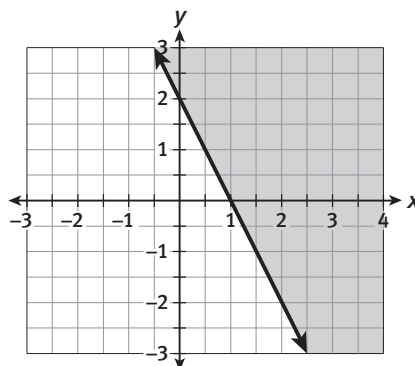
Check Your Understanding

5. Graph the linear inequality $x < -3$ on the coordinate plane.
6. Graph the linear inequality $y > 2$ on the coordinate plane.
7. Graph the linear inequality $x \geq 0$ on the coordinate plane.
8. Write an inequality whose solutions are all points in the second and third quadrants.

LESSON 16-2 PRACTICE

Graph each inequality on the coordinate plane.

9. $x - y \leq 4$
10. $2x - y > 1$
11. $y \geq 3x + 7$
12. $-x + 6 > y$
13. **Make sense of problems.** Write the inequality whose solutions are shown in the graph.



ACTIVITY 16 PRACTICE

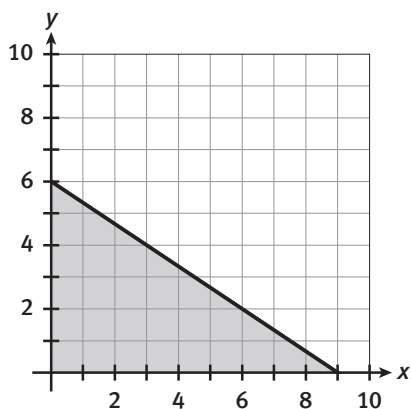
Write your answers on notebook paper.

Show your work.

Lesson 16-1

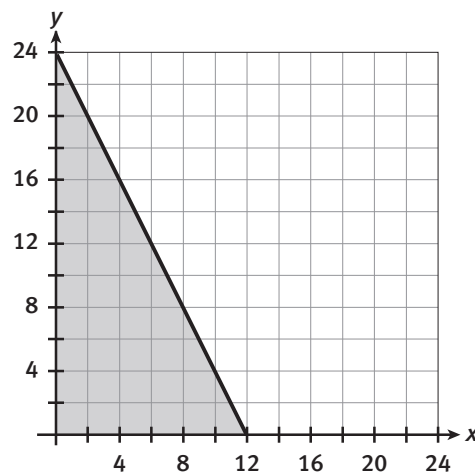
- Which ordered pairs are solutions of the inequality $5y - 3x \leq 7$?

A. (0, 0)	B. (3, 5)
C. (-2, -5)	D. (1, 2.5)
E. (5, -3)	
- Apple juice costs \$2 per bottle, and cranberry juice costs \$3 per bottle. Tamiko has at most \$18 with which to buy drinks for a club picnic. She lets x represent the number of bottles of apple juice and lets y represent the number of bottles of cranberry juice. Then she graphs the inequality $2x + 3y \leq 18$, as shown below.



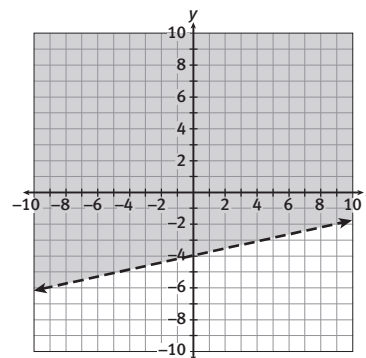
- Tamiko states that the graph does not help her decide how many bottles of each type of juice to buy, because there are infinitely many solutions. Do you agree or disagree? Why?
- Suppose Tamiko decides to buy two bottles of apple juice. Explain how she can use the graph to determine the possible numbers of bottles of cranberry juice she can buy.

- Describe a real-world situation that can be represented by the inequality shown in the graph.

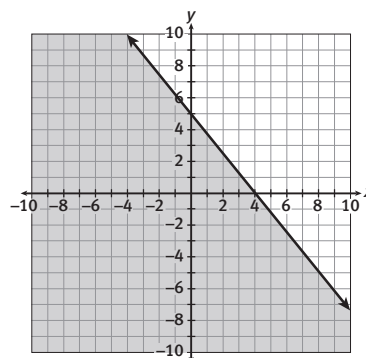


Lesson 16-2

- Write an inequality for the half-plane. Is the half-plane open or closed?



- Write an inequality for the half-plane. Is the half-plane open or closed?



- Sketch a graph of the inequality $y \geq -\frac{2}{5}x + 2$.
- Sketch a graph of the inequality $3y > 7x - 15$.

ACTIVITY 16

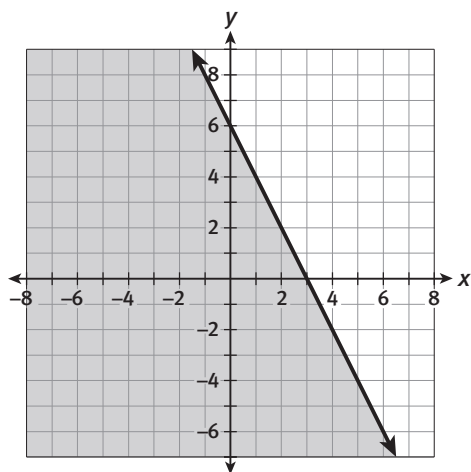
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Inequalities in Two Variables

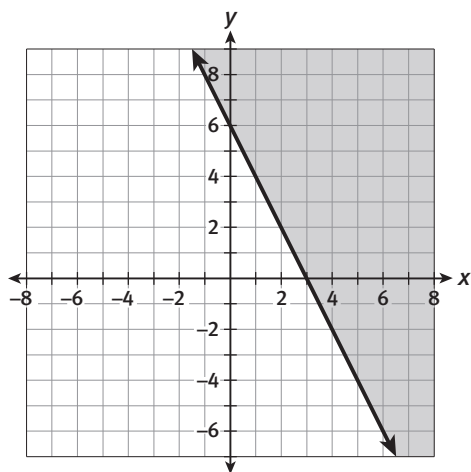
Shared Storage

8. Which inequality represents all of the points in the first and fourth quadrants?
- A. $x < 0$ B. $x > 0$
 C. $y < 0$ D. $y > 0$
9. There are at most 30 students in Mr. Moreno's history class.
- Write an inequality in two variables that represents the possible numbers of boys b and girls g in the class.
 - Graph the inequality on a coordinate plane.
 - Explain whether your graph has a solid boundary line or a dashed boundary line.
 - Choose a point in the shaded region of your graph and explain what the point represents.
10. Which graph represents the solutions of the inequality $2x - y \geq 6$?

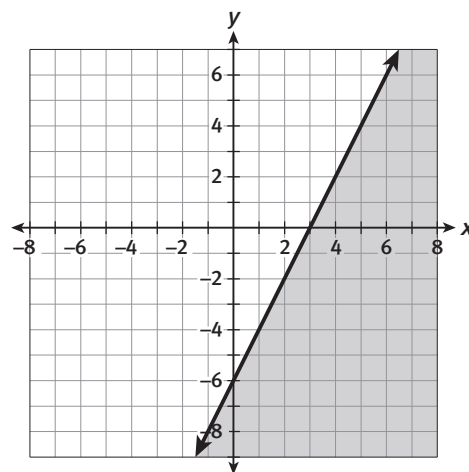
A.



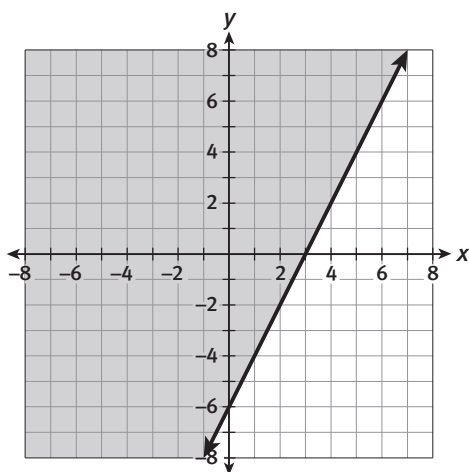
B.



C.



D.



11. Tickets for the school play cost \$3 for students and \$6 for adults. The drama club hopes to bring in at least \$450 in sales. The auditorium has 120 seats. Let a represent the number of adult tickets and s represent the number of student tickets.
- Write an inequality in two variables that represents the desired ticket sales.
 - Write an inequality in two variables that represents the possible numbers of tickets that can be sold.
 - Sketch both inequalities on the same grid. What does the intersection of the two graphs represent?
12. When is the boundary line of the graph of an inequality in two variables part of the solution?

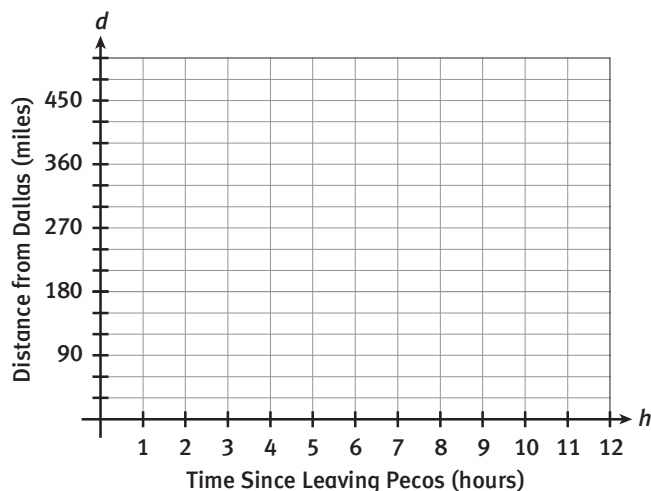
MATHEMATICAL PRACTICES

Look For and Make Use of Structure

13. Graph the inequality $x < 3$ on a number line and on the coordinate plane. Describe the differences in the graphs.

My Notes

4. Graph the equation you wrote in Item 3.



5. Roy leaves Dallas one-half hour before Travis leaves Pecos. In terms of the hours h since Travis left Pecos, write an expression that represents the time since Roy left Dallas.
6. Roy travels 60 mi/h from Dallas to Pecos. Write an equation that expresses Roy's distance d from Dallas as a function of the hours h since Travis left Pecos.
7. Graph the equation from Item 6 on the grid in Item 4.
8. Identify the intersection point of the two lines. Describe the information these coordinates provide.

Lesson 17-1

The Graphing Method

ACTIVITY 17

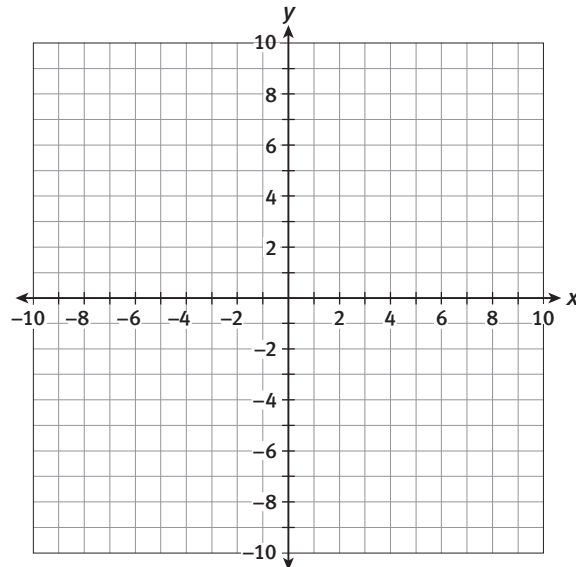
continued

The two equations you wrote in Items 3 and 6 form a **system of linear equations**.

To determine the solution of a system of linear equations, you must identify all the ordered pairs that make both equations true. One method is to graph each equation and determine the intersection point.

9. Graph each system of linear equations. Give each solution as an ordered pair. Check that the point of intersection is a solution of both equations by substituting the solution values into the equations.

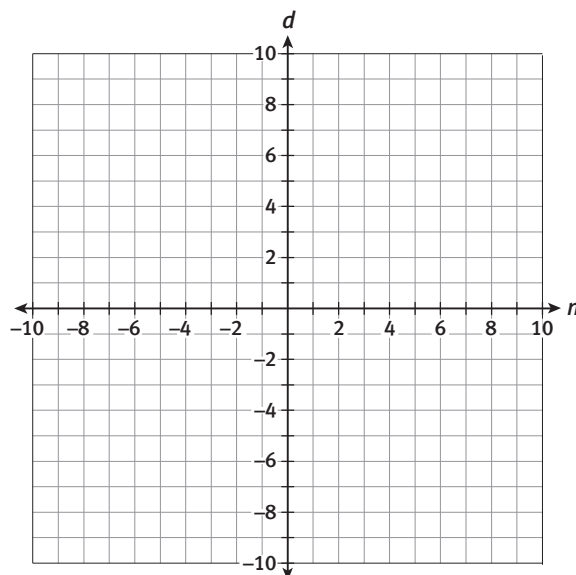
a. $y = 2x - 10$
 $y = -3x + 5$



- b. **Reason abstractly.** Edgar has nine coins in his pocket. All of the coins are nickels or dimes and are worth a total of \$0.55. The system shown below represents this situation. How many of each type of coin does Edgar have in his pocket?

$$n + d = 9$$

$$n + 2d = 11$$



My Notes

MATH TERMS

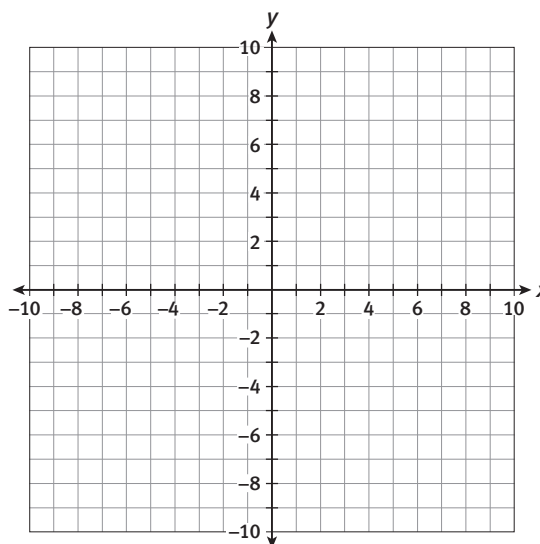
Two or more linear equations with the same variables form a **system of linear equations**.

TECHNOLOGY TIP

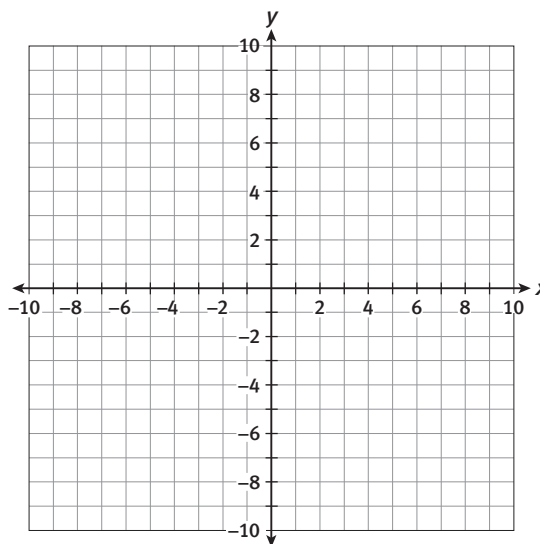
You can graph each equation on a graphing calculator and use the built-in commands to determine the intersection point.

My Notes

c. $y = 3x - 5$
 $y = -2x + 4$



d. $3x + y = 1$
 $6x + 2y = 10$



e. What made finding the solutions to Parts (c) and (d) challenging?

10. Lena graphs the system $x - 2y = 3$ and $2x - y = -3$ and determines the solution to be $(1, -1)$. She checks her solution algebraically and decides the solution is correct. Explain her error.

Check Your Understanding

11. Solve each system.

a. $y = -2x + 5$

$y = \frac{1}{8}x - \frac{7}{2}$

c. $y = -2x + 3$

$y = x$

b. $3x - y = 5$

$4x - 2y = 4$

d. $2x + y = 5$

$4x - 1 = y$

12. Roberto has eight coins that are all dimes or nickels. They are worth \$0.50. The system $n + d = 8$ and $n + 2d = 10$ represents this situation. Graph the system to determine how many of each coin Roberto has.

LESSON 17-1 PRACTICE

13. Solve each system.

a. $y = 2x + 2$

$y = -2x - 6$

c. $3x + 2y = 6$

$x - y = -3$

b. $y = \frac{1}{3}x - 2$

$y = -x + 2$

d. $y = -2$

$2y = -x - 1$

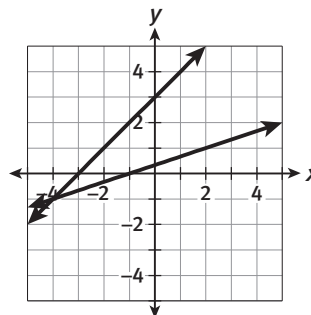
14. Sandeep ordered peanuts and raisins for his bakery. He ordered a total of eight pounds of these ingredients. Peanuts cost \$1 per pound, and raisins cost \$2 per pound. He spent a total of \$10. The system $p + r = 8$ and $p + 2r = 10$ represents this situation. Graph the system to determine how many pounds of peanuts and raisins Sandeep ordered.

15. Critique the reasoning of others.

Kyla was asked to solve the system of equations below. She made the graph shown and stated that the solution of the system is $(-4, -1)$. Is Kyla correct? Justify your response and identify Kyla's errors, if they exist.

$x + y = 3$

$x - 3y = -1$



My Notes

My Notes

Learning Targets:

- Solve a system of linear equations using a table or the substitution method.
- Interpret the solution of a system of linear equations.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Note Taking, Marking the Text, Guess and Check, Simplify the Problem

On another trip, Travis is traveling from Pecos to Dallas, and Roy is driving from Dallas to Pecos. They agree to meet for lunch along the way. Each driver averages 60 mi/h but Roy leaves 1.5 hours before his brother does. To determine when and where they will meet, you will solve this system of linear equations.

$$d = 450 - 60h$$

$$d = 60h + 90$$

1. What do the coefficient 60 and the constants 90 and 450 represent in the context of the problem?

2. Which equation represents Roy's distance from Dallas? How do you know?

In addition to graphing, a system of equations can be solved by first making a table of values. Then look for an ordered pair that is common to both equations.

3. Complete each table.

$$d = 450 - 60h$$

<i>h</i>	<i>d</i>
0	
1	
2	
3	
4	

$$d = 60h + 90$$

<i>h</i>	<i>d</i>
0	
1	
2	
3	
4	

4. What ordered pair do the two equations have in common?

Lesson 17-2

Using Tables and the Substitution Method

ACTIVITY 17

continued

5. **Use appropriate tools strategically.** Graph the equations on a graphing calculator. Identify the solution of the linear system. Describe its meaning in terms of the situation.
6. Is it possible that the intersection point from a graph of a linear system does not show up on a table of values? How could the solution be determined if it is not shown in the table?

My Notes

TECHNOLOGY TIP

You can use the table feature of a graphing calculator to quickly generate a table of values.

Check Your Understanding

7. Solve each system.
- a. $y = 2x + 6$
 $y = -3x + 16$
- b. $x + y = 8$
 $3x + 2y = 14$
- c. $y = 100 - 2x$
 $y = 20 + 6x$
- d. $2x + y = 6$
 $2x + 3y = 8$
8. What challenges did you encounter when solving these systems of linear equations?

Sometimes it is difficult to solve a system of equations by graphing or by using tables of values, and another solution method is necessary.

On another trip, Roy leaves from Pecos one hour before his brother and averages 55 mi/h. Travis leaves from Dallas during rush hour so he averages only 45 mi/h.

This system of linear equations represents each brother's distance d from Dallas, h hours after Roy leaves Pecos.

$$\text{Roy: } d = 450 - 55h$$

$$\text{Travis: } d = 45(h - 1)$$

Solve the system by finding when Travis's distance from Dallas is the same as Roy's distance.

$$\text{Travis's distance} = \text{Roy's distance}$$

$$45(h - 1) = 450 - 55h$$

9. Solve the equation for h and show your work.

My Notes

10. Reason quantitatively. What does the answer to Item 9 represent? Is your answer reasonable?

- 11.** Use a graphing calculator to graph the system above.
- What is the intersection point of the graphs? How would you use this point to answer Item 9?
 - The second coordinate of the intersection point represents the distance that the brothers are from Dallas. How could you confirm this using the equations?

Another method for solving systems of equations is the **substitution method**, in which one equation is solved for one of the variables. Then the expression for that variable is substituted into the other equation.

Example A

For a Valentine's Day dance, tickets for couples cost \$12 and tickets for individuals cost \$8. Suppose 250 students attended the dance, and \$1580 was collected from ticket sales. How many of each type of ticket was sold?

Step 1: Let x = number of couples, and y = number of individuals.

Step 2: Write an equation to represent the number of people attending.

$$2x + y = 250 \qquad \text{The number of attendees is 250.}$$

Write another equation to represent the money collected.

$$12x + 8y = 1580 \qquad \text{The total ticket sales is \$1580.}$$

Step 3: Use substitution to solve this system.

$$2x + y = 250 \qquad \text{Solve the first equation for } y.$$

$$y = 250 - 2x$$

$$12x + 8(250 - 2x) = 1580 \qquad \text{Substitute for } y \text{ in the second equation.}$$

$$12x + 2000 - 16x = 1580 \qquad \text{Solve for } x.$$

$$-4x = -420$$

$$x = 105$$

Lesson 17-2

Using Tables and the Substitution Method

ACTIVITY 17

continued

Step 4: Substitute the value of x into one of the original equations to find y .

$$2x + y = 250$$

$$2(105) + y = 250 \quad \text{Substitute 105 for } x.$$

$$210 + y = 250$$

$$y = 40$$

Solution: For the dance, 105 couples' tickets and 40 individual tickets were sold.

Try These A

Solve each system using substitution.

a. $x + 2y = 8$ and $3x - 4y = 4$

b. $5x - 2y = 0$ and $3x + y = -1$

c. Patty and Toby live 345 miles apart. They decide to drive to meet one another. Patty leaves at noon, traveling at an average rate of 45 mi/h, and Toby leaves at 3:00 P.M., traveling at an average speed of 60 mi/h. At what time will they meet?

Check Your Understanding

Solve the systems by any method. Explain why you chose the method you did.

12. $x - 5y = 2$ and $x + y = 8$

13. $2y = x$ and $x - 7y = 10$

14. Theo buys a slice of pizza and a bottle of water for \$3. Ralph buys 3 slices of pizza and 2 bottles of water for \$8. How much does a slice of pizza cost?

My Notes

My Notes

LESSON 17-2 PRACTICE

15. Delano was asked to solve the system $y = 2x - 4.5$ and $y = -x + 6$. He made the tables shown below.

$$y = 2x - 4.5$$

x	y
0	-4.5
1	-2.5
2	-0.5
3	1.5
4	3.5
5	5.5

$$y = -x + 6$$

x	y
0	6
1	5
2	4
3	3
4	2
5	1

- Are any of the ordered pairs in the tables solutions of the system? Why or why not?
 - What can you conclude about the solution of the system? Explain your reasoning.
 - Would you choose to solve the system graphically or algebraically? Justify your choice.
16. A rock-climbing gym called Rock-and-Roll charges \$2.75 to rent shoes, and \$3 per hour to climb. A competing gym, Climb the Walls, charges \$4.25 to rent shoes, and \$2.50 per hour to climb.
- Write an equation that gives the cost y of renting shoes and climbing for x hours at Rock-and-Roll.
 - Write an equation that gives the cost y of renting shoes and climbing for x hours at Climb the Walls.
 - Solve the system of the two equations you wrote in Parts (a) and (b) using any method. Why did you choose the solution method you did?
 - What does your solution to the system represent in the context of the problem?
 - Construct viable arguments.** Suppose you plan to rent shoes and go rock climbing for 4 hours. Which gym offers a better deal in this case? Justify your response.

Learning Targets:

- Use the elimination method to solve a system of linear equations.
- Write a system of linear equations to model a situation.

SUGGESTED LEARNING STRATEGIES: Note Taking, Discussion Groups, Critique Reasoning, Vocabulary Organizer, Marking the Text

Elimination is another algebraic method that may be used to solve a system of equations. Two equations can be combined to yield a third equation that is also true. The **elimination method** creates like terms that add to zero.

Example A

Solve the system using the elimination method: $4x - 5y = 30$
 $3x + 4y = 7$

Step 1: To solve this system of equations by elimination, decide to eliminate the y variable.

Original system	Multiply the first equation by 4.	Add the two equations to eliminate y .
	Multiply the second equation by 5.	

$$\begin{array}{rcl}
 4x - 5y = 30 & \rightarrow & 4(4x - 5y) = 4(30) \rightarrow 16x - 20y = 120 \\
 3x + 4y = 7 & \rightarrow & 5(3x + 4y) = 5(7) \rightarrow 15x + 20y = 35 \\
 & & \hline
 & & 31x = 155 \\
 & & \text{Solve for } x. \qquad x = 5
 \end{array}$$

Step 2: Find y by substituting the value of x into one of the original equations.

$$\begin{array}{rcl}
 4x - 5y = 30 & & \\
 4(5) - 5y = 30 & \text{Substitute 5 for } x. & \\
 20 - 5y = 30 & & \\
 -5y = 10 & & \\
 y = -2 & &
 \end{array}$$

Step 3: Check $(5, -2)$ in the second equation $3x + 4y = 7$.

$$\begin{array}{rcl}
 3x + 4y = 7 & & \\
 3(5) + 4(-2) ? 7 & & \\
 15 - 8 ? 7 & & \\
 7 = 7 \quad \text{check} & &
 \end{array}$$

Solution: The solution is $(5, -2)$.

Try These A

Solve each system using elimination.

- a. $3x - 2y = -21$ b. $7x + 5y = 9$
 $2x + 5y = 5$ $4x - 3y = 11$

My Notes

MATH TERMS

The **elimination method**, also called the linear combination method, for solving a system of two linear equations involves *eliminating* one variable. To eliminate one variable, multiply each equation in the system by an appropriate number so that the terms for one of the variables will combine to zero when the equations are added. Then substitute the value of the known variable to find the value of the unknown variable. The ordered pair is the *solution* of the system.

My Notes

- Phuong has \$2.00 in nickels and dimes in his bank. The number of dimes is five more than twice the number of nickels. How many of each type of coin are in his bank?

Example B

Noah is given two beakers of saline solution in chemistry class. One contains a 3% saline solution and the other an 8% saline solution. How much of each type of solution will Noah need to mix to create 150 mL of a 5% solution?

Step 1: To solve this problem, write and solve a system of linear equations.

Let x = number of mL of 3% solution.

Let y = number of mL of 8% solution.

Step 2: Write one equation based on the amounts of liquid being mixed. Write another equation on the amount of saline in the final solution.

$$x + y = 150 \quad \text{The amount of the mixture is 150 mL.}$$

$$0.03x + 0.08y = 0.05(150) \quad \text{The amount of saline in the mixture is 5%.$$

Step 3: To solve this system of equations by elimination, decide to eliminate the x variable.

$-3(x + y) = -3(150)$	Multiply the first equation by -3 .
$100(0.03x + 0.08y) = 100(7.5)$	Multiply the second equation by 100 to remove decimals.
$-3x - 3y = -450$	
$3x + 8y = 750$	
$5y = 300$	Add the two equations to eliminate x .
$y = 60$	Solve for y .

Step 4: Find the value of the eliminated variable x by using one of the original equations.

$$x + y = 150$$

$$x + 60 = 150 \quad \text{Substitute 60 for } y.$$

$$x = 90 \quad \text{Subtract 60 from both sides.}$$

Step 5: Check your answers by substituting into the original second equation.

$$0.03x + 0.08y = 0.05(150)$$

$$0.03(90) + 0.08(60) \stackrel{?}{=} 0.05(150) \quad \text{Substitute 90 for } x \text{ and 60 for } y.$$

$$2.7 + 4.8 \stackrel{?}{=} 7.5$$

$$7.5 = 7.5 \quad \text{check}$$

Solution: Noah needs 90 mL of 3% solution mixed with 60 mL of 8% solution to make 150 mL of the 5% solution.

Lesson 17-3

The Elimination Method

ACTIVITY 17

continued

Try These B

Solve each system using elimination.

a. $7x + 5y = -1$
 $4x - y = -16$

- b. **Make sense of problems.** Mary had \$25,000 to invest. She invested part of that amount at 3% interest and part at 5% interest for one year. The amount of interest she earned for both investments was \$1100. How much was invested at each rate?

2. Sylvia wants to mix 100 pounds of Breakfast Blend coffee that will sell for \$25 per pound. She is using two types of coffee to create the mixture. Kona coffee sells for \$51 per pound and Columbian coffee sells for \$11 per pound. How many pounds of each type of coffee should Sylvia use?

Check Your Understanding

3. Fay wants to solve the system $2x - 3y = 5$ and $3x + 2y = -5$ using elimination. She multiplies the second equation by 1.5 to get $4.5x + 3y = -7.5$.
- a. Do you think Fay's approach is correct? Explain why or why not.
- b. Describe how Fay could have multiplied to avoid decimal coefficients.

LESSON 17-3 PRACTICE

For each situation, write and solve a system of equations.

4. **Attend to precision.** A pharmacist has a 10% alcohol solution and a 25% alcohol solution. How many milliliters of each solution will she need to mix together in order to have 200 mL of a 20% alcohol solution?
5. Alyssa invested a total of \$1500 in two accounts. One account paid 2% annual interest, and the other account paid 4% annual interest. After one year, Alyssa earned a total of \$44 interest. How much did she invest in each account?
6. Kendall is Jamal's older brother. The sum of their ages is 39. The difference of their ages is 9. How old are Kendall and Jamal?
7. Yolanda wants to solve the system shown below.

$$3x - 4y = 5$$
$$2x + 3y = -2$$

She decides to use the elimination method to eliminate the x variable. Describe how she can do this.

My Notes

MATH TIP

Recall the formula for simple interest,

$$i = prt,$$

where

i = interest earned

p = principal amount

r = rate

t = time in years

My Notes

MATH TERMS

Objects that are coincident lie in the same place. **Coincident** lines occupy the same location in the plane and pass through the same set of ordered pairs.

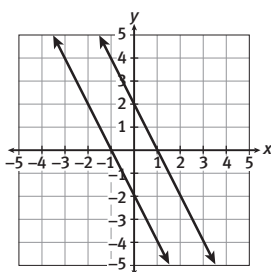
Learning Targets:

- Explain when a system of linear equations has no solution.
- Explain when a system of linear equations has infinitely many solutions.

SUGGESTED LEARNING STRATEGIES: Create Representations, Marking the Text, Close Reading, Think-Pair-Share

The graph of a system of linear inequalities does not always result in a unique intersection point. **Parallel** lines have graphs that do not intersect. **Coincident** lines have graphs that intersect infinitely many times.

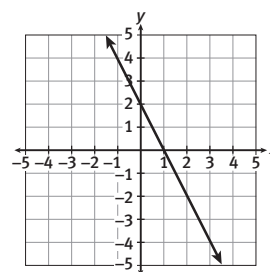
A System of Parallel Lines
No Solution



$$y = -2x + 2$$

$$y = -2x - 2$$

A System of Coincident Lines
Infinitely Many Solutions



$$y = -2x + 2$$

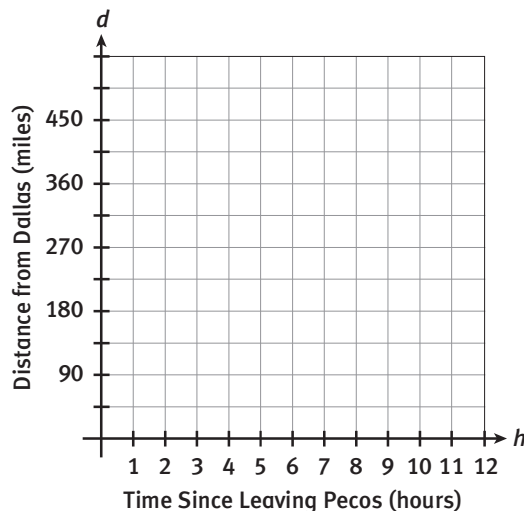
$$6x = 6 - 3y$$

On a particular trip, Roy leaves from Pecos driving 55 mi/h. After 20 minutes, Travis realizes Roy forgot two cases of peaches. In 4 minutes, Travis loads the cases into his truck and heads out after Roy, traveling 55 mi/h.

This system of equations represents each brother's distance d from Dallas h hours after Roy leaves Pecos.

$$\text{Roy: } d = 450 - 55h \quad \text{Travis: } d = 450 - 55(h - 0.4)$$

1. **Model with mathematics.** Graph the system of equations. Describe the graph.



Lesson 17-4

Systems Without a Unique Solution

ACTIVITY 17

continued

2. What information about Travis and Roy does the graph provide?

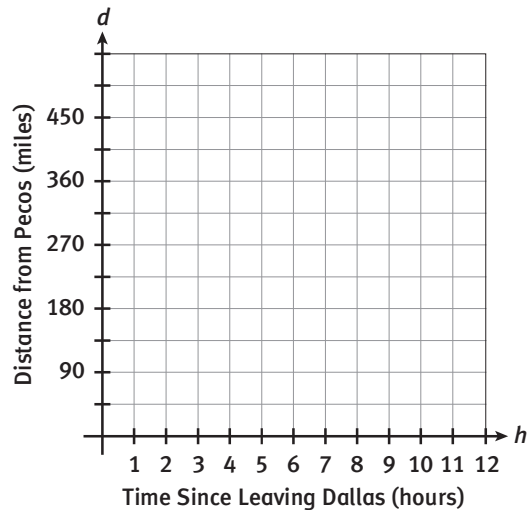
3. How many solutions exist to the system of equations? How is this shown in the graph?

The system below represents a return trip from Dallas to Pecos, where d represents the distance from Pecos h hours after Roy leaves Dallas.

$$\text{Travis: } d = 450 - 60h$$

$$\text{Roy: } d = -10(6h - 45)$$

4. **Model with mathematics.** Graph the system of equations. Describe the graph.



5. What information about Travis and Roy does the graph provide?

6. How many solutions exist to the system of equations? How is this shown in the graph?

My Notes

My Notes

Check Your Understanding

7. Solve each system.
- a. $y = 50 + 3x$
 $y = 100 - 2x$
- b. $x + 3y = 9$
 $-3x + 2y = 8$
8. Tom leaves for Los Angeles averaging 65 mi/h. Michelle leaves for Los Angeles one hour later than Tom from the same location. She travels the same route averaging 70 mi/h. When will she pass Tom?
9. Juan bought a house for \$200,000 and each year its value increases by \$10,000. Tia bought a house for \$350,000 and its value is decreasing annually by \$5000. When will the two homes be worth the same amount of money?
10. Solve each system by graphing. If the lines are parallel, write *no solution*. If the lines are coincident, write *infinitely many solutions*.
- a. $4x + 2y = 10$
 $y = -2x + 5$
- b. $y - 2x = 4$
 $y = 5 + 2x$

LESSON 17-4 PRACTICE

Speedy Plumber charges \$75 for a house call and \$40 per hour for work done during the visit. Drains-R-Us charges \$35 for a house call and \$60 per hour for work done during the visit. Use this information for Items 11–13.

11. **Reason quantitatively.** Write and solve a system of equations to determine how many hours of work result in the same total cost for a house call from either company. What is the cost in this case? How many hours must each company work to charge the same amount?
12. Eliza has a coupon for \$40 off the fee for a house call from Speedy Plumber. How does this change your answer to Item 11? Is the total cost for the two companies ever the same? If so, after how many hours?
13. Tyrone has a coupon that lowers the hourly rate for Drains-R-Us to \$40 per hour. How does this change your answer to Item 11? Is the total cost for the two companies ever the same? If so, after how many hours?

Learning Targets:



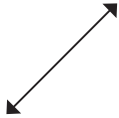
- Determine the number of solutions of a system of equations.
- Classify a system of linear equations as independent or dependent and as consistent or inconsistent.

SUGGESTED LEARNING STRATEGIES: Summarizing, Paraphrasing, Sharing and Responding, Interactive Word Wall, Predict and Confirm

When a system of two linear equations in two variables is solved, three possible relationships can occur.

- Two distinct lines that intersect produce one ordered pair as the solution.
- Two distinct parallel lines that do not intersect produce no solutions.
- Two lines that are coincident produce the same solution set—an infinite set of ordered pairs that satisfy both equations.

The three systems in the chart represent each of the possible relationships described above.

Relationship of Lines	Sketch	Number of Solutions
two intersecting lines		one solution
two parallel lines		no solution
two coincident lines		infinitely many solutions

- Use the system $\begin{cases} 4x - 2y = 21 \\ y - 2x = 10 \end{cases}$ to answer Parts (a)–(d) below.
 - Make use of structure.** Write each equation in the system in **slope-intercept form**. Compare the slopes and y -intercepts.
 - Make a conjecture about the graph and the solution of this system.

My Notes

MATH TERMS

The **slope-intercept form** of a linear equation is $y = mx + b$; m is the slope of the line and $(0, b)$ is the y -intercept.

My Notes

- c. Check the conjecture from Part (b) by graphing the system on a graphing calculator. Revise the conjecture if necessary.
- d. Solve the system using either the substitution method or the elimination method. Describe the result.
2. Use the system $\begin{cases} 2x = -y + 1 \\ 6x + 3y = 3 \end{cases}$ to answer Parts (a)–(d) below.
- a. Write each equation in the system in slope-intercept form. Compare the slopes and y -intercepts.
- b. Make a conjecture about the graph and the solution of this system.
- c. Check the conjecture from Part (b) by graphing the system on a graphing calculator. Revise the conjecture if necessary.
- d. Solve the system using either the substitution method or the elimination method. Describe the result.
3. Write each equation in the system $\begin{cases} 2x - y = 6 \\ x = 6 - y \end{cases}$ in slope-intercept form.
4. **Construct viable arguments.** Without graphing or solving, describe the solution of the system in Item 3. Justify your response.
5. Verify your answer to Item 4 by solving the system using any method. Revise your answer to Item 4 if necessary.

Systems of linear equations are classified by the relationships of their lines.

- Systems that produce two distinct lines when graphed are **independent**. Systems that produce coincident lines are **dependent**.
 - Systems that have no solution are **inconsistent**. Systems that have at least one solution are **consistent**.
6. Classify the systems in Items 1, 2, and 3.

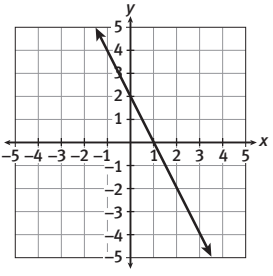
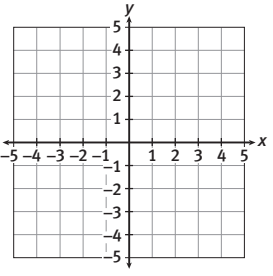
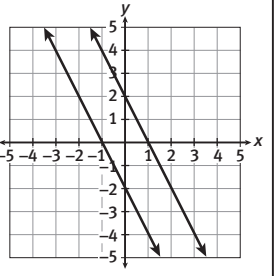
Lesson 17-5

Classifying Systems of Equations

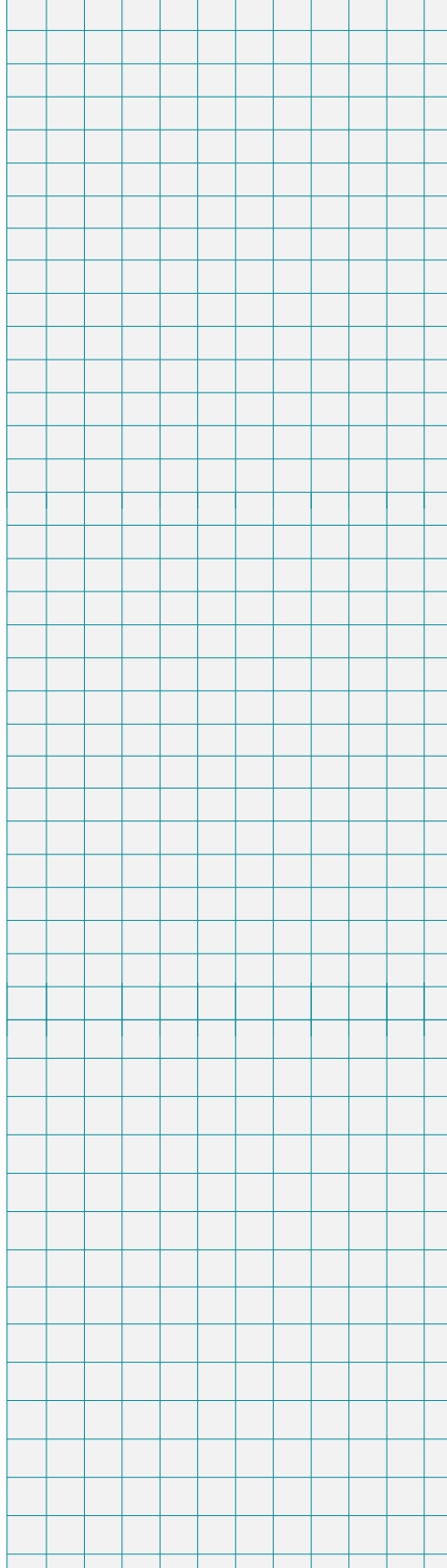
ACTIVITY 17

continued

7. For each system below, complete the table with the information requested.

The Nature of Solutions to a System of Two Linear Equations		
Equations in Standard Form:		
$2x + y = 2$ $6x + 3y = 6$	$2x + y = 2$ $x + y = 3$	$2x + y = 2$ $4x + 2y = -4$
Graph Each System:		
		
Write the Number of Solutions:		
Write the Relationship of the Lines:		
Solve Algebraically:		
Write the Equations in Slope-Intercept Form:		
Compare the Slopes and y-intercepts:		
Classify the System:		

My Notes



My Notes

Check Your Understanding

For each system below:

- a. Tell how many solutions the system has.
- b. Describe the graph.
- c. Classify the system.

8. $2x - 2y = 6$
 $y - x = -3$

9. $y = 1.5x + 5$
 $3x - 2y = 10$

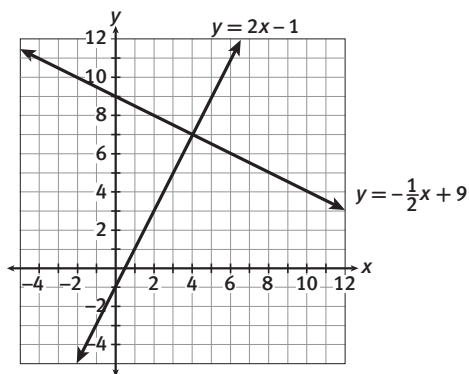
10. $y = \frac{2}{3}x + 1$
 $4x - 6y = -6$

11. $3x + 4y = 1$
 $2x - 5y = 16$

LESSON 17-5 PRACTICE

12. Solve the system $3x + 4y = 8$ and $y = \frac{3}{4}x - 2$ using any method. Classify the system.

13. Approximate the point of intersection for the system of linear equations graphed below. Verify algebraically using substitution or elimination that the selected point is a solution for the system.



14. Monica claims that the system $3x + y = 16$ and $2x + 2y = 12$ has the same solutions as the system $6x + 2y = 32$ and $2x + 2y = 12$. Explain how you can tell whether Monica is correct without solving the systems.

15. Find the solution of the system $y = -\frac{2}{5}x + 1$ and $2x + 5y = 3$ by any method. Classify the system.

16. **Critique the reasoning of others.** Kristen graphed the system $y = 3x + 5$ and $10y = 30x + 51$ on her graphing calculator. She saw a single line on the screen and concluded that the system was dependent and consistent. Do you agree or disagree? Explain.

ACTIVITY 17 PRACTICE

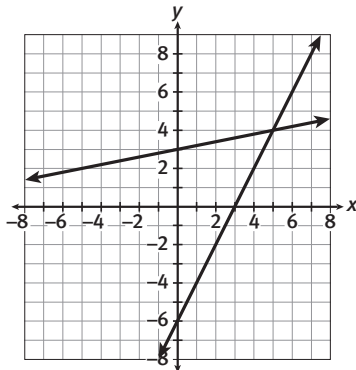
Write your answers on notebook paper. Show your work.

Lesson 17-1

- Solve each system of linear equations.
 - $y = 3x - 4$
 $y = \frac{2}{5}x + 9$
 - $x + y = 7$
 $x - 3y = -1$
- Which ordered pair is a solution to the system shown at right?

$$y = \frac{2}{3}x + 3$$

$$y = -3x + 14$$
 - $(-3, 1)$
 - $(5, -1)$
 - $(3, 5)$
 - $(5, 3)$
- Which system's solution is represented by the graphs shown below?



- $y = 2x + 3$
 $y = \frac{1}{5}x - 6$
- $y = 2x - 6$
 $y = \frac{1}{5}x + 3$
- $y = 5x - 6$
 $y = \frac{1}{2}x + 3$
- $y = 5x + 3$
 $y = \frac{1}{2}x - 6$

Lesson 17-2

- A bushel of apples currently costs \$10 and the price is increasing by \$0.50 per week. A bushel of pears currently costs \$15 and the price is decreasing by \$0.25 per week. Which system of linear equations could be used to determine when the two fruits will cost the same amount per bushel?
 - $y = 0.5x + 10$
 $y = -0.25x + 15$
 - $y = 0.25x + 15$
 $y = -0.5x + 10$
 - $y = 10x + 0.5$
 $y = 15x - 0.25$
 - $y = 5x + 3$
 $y = -\frac{1}{2}x - 6$
- Ray starts walking to school at a rate of 2 mi/h. Ten minutes later, his sister runs after him with his lunch, averaging 6 mi/h.
 - Write a system of linear equations to represent this situation.
 - Solve the system to determine how much time it took Ray's sister to catch up to him.
- Colleen is in charge of ordering office supplies for her company. Last month she ordered ink cartridges and toner cartridges for the office printers. The cost of a toner cartridge is \$19.50 more than the cost of an ink cartridge. Colleen ordered 11 ink cartridges and 4 toner cartridges, and the total cost was \$460.50. Write and solve a system of equations to find the cost of each cartridge.

ACTIVITY 17*continued***Solving Systems of Linear Equations**
A Tale of Two Truckers

7. At Rocking Horse Ranch, admission is \$45 and trail rides are \$22.50 per hour. At Saddlecreek Ranch, admission is \$30 and trail rides are \$35 per hour.
- Write a system of equations that shows the total cost c for a trail ride that lasts h hours at each ranch.
 - Solve your system from Part (a). Interpret the meaning of the solution in the context of the problem.
 - Janelle has \$125, and she wants to go on a three-hour trail ride. Which ranch should Janelle choose? Justify your answer.

Lesson 17-3

8. Lawrence has 10 coins in his pocket. One coin is a quarter, and the others are all nickels or dimes. The coins are worth 90 cents.
- Write a system that represents this situation.
 - Solve the system to determine the number of dimes and nickels in Lawrence's pocket.
9. Pedro placed an order with an online nursery for 6 apple trees and 5 azaleas and the order came to \$147. The next order for 3 apple trees and 4 azaleas came to \$96. What was the unit cost for each apple tree and for each azalea?
10. Jeremiah scored 28 points in yesterday's basketball game. He made a total of 17 baskets. Some of the baskets were field goals (worth two points) and the rest were free throws (worth one point). Write and solve a system of equations to find the number of field goals and the number of free throws that Jeremiah made.

Lesson 17-4

11. Graph each system of linear equations and describe the solutions.
- $x + y = 3$ and $2x + 2y = 6$
 - $2x + 3y = 6$ and $-x + y = -3$
 - $x - 4y = 1$ and $2x - 9 = 8y$
12. Ming graphs a system of linear equations on his calculator. When he looks at the result, he sees only one line. Assuming he graphed the equations correctly, what could this mean?
- The system has infinitely many solutions.
 - The system has no solution.
 - The system has exactly one solution.
 - Every possible ordered pair (x, y) is a solution.

Lesson 17-5

13. Which is the best way to classify the system $y = 1.5x - 2$ and $3y = 4.5x - 7$?
- Independent and inconsistent
 - Independent and consistent
 - Dependent and inconsistent
 - Dependent and consistent
14. Solve the system $3x + 4y = 8$ and $y = -\frac{3}{4}x + 2$ and classify the system.
15. The equation $3x - y = -1$ is part of a system of linear equations that is dependent and consistent. Write an equation that could be the other equation in the system.
16. Three friends decide to increase their exercise programs at the same time. Carolyn walks 7 miles per week and decides to increase the number of miles she walks by 1.5 miles per week. Eduardo walks 3 miles per week and decides to increase the number of miles he walks by 3.5 miles per week. Kendra walks 5 miles per week and decides to increase the number of miles she walks by 3.5 miles per week.
- Write and solve a system of equations to determine how many weeks it will be until Carolyn and Eduardo are walking the same distance each week.
 - Write a system of equations you could use to determine how many weeks it will be until Eduardo and Kendra are walking the same distance each week.
 - Classify the systems you wrote in Parts (a) and (b).
 - Describe what would happen if you tried to solve the system you wrote in Part (b) using the graphing method

MATHEMATICAL PRACTICES**Look for and Make Use of Structure**

17. Explain how you can determine whether a system of two linear equations has a unique solution by examining the equations.

Solving Systems of Linear Inequalities

Which Region Is It?

Lesson 18-1 Representing the Solution of a System of Inequalities

Learning Targets:

- Determine whether an ordered pair is a solution of a system of linear inequalities.
- Graph the solutions of a system of linear inequalities.

SUGGESTED LEARNING STRATEGIES: Create Representations, Look for a Pattern, Discussion Groups, Quickwrite, Graphic Organizer

1. Graph each inequality on the number lines and grids.

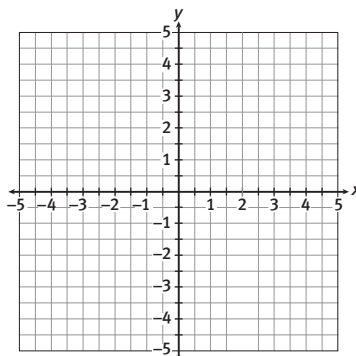
Inequality	Graph all x	Graph all (x, y)
$x < 2$		
$x \geq -3$		
$x < 2$ and $x \geq -3$		

2. Compare and contrast the graphs you made in the third row of the table. In your explanation, compare the graphs to those in the first two rows and use the following words: dimension, half-line, half-plane, open, closed, and intersection.

My Notes

My Notes

3. On the coordinate grid, graph the solutions common to the inequalities $y \leq 4$ and $y > 1$.



MATH TERMS

A **system of linear inequalities** consists of two or more linear inequalities with the same variables.

Solving a **system of linear inequalities** means finding all solutions that are common to all inequalities in the system.

4. **Reason quantitatively.** For the inequalities below, complete the table showing whether each ordered pair is a solution of the system of inequalities by deciding whether the ordered pair is a solution of **both** inequalities. Justify your responses.

$$x + y > 2$$

$$2x - y \geq -5$$

Ordered Pair	Is it a Solution?	Why or Why Not?
$(-2, -3)$		
$(3, 2)$		
$(3, -1)$		
$(0, 5)$		

5. **Make use of structure.** Find two more solutions of the system in Item 4.

Lesson 18-1

Representing the Solution of a System of Inequalities

ACTIVITY 18

continued

Since a system of inequalities has infinitely many solutions, you can represent all solutions using a graph. To solve a system of inequalities, graph each inequality on the same coordinate grid by shading a half-plane. The region that is the intersection of the two shaded half-planes, called the **solution region**, represents all solutions of the system.

My Notes

MATH TERMS

A **solution region** is the part of the coordinate plane in which the ordered pairs are solutions to all inequalities in a system.

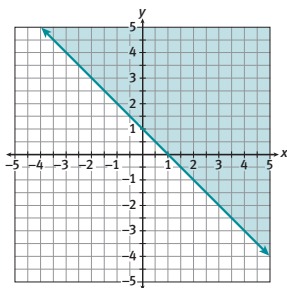
Example A

Solve the system of inequalities.

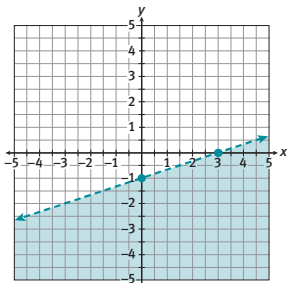
$$x + y \geq 1$$

$$x - 3y > 3$$

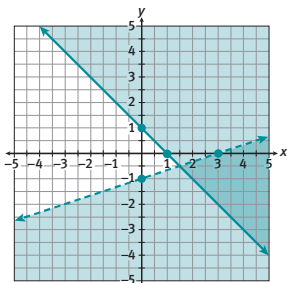
Step 1: First, graph $x + y \geq 1$.



Step 2: Next, graph $x - 3y > 3$.



Step 3: Identify the solution region.



The solution region is the double shaded region.

Solution: The solution region is the double shaded region shown in Step 3.

MATH TIP

To graph an inequality, it may be helpful to solve the inequality for y first.

MATH TIP

When finding the solution region for a system of inequalities, it may be helpful to shade each half-plane with a different pattern or color.

My Notes

Try These A

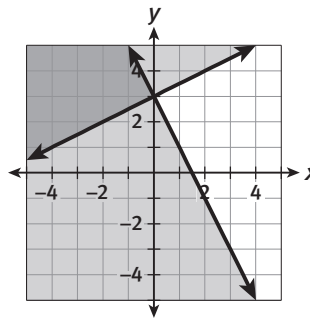
Solve each system of inequalities using the coordinate grids in the *My Notes* section.

- a. $x + y > 2$ b. $y \geq x - 1$ c. $2x + 3y > 6$
 $2x - y \geq -5$ $y \leq -\frac{1}{2}x + 2$ $x - 2y < 4$

6. The system in Try These Part (a) is the same as the system in Item 4. Plot the points listed in Items 4 and 5 on the graph you made for Try These Part (a). Where do the points that are solutions lie? Where do points that are not solutions lie? Give three additional points that are solutions.

Check Your Understanding

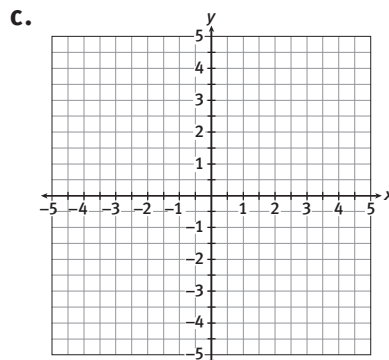
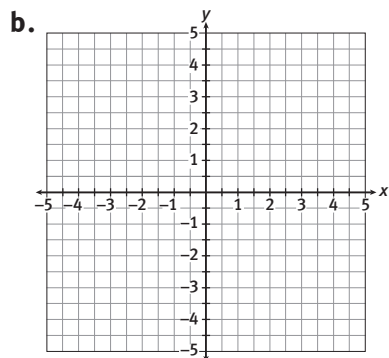
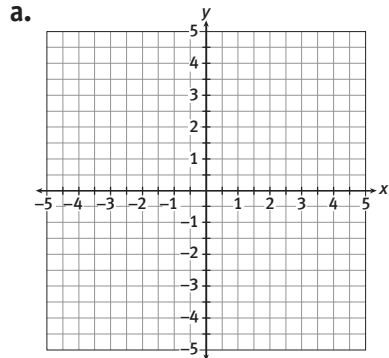
7. The graph below shows the solution of a system of inequalities.



- a. Name an ordered pair that is a solution of the system.
 b. Name an ordered pair that is **not** a solution of the system.
 c. Is the ordered pair $(0, 3)$ a solution of the system? Explain how you know.

Solve each system of inequalities.

8. $y < 2x - 1$ 9. $3x + y < 3$ 10. $y \geq \frac{2}{3}x + 4$
 $y \geq -x$ $x - y > 1$ $x + 2y < 6$



Lesson 18-1

Representing the Solution of a System of Inequalities

ACTIVITY 18

continued

LESSON 18-1 PRACTICE

Solve each system of inequalities.

11. $y \geq x - 3$
 $y \leq \frac{1}{2}x + 1$

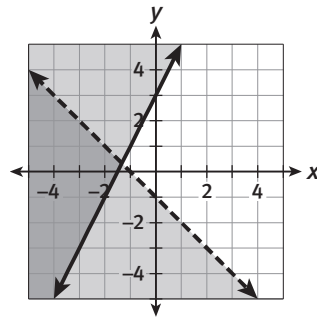
12. $3x + 3y > 1$
 $2y < 11$

13. $y < \frac{1}{3}x$
 $x + y \geq -2$

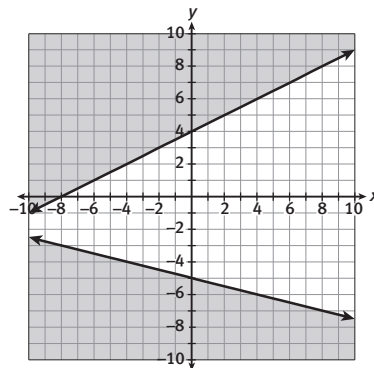
14. Name three ordered pairs that are solutions of the system of inequalities shown below.

$$y < 4x + 4$$
$$x - y > 3$$

15. Write a system of inequalities whose solution is shown by the overlapping regions in the graph below.



16. **Critique the reasoning of others.** A student was asked to solve the system $y \geq \frac{1}{2}x + 4$ and $x + 4y \leq -20$. She made the graph shown below. She noticed that the two shaded regions do not overlap, so she concluded that the system of inequalities has no solution. Do you agree or disagree? Justify your reasoning.



My Notes

My Notes

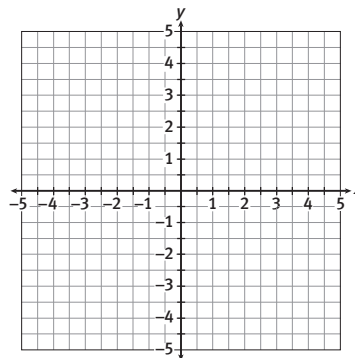
Learning Targets:

- Identify solutions to systems of linear inequalities when the solution region is determined by parallel lines.
- Interpret solutions of systems of linear inequalities.

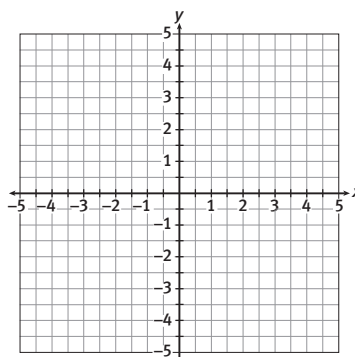
SUGGESTED LEARNING STRATEGIES: Create Representations, Look for a Pattern, Discussion Groups, Close Reading, Marking the Text

1. Determine the solutions to the systems of inequalities by graphing.

a. $y > \frac{1}{2}x + 2$
 $x - 2y > 8$



b. $3x + y < 3$
 $y + 2 \geq -3x$



2. Compare and contrast the systems in Items 1a and 1b.

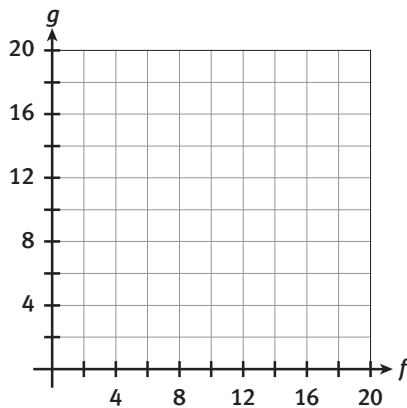
Lesson 18-2

Interpreting the Solution of a System of Inequalities

ACTIVITY 18

continued

3. Ray plays on the basketball team. Last week, he scored all of his points from free throws (worth one point each) and field goals (worth two points each). He has forgotten how many points he scored, but he remembers some facts from the game.
- a. Reason abstractly.** Let f represent the number of free throws and g represent the number of field goals. Write an inequality for each of the facts below.
- Ray scored fewer than 20 points.
- Ray made fewer than six free throws.
- At most, Ray made twice as many free throws as he made field goals.
- b.** Graph the solutions to the system represented by your inequalities from Part (a).



- c. Reason quantitatively.** In the solution region you graphed in Part (b), not every point makes sense in this context. Give two solutions that make sense in the context of the problem and one that does not. Explain your reasoning.

My Notes

Check Your Understanding

4. Use guess and check to identify an ordered pair that is a solution to the system of inequalities.

$$3x + y \geq 6$$

$$x + 3y < 3$$

5. Solve the system in Item 4 by graphing. Confirm that the ordered pair you wrote in Item 4 is a solution. Explain.

6. Solve the system of linear inequalities.

$$y > 2x - 3$$

$$y < 4x - 3$$

7. Write a system of inequalities whose solution region is all the points in the third quadrant.

LESSON 18-2 PRACTICE

Catherine bought apples and pears for a school picnic. The total number of pieces of fruit that she bought was less than 100. She bought more apples than pears. Use this information for Items 8–11.

8. **Model with mathematics.** Write a system of inequalities that describes the situation. Let x represent the number of apples Catherine bought and let y represent the number of pears Catherine bought.
9. Graph the solutions of the system you wrote in Item 8.
10. Give two ordered pairs from the solution region that make sense in the context of the problem.
11. Give an ordered pair from the solution region that does not make sense in the context of the problem. Explain why the ordered pair does not make sense.
12. Describe the solution region of the system of the inequalities $-x + 2y \leq 6$ and $2y \geq x + 6$.

ACTIVITY 18 PRACTICE

Write your answers on notebook paper. Show your work.

Lesson 18-1

1. Determine which of the following ordered pairs are solutions to each of the given systems of inequalities.

$$\{(5, 3), (-2, 1), (1, 2), (2, -3), (3, 5), (-2, 3), (2, 0)\}$$

a. $y < -x + 3$
 $y \geq x - 2$

b. $2x - y \leq 0$
 $y > -\frac{1}{2}x + 1$

c. $3x - y > 6$
 $y \leq 3$

2. Graph each of the systems of inequalities in Item 1. Graph the points that you chose as solutions from Item 1 on the same coordinate grid to verify that they are solutions.
3. Identify four ordered pairs that are solutions to the following system of equations. Explain how you chose your points.

$$4x + 3y \geq -12$$

$$2x - y < 4$$

4. Solve the following systems of inequalities.

a. $2x + 3y \geq 15$
 $5x - y \geq 3$

b. $x - 4y \geq 4$
 $4y - x > 8$

5. Tickets for the school play cost \$3 for students and \$6 for adults. The drama club hopes to bring in at least \$450 in sales, and the auditorium has 120 seats. Let a represent the number of adult tickets, and let s represent the number of student tickets.

- a. Write a system of inequalities representing this situation.
- b. Show the solutions to the system of inequalities by graphing.
- c. If the show sells out, what is the greatest number of student tickets that could be sold to get the desired amount of sales?

6. Which system of inequalities represents all of the points in the second quadrant?

A. $x > 0, y < 0$

B. $x > 0, y > 0$

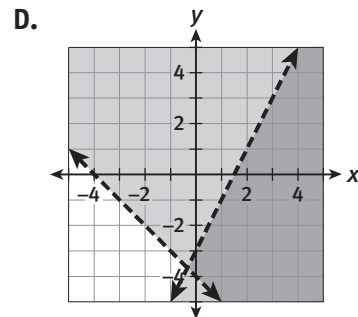
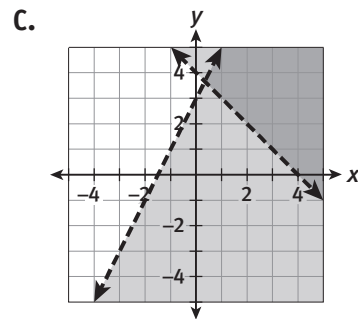
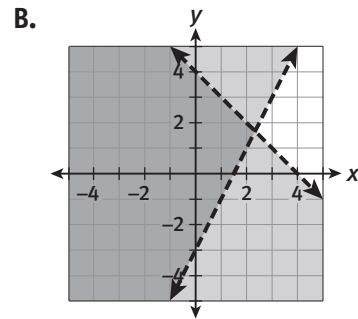
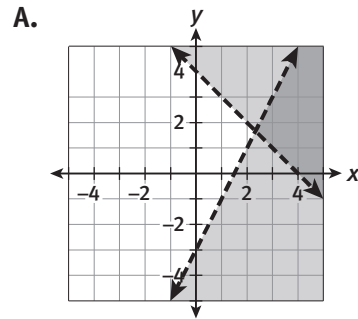
C. $x < 0, y < 0$

D. $x < 0, y > 0$

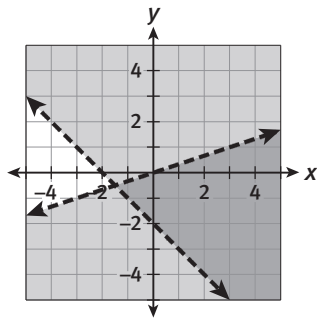
7. Which graph represents the system of inequalities shown?

$$2x - y > 3$$

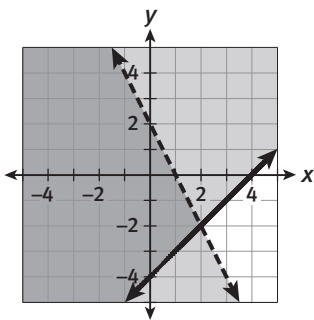
$$x + y > 4$$



8. Which system of inequalities has the solution region shown in the graph below?



- A. $3y < x$
 $x + y > 2$
- B. $3y < x$
 $x + y > -2$
- C. $3y > x$
 $x + y > 2$
- D. $3y > x$
 $x + y > -2$
9. The graph shows the solution of a system of inequalities. Which statement is true?



- A. $(2, -2)$ is a solution.
- B. The origin is not a solution.
- C. Any ordered pair with a negative x -coordinate is a solution.
- D. $(2, -4)$ is not a solution.

Lesson 18-2

10. Consider the system of inequalities shown below.

$$\begin{aligned} x + 2y &\leq 6 \\ x + 2y &\geq -2 \end{aligned}$$

- a. Graph the solution of the system of inequalities.
- b. Describe the solution region.
- c. Name three ordered pairs that are solutions of the system.
- d. How would the solution be different if the inequality signs were reversed? That is, what is the solution of the system $x + 2y \geq 6$ and $x + 2y \leq -2$? Explain why your answer makes sense.
11. Connor bought used books from a Web site. The paperbacks cost \$2 each, and the hardcovers cost \$3 each. He spent no more than \$90 on the books, and he bought no more than 35 books.
- a. Write a system of inequalities to represent the situation. State what the variables represent.
- b. Graph the solution of the system of inequalities you wrote in Part (a).
- c. Name two ordered pairs that are solutions. Interpret the meaning of each ordered pair in the context of the problem.
- d. Suppose you know that Connor spent exactly \$90 and that he bought exactly 35 books. What can you conclude in this case? Why?
12. a. Write a system of linear inequalities that has no solutions.
- b. Is it possible for a system of linear inequalities with no solutions to have nonparallel boundary lines? If so, explain why and give an example. If not, explain why not. (Hint: Consider coincident boundary lines.)

MATHEMATICAL PRACTICES

Attend to Precision

13. Explain why graphing is the preferred method of representing the solutions of a system of linear inequalities.