Lesson 22-3
Graphs of Exponential Functions

My Notes

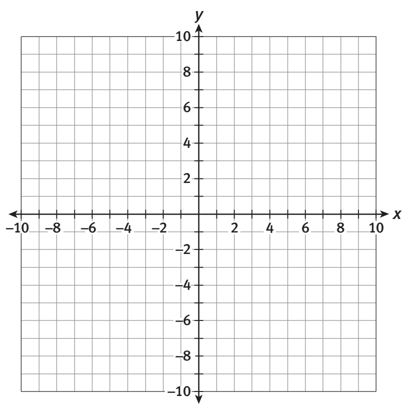
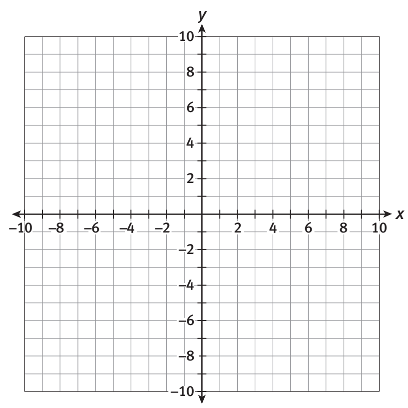
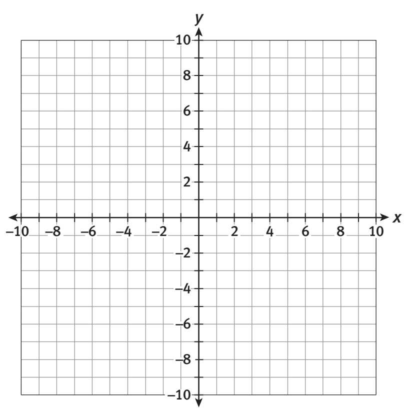
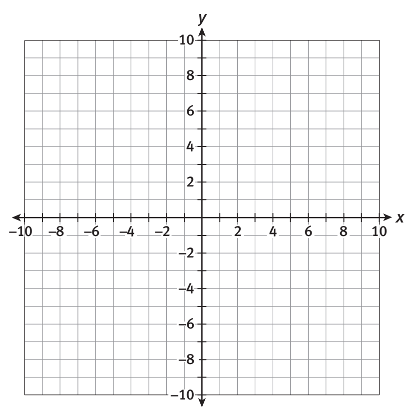
Learning Targets:

- Describe key features of graphs of exponential functions.
- Compare graphs of exponential and linear functions.

SUGGESTED LEARNING STRATEGIES: Predict and Confirm, Discussion Groups, Create Representations, Look for a Pattern, Sharing and Responding, Summarizing

Recall that an exponential function is a function of the form $f(x) = ab^x$, where $a \neq 0$, $b > 0$, and $b \neq 1$.

1. Use a graphing calculator to graph each function. Sketch each graph on the coordinate grid provided.

$a > 0$	$a < 0$
a. $y = 3 \cdot 2^x$ 	**b.** $y = (-3) \cdot 2^x$ 
c. $y = 3 \cdot (0.5^x)$ 	**d.** $y = -2 \cdot (0.5^x)$ 

2. Compare and contrast the graphs and equations in Items 1a and 1b above.

a. How are the equations similar and different?

My Notes

b. Use words like *increasing*, *decreasing*, *positive*, *negative*, *domain*, and *range* to describe the similarities and differences in the graphs.

c. What connections can be made between the graphs and their equations?

3. Compare and contrast the graphs and equations in Items 1c and 1d.

a. How are the equations similar and different?

b. Use words like *increasing*, *decreasing*, *positive*, *negative*, *domain*, and *range* to describe the similarities and differences between the graphs.

c. What connections can be made between the graphs and their equations?

4. Describe the effects of the values of a and b on the graph of the exponential function $f(x) = ab^x$.

a. Describe the graph of an exponential function when $a > 0$.

b. Describe the graph of an exponential function when $a < 0$.

c. Describe the graph of an exponential function when $b > 1$.

d. Describe the graph of an exponential function when $0 < b < 1$.

Lesson 22-3
Graphs of Exponential Functions

My Notes

Check Your Understanding

5. Describe the values of a and b for which the exponential function $f(x) = ab^x$ is always positive.

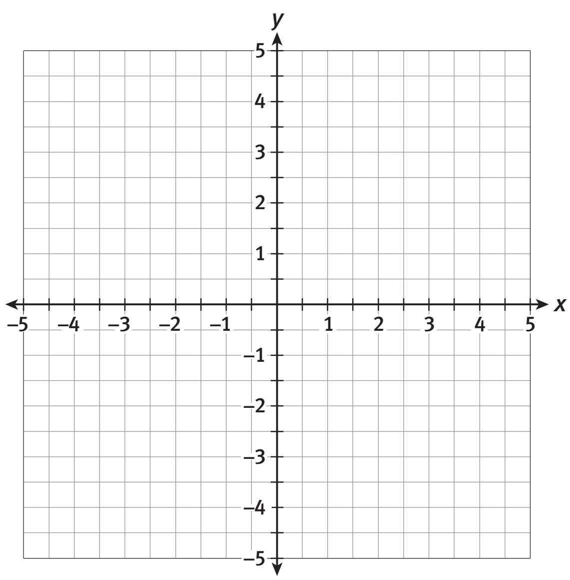
6. Describe the values of a and b for which the exponential function $f(x) = ab^x$ is increasing.

7. Let $f(x) = 2x$ and $g(x) = 2^x$. Complete the tables below for each function.

x	0	1	2	3	4	5
$f(x)$						

x	0	1	2	3	4	5
$g(x)$						

8. Graph $f(x)$ and $g(x)$ below.



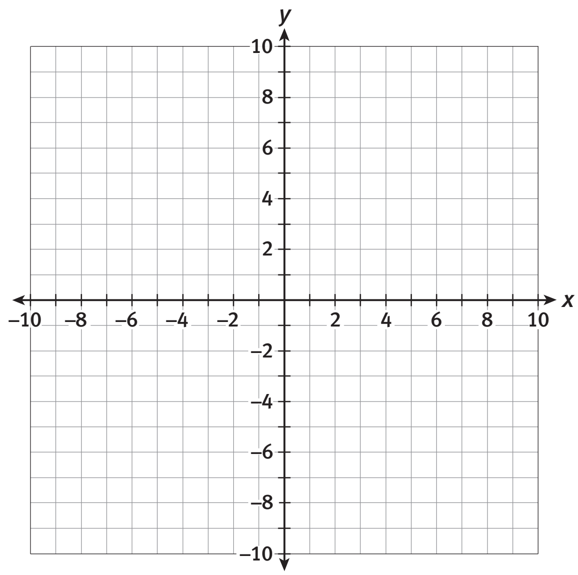
9. Examine the graphs of $f(x)$ and $g(x)$. Compare the values of each function from $x = -2$ through $x = 2$. Which function is greater on this interval?

10. Examine the values of $f(x)$ and $g(x)$ for $x > 2$.
a. Which function is greater on this interval?

My Notes

b. Do you think this will continue to be true as x continues to increase? Explain your reasoning.

11. To take a closer look at the graphs of $f(x)$ and $g(x)$ for larger values of x, regraph the two functions below. Note the new scale.



12. Does the new graph support the prediction you made in Item 10b?

13. Which function increases faster, $f(x)$ or $g(x)$? Explain your reasoning using the graph and the tables.

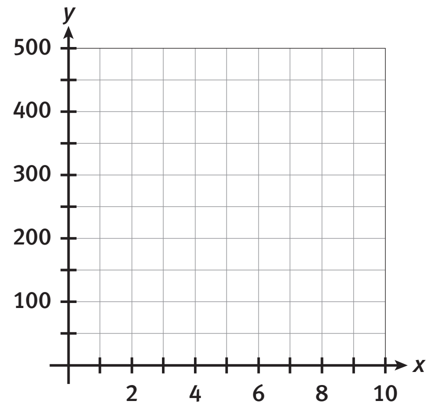
Alex believes that for the linear function $f(x) = 50x$ and the exponential function $g(x) = 2^x$, the value of $f(x)$ is always greater than the value of $g(x)$.

Glenda believes that for a linear function $f(x)$ to always be greater than an exponential function $g(x)$, the graph of $f(x)$ must be very steep while the graph of $g(x)$ must be very flat. She proposes graphing $f(x) = 50x$ and $g(x) = 1.1^x$ to test her conjecture.

14. a. Test Alex's conjecture by graphing $f(x) = 50x$ and $g(x) = 2^x$ on your graphing calculator. Do you agree or disagree with Alex's conjecture? Explain your reasoning.

My Notes

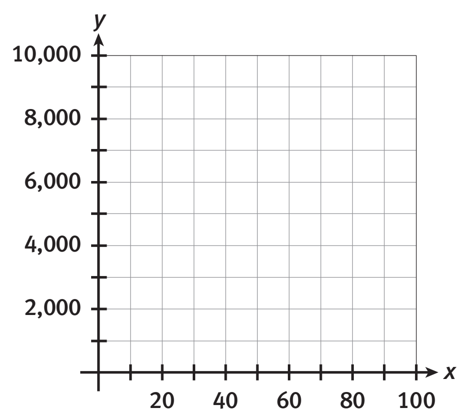
b. **Use appropriate tools strategically.** Now adjust your viewing window to match the coordinate plane below. Sketch the graphs of $f(x)$ and $g(x)$.



15. Should Alex revise his conjecture? Use the graph in Item 14b to explain.

16. a. Test Glenda's conjecture by graphing $f(x) = 50x$ and $g(x) = 1.1^x$ on your graphing calculator. Do you agree with Glenda's conjecture?

b. Now adjust your viewing window to match the coordinate plane below. Sketch the graphs of $f(x)$ and $g(x)$.



17. Should Glenda revise her conjecture? Use the graph in Item 16b to support your response.

18. **Attend to precision.** Is an exponential function always greater than a linear function? Explain your reasoning.

TECHNOLOGY TIP

Use the axis labels on the blank grid to determine how to set the viewing window on your calculator.

My Notes

Check Your Understanding

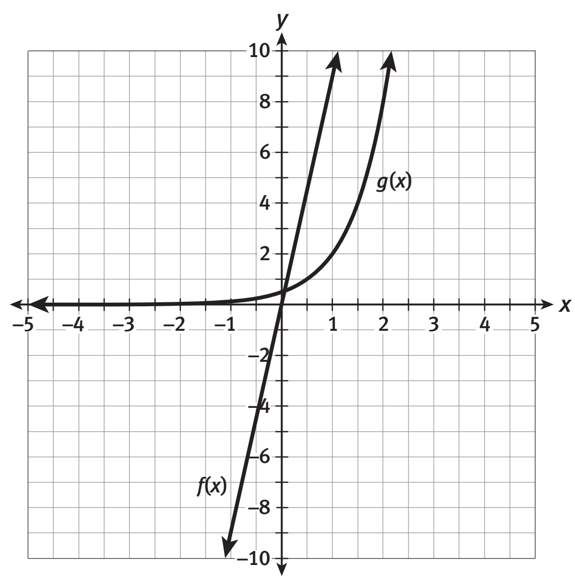
Use the functions $a(x) = 25x$ and $b(x) = 5 \cdot 3^x$ for Items 19 and 20.

19. Without graphing, tell which function increases more quickly. Explain your reasoning.

20. Use your graphing calculator to justify your answer to Item 19. Sketch the graphs from your calculator, and be sure to label your viewing window.

LESSON 22-3 PRACTICE

Isaac graphs $f(x) = 0.5 \cdot 4^x$ and $g(x) = 9x$. His graphs are shown below.



21. Isaac states that $f(x)$ will always be less than $g(x)$. Explain Isaac's error.

22. Describe the relationship between the graphs of $f(x)$ and $g(x)$. Make a new graph to support your answer.

23. Make sense of problems. The math club has only 10 members and wants to increase its membership.

- Julia proposes a goal of recruiting 2 new members each month. If the club meets this goal, the function $y = 2x + 10$ will give the total number of members y after x months.
- Jorge proposes a goal to increase membership by 10% each month. If the club meets this goal, the function $y = 10 \cdot 1.1^x$ will give the total number of members y after x months.

Club members want to choose the goal that will cause the membership to grow more quickly. Assume that the club will meet the recruitment goal that they choose. Which proposal should they choose? Use a graph to support your answer.

ACTIVITY 22 PRACTICE

Write your answers on notebook paper. Show your work.

Lesson 22-1

In January of this year, a clothing store earned \$175,000. Since then, earnings have increased by 10% each month. A function that models the store's earnings after m months is $e(m) = 175{,}000 \cdot (1.1)^m$. Use this information for Items 1–3.

1. Copy and complete the table.

Months After January (m)	Earnings $e(m)$
0	\$175,000
1	
2	
3	
4	

2. Make a graph of the function.

3. Predict the store's earnings after 9 months.

4. A scientist studying a bacteria population recorded the data in the table below.

Time (min)	0	1	2	3
Number of Bacteria	8	20	50	125

Is the number of bacteria growing exponentially? Justify your response.

5. The function $f(x) = 3 \cdot b^x$ is an exponential growth function. Which statement about the value of b is true?

A. Because $f(x)$ is an exponential growth function, b must be positive.
B. Because $f(x)$ is an exponential growth function, b must be greater than 1.
C. Because $f(x)$ is an exponential growth function, b must be between 0 and 1.
D. The function represents exponential growth because $3 > 1$, so b can have any value.

Lesson 22-2

A new car *depreciates*, or loses value, each year after it is purchased. A general rule is that a car loses 15% of its value each year.

Christopher bought a new car for \$25,000. A function that models the value of Christopher's car after t years is $v(t) = 25{,}000 \cdot (0.85)^t$. Use this information for Items 6–8.

6. Copy and complete the table.

Years After Purchase (t)	Value of Car $v(t)$
0	\$25,000
1	
2	
3	
4	

7. Make a graph of the function.

8. Predict the value of Christopher's car after 10 years.

For Items 9–12, graph each function and tell whether it represents exponential growth, exponential decay, or neither.

9. $y = (2.5)^x$

10. $y = -0.75x$

11. $y = 3(1.5)^x$

12. $y = 80(0.25)^x$

For Items 13–15, tell whether each function represents exponential growth, exponential decay, or neither. Justify your responses.

13.

x	0	1	2	3
y	2	60	118	176

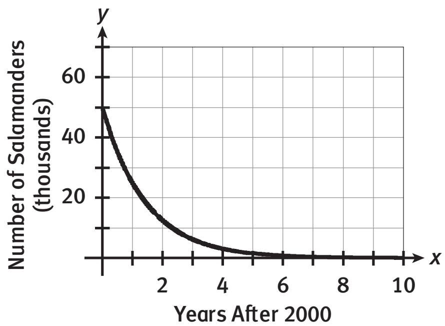
14.

x	0	1	2	3
y	25	5	1	0.2

15.

x	0	2	4	6
y	1	9	81	729

16. A wildlife biologist is studying an endangered species of salamander in a particular region. She finds the following data.



What was the initial number of salamanders in 2000?

17. Write a function that represents exponential decay. Explain how you know that your function represents exponential decay.

Lesson 22-3

Graph each of the following functions. Identify the values of a and b, and describe how these values affect the graphs.

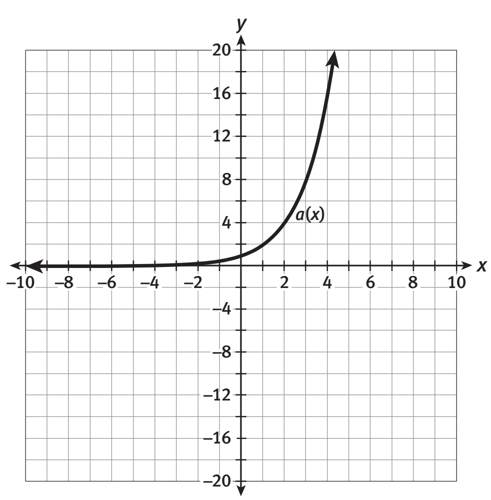
18. $y = -4(2)^x$

19. $y = -1(2.5)^x$

20. $y = 1.5(2)^x$

21. $y = 0.5(0.2)^x$

For Items 22–29, use a graphing calculator to graph each function. Compare each function to $a(x) = 2^x$, graphed below. Describe the similarities and differences between the graphs.



22. $f(x) = 0.5 \cdot 5^x$

23. $f(x) = 2 \cdot (1.1)^x$

24. $f(x) = 12^x$

25. $f(x) = 0.25 \cdot 4^x$

26. $f(x) = -3 \cdot 6^x$

27. $f(x) = -1 \cdot (0.3)^x$

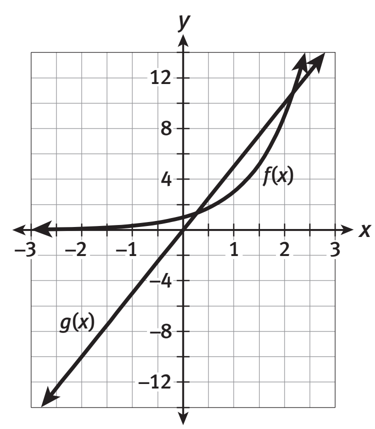
28. $f(x) = 0.1 \cdot 2^x$

29. $f(x) = (0.5)^x$

30. Which function increases the fastest?

A. $y = 104x$
B. $y = -2 \cdot 15^x$
C. $y = 12^x$
D. $y = -220x$

31. Examine the graphs of $f(x) = 3^x$ and $g(x) = 5x$, shown below.



a. Estimate the values of x for which $f(x)$ is greater than $g(x)$.
b. Estimate the values of x for which $g(x)$ is greater than $f(x)$.

MATHEMATICAL PRACTICES
Reason Abstractly and Quantitatively

32. Why can't the value of a in an exponential function be 0? Why can't the value of b be equal to 1?

Modeling with Exponential Functions

ACTIVITY 23

Growing, Growing, Gone

Lesson 23-1 Compound Interest

My Notes

Learning Target:

- Create an exponential function to model compound interest.

SUGGESTED LEARNING STRATEGIES: Create Representations, Look for a Pattern, Predict and Confirm, Discussion Groups, Think-Pair-Share, Critique Reasoning

Madison received $10,000 in gift money when she graduated from college. She deposits the money into an account that pays 5% **compound interest** annually.

MATH TERMS

Compound interest is interest, or money paid by a bank to an account holder, that is earned on both initial account funds, or *principal*, and previously earned interest.

1. To find the total amount of money in her account after the first year, Madison must add the interest earned in the first year to the initial amount deposited.

a. Calculate the earned interest for the first year by multiplying the amount of Madison's deposit by the interest rate of 5%.

b. Including interest, how much money did Madison have in her account at the end of the first year?

2. Madison wants to record the amount of money she will have in her account at the end of each year. Complete the table. Round amounts to the nearest cent.

Year	Account Balance
0	$10,000.00
1	$10,500.00
2	$11,025.00
3	
4	
5	
6	
7	
8	
9	
10	

My Notes

The amount of money in the account increases by a constant growth factor each year.

3. Identify the constant growth factor to the nearest hundredth.

4. How is the interest rate on Madison's account related to the constant growth factor in Item 3?

5. Instead of calculating the amount of money in the account after each year, write an expression for each amount of money using $10,000 and repeated multiplication of the constant factor. Then rewrite each expression using exponents.

Year	Account Balance
0	\$10,000.00
1	$\$10{,}000 \cdot 1.05 = \$10{,}000 \cdot 1.05^1$
2	$(\$10{,}000 \cdot 1.05) \cdot 1.05 = \$10{,}000 \cdot 1.05^2$
3	
4	
5	
6	

6. Describe the relationship between the year number and the exponential expression.

7. Write an expression to represent the amount of money in the account at the end of Year 8.

8. Let t equal the number of years.

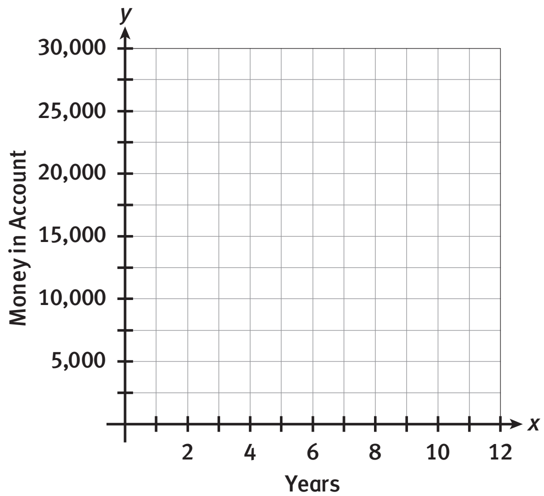
a. **Express regularity in repeated reasoning.** Write an expression to represent the amount of money in the account after t years.

b. Evaluate the expression for $t = 6$ to confirm that the expression is correct.

c. Evaluate the expression for $t = 10$.

9. Write your expression as a function $m(t)$, where $m(t)$ is the total amount of money in Madison's account after t years.

10. Use the data from the table in Item 2 to graph the function.



11. Describe the function as linear or non-linear. Justify your response.

12. Identify the reasonable domain and range. Explain your reasoning.

13. Madison's future plans include purchasing a home. She estimates that she will need at least $20,000 for a down payment. Determine the year in which Madison will have enough funds in her account for the down payment.

At the same time that Madison opens her account, her friend Frank deposits $10,000 in an account with an annual compound interest rate of 6%.

14. Write a new function to represent the total funds in Frank's account, $f(t)$, after t years.

15. Predict how the graph of Frank's bank account balance will differ from the graph of Madison's account balance.

My Notes

16. Create a table of values for $f(t)$, rounding to the nearest dollar. Then graph $f(t)$ on the grid in Item 10. Confirm or revise your prediction in Item 15 using the table and graph.

Year, t	Funds in Frank's Account, $f(t)$
0	\$10,000
1	\$10,600
2	\$11,236
3	
4	
5	
6	
7	
8	
9	
10	

At the same time that Madison and Frank open their accounts, another friend, Kasey, opens a savings account in a different bank. Kasey deposits \$12,000 at an annual compound interest rate of 4%.

17. How does Kasey's situation change the function? Write a new function $k(t)$ to represent Kasey's account balance at any year t.

18. **Critique the reasoning of others.** Kasey believes that since she started her account with more money than Madison or Frank, she will always have more money in her account than either of them, even though her interest rate is lower. Is Kasey correct? Justify your response using a table, graph, or both.

My Notes

19. Over a long period of time, does the initial deposit or the interest rate have a greater effect on the amount of money in an account that has interest compounded yearly? Explain your reasoning.

Most savings institutions offer compounding intervals other than annual compounding. For example, a bank that offers *quarterly compounding* computes interest on an account every quarter; that is, at the end of every 3 months. Instead of computing the interest once each year, interest is computed four times each year. If a bank advertises that it is offering 8% interest compounded quarterly, 8% is not the actual growth factor. Instead, the bank will use $\frac{8\%}{4} = 2\%$ to determine the quarterly growth factor.

CONNECT TO FINANCE

Interest can be compounded semiannually (every 6 months), quarterly (every 3 months), monthly, and daily.

20. What is the quarterly interest rate for an account with an annual interest rate of 5%, compounded quarterly?

21. Suppose that Madison invested her $10,000 in the account described in Item 20.

a. In the table below, determine Madison's account balance after the specified times since her initial deposit.

Time Since Initial Deposit	Number of Times Interest Has Been Compounded	Account Balance
3 months		
6 months		
9 months		
1 year		
4 years		
t years		

b. Write a function $A(t)$ to represent the balance in Madison's account after t years.

c. Calculate the balance in Madison's account after 20 years.

My Notes

MATH TIP

For a given annual interest rate, properties of exponents can be used to approximate equivalent semi-annual, quarterly, monthly, and daily interest rates. For example, the function $f(t) = 5000(1.03)^t$ is used to approximate the balance in an account with an initial deposit of \$5000 and an annual interest rate of 3%. $f(t)$ can be rewritten as

$f(t) = 5000\left(1.03^{\frac{1}{12}}\right)^{12t}$ and is

equivalent to the function $g(t) = 5000(1.0025)^{12t}$, which reveals that the approximate equivalent monthly interest rate is 0.25%.

$f(t) = 5000\left(1.03^{\frac{1}{4}}\right)^{4t}$ is equivalent to

$h(t) = 5000(1.0074)^{4t}$, which reveals that the approximate equivalent quarterly interest rate is 0.74%.

22. For the compounding periods given below, write a function to represent the balance in Madison's account after *t* years. Then calculate the balance in the account after 20 years. She is investing \$10,000 at a rate of 5% annual compound interest.

a. Yearly:

b. Quarterly:

c. Monthly:

d. Daily (assume there are 365 days in a year):

23. What is the effect of the compounding period on the amount of money in the account after 20 years as the number of times the interest is compounded each year increases?

Check Your Understanding

24. Write a function that gives the amount of money in Frank's account after *t* years when 6% annual interest is compounded monthly.

25. Create a table and a graph for the function in Item 24. Be sure to label the units on the *x*-axis correctly.

LESSON 23-1 PRACTICE

Model with mathematics. Nick deposits \$5000 into an account with a 4% annual interest rate, compounded annually.

26. Create a table showing the amount of money in Nick's account after 0–8 years.

27. Write a function that gives the amount of money in Nick's account after *t* years. Identify the reasonable domain and range.

28. Create a graph of your function.

29. Explain how Nick's account balance would be different if he deposited his money into an account that pays 2% annual interest, compounded annually. Graph this situation on the same coordinate plane that you used in Item 28. Describe the similarities and differences between the graphs.

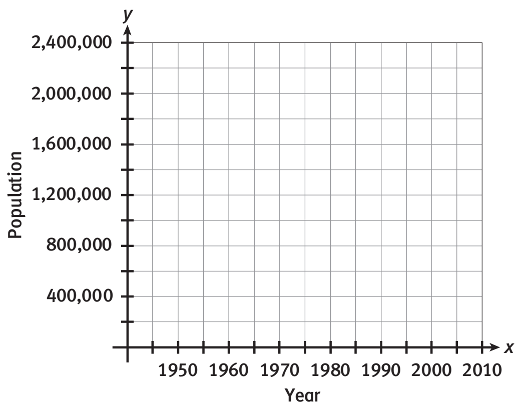
Lesson 23-2
Population Growth

Learning Targets:
- Create an exponential function to fit population data.
- Interpret values in an exponential function.

SUGGESTED LEARNING STRATEGIES: Create Representations, Look for a Pattern, Predict and Confirm, Think Aloud, Sharing and Responding, Construct Arguments

The population of Nevada since 1950 is shown in the table in the *My Notes* section.

1. Graph the data from the table.



2. Use the table and the graph to explain why the data are not linear.

3. a. Complete the table by finding the approximate ratio between the populations in each decade.

Decades Since 1950	Resident Population	Ratio
0	160,083	--
1	285,278	$\frac{285{,}278}{160{,}083} \approx 1.782$
2	488,738	$\frac{488{,}738}{285{,}278} \approx$
3	800,508	
4	1,201,833	
5	1,998,257	

b. Explain how the table shows that the data are not exponential.

The data are not exactly exponential, but the shape of the graph resembles an exponential curve. Also, the table in Item 3a shows a near-constant factor. These suggest that the data are approximately exponential. Use **exponential regression** to find an exponential function that models the data.

My Notes

Year	Resident Population
1950	160,083
1960	285,278
1970	488,738
1980	800,508
1990	1,201,833
2000	1,998,257

MATH TERMS

Exponential regression is a method used to find an exponential function that models a set of data.

My Notes

TECHNOLOGY TIP

On a Texas Instruments (TI) calculator, perform exponential regression using the ExpReg function.

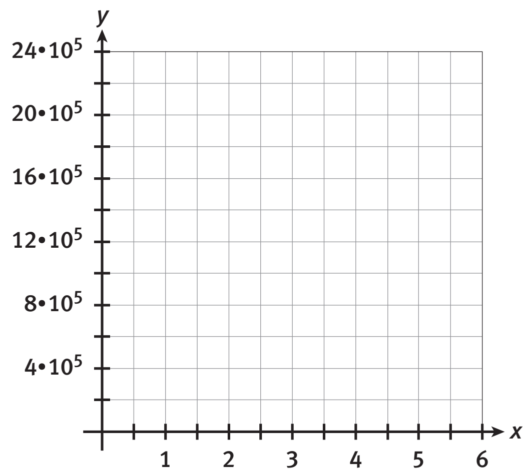
4. Use a graphing calculator to determine the exponential regression equation to model the relationship between the decades since 1950 and the population.

a. The calculator returns two values, a and b. Write these values below. Round a to the nearest whole number and b to the nearest thousandth, if necessary.

$a =$ $b =$

b. The general form of an exponential function is $y = ab^x$. Use this general form and the values of a and b from Part (a) to write an exponential function that models Nevada's population growth.

c. Use a graphing calculator to graph the data points and the function from Part (b). Sketch the graph and the data points below. Is the exponential function a good approximation of the data? Explain.



5. **Reason abstractly.** What does the value of b tell you about Nevada's population growth?

6. Interpret the value of a in terms of Nevada's population. How is this value related to the graph?

7. What do the domain values represent?

8. What would the x-intercept represent in terms of Nevada's population? Does the graph have an x-intercept? Explain.

My Notes

TECHNOLOGY TIP

Use the table function on a graphing calculator to determine the value of the function when $x = 4.5$.

9. **Make sense of problems.** Describe how to estimate the population of Nevada in 1995 using each of the following:
   a. the function identified in Item 4b
   b. the graph of the function
   c. a table

10. Estimate the population in 1995. Which method did you use, and why?

11. a. Estimate Nevada's population in 2010.
    b. **Construct viable arguments.** Which estimate do you think is likely to be more accurate, your estimate of the population in 1995 or in 2010? Explain.

The function for the growth rate of Nevada's population estimates the growth per decade. You can use this rate to estimate the growth per year, or the annual growth rate.

12. Let n be the number of years since 1950. Write an equation that gives the number of years n in x decades. Solve your equation for x.

13. Rewrite the function that models Nevada's population from Item 4b. Then write the function again, but replace x with the equivalent expression for x from Item 12.

14. Simplify to write the function in the form $y = ab^n$.

$$y = 170,377 \cdot (1.645)^{\frac{n}{10}} = 170,377 \cdot \left(1.645^{\frac{1}{10}}\right)^{\square}$$

$$\approx 170,377 \cdot (\quad)^n$$

My Notes

15. What is the approximate annual growth rate of Nevada's population? How do you know?

16. To find the approximate population of Nevada in 2013, what value should you use for n? Explain.

17. Use the function from Item 14 to find the approximate population of Nevada for the year 2013.

18. Compare the approximate population for 2013 that you found in Item 17 to the approximate population you found for 2010 in Item 11. Does your estimate for 2013 seem reasonable? Why or why not?

Check Your Understanding

19. Create a graph showing the annual growth of Nevada's population.

20. Describe the similarities and differences between the graph in Item 19 and the previous graph of Nevada's population from Item 4c.

LESSON 23-2 PRACTICE

The population of Texas from 1950 to 2000 is shown in the table below.

Year	Resident Population
1950	7,711,194
1960	9,579,677
1970	11,198,655
1980	14,225,513
1990	16,986,510
2000	20,851,820

21. **Use appropriate tools strategically.** Use a graphing calculator to find a function that models Texas's population growth.

22. Create a graph showing the actual population from the table and the approximate population from the function in Item 21.

23. Is the function a good fit for the data? Why or why not?

24. Describe the meanings of the domain, range, y-intercept, and x-intercept in the context of Texas's population growth.

ACTIVITY 23 PRACTICE

Write your answers on notebook paper. Show your work.

Lesson 23-1

1. Four friends deposited money into savings accounts. The amount of money in each account is given by the functions below.

   Marisol: $m(t) = 100 \cdot (1.01)^t$
   Iris: $i(t) = 200 \cdot (1.04)^t$
   Brenda: $b(t) = 300 \cdot (1.05)^t$
   José: $j(t) = 400 \cdot (1.03)^t$

   Which statement is correct?
   A. José has the greatest interest rate.
   B. Brenda has the greatest initial deposit.
   C. The person with the least initial deposit also has the least interest rate.
   D. The person with the greatest initial deposit also has the greatest interest rate.

Darius makes an initial deposit into a bank account, and then earns interest on his account. He records the amount of money in his account each year in the table below. Use this table for Items 2–5.

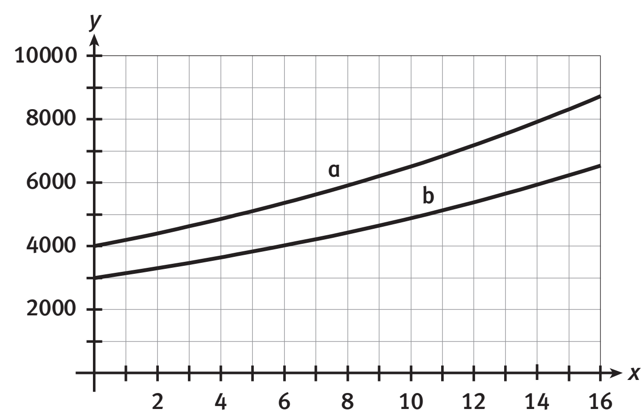
Year	Amount
0	\$4000.00
1	\$4120.00
2	\$4243.60
3	\$4370.91
4	\$4502.04

2. Make a graph showing the amount of money in Darius's account each year.
3. Identify the constant factor. Round to the nearest hundredth.
4. Identify the reasonable domain and range. Explain your answers.
5. What is the annual interest rate? How do you know?

The amount of money y in Jesse's checking account t years after the account was opened is given by the function $j(t) = 15{,}000 \cdot (1.02)^t$. Use this information for Items 6–10.

6. What was the initial amount of money deposited in Jesse's account?
7. What is the annual interest rate?
8. Create a graph of the amount of money in Jesse's checking account.
9. Interpret the meaning of the y-intercept in the context of Jesse's account.
10. Find the amount of money in the account after 4 years.

The two graphs on the coordinate grid below represent the amounts of money in two different savings accounts. Graph a represents the amount of money in Allison's account, and graph b represents the amount of money in Boris's account. Use the graph for Items 11–13.



11. Whose account had a higher initial deposit? Use the graph to justify your answer.
12. What was the amount of Allison's initial deposit?
13. Identify the reasonable domain and range for each function, and explain your answers.

Maria's bank offers two types of savings accounts. The first has an annual interest rate of 8% compounded annually. The second also has an annual interest rate of 8%, but it is compounded monthly. She is going to open an account by depositing $1000. Use this information for Items 14–19.

14. If Maria chooses the first account, determine the amount of money she will have in the account after 3 years.

15. Write a function that gives the amount of money in the first account after t years.

16. Write a function that gives the amount of money in the second account after t years.

17. What is the *monthly* interest rate for the second account?

18. If Maria chooses the second account, determine the amount of money she will have in the account after 1 year.

19. After 10 years, which account will have the higher balance?

Lesson 23-2

20. Which function is the best model for the data in the table?

x	y
0	19
1	44.5
2	112
3	282
4	704

A. $y = 172x + 18$ **B.** $y = 44x^2$
C. $y = 44x$ **D.** $y = 172 \cdot 18^x$

For Items 21 and 22, tell whether an exponential function would be a good model for each data set. Explain your answers.

21.

x	0	1	2	3	4
y	33	58	120	247	506

22.

x	0	3	6	9	4
y	16.5	12	8.5	4.6	0

The head circumference of an infant is measured and recorded to track the infant's growth and development. Nathan's head circumferences from age 3 months through 12 months are recorded in the table below. Use the table for Items 23–27.

Age (months)	Head Circumference (cm)
3	43
4	44
5	44.7
6	45.2
7	45.8
8	46.3
9	47.1
10	47.6
11	48.0
12	48.3

23. Use a graphing calculator to find an exponential function to approximate Nathan's head circumference.

24. Identify the reasonable domain and range for the function in Item 23. Explain your answers.

25. Create a graph showing Nathan's head circumference.

26. Determine the growth rate, and explain how you found your answer.

27. Interpret the meaning of the y-intercept in the context of Nathan's head circumference.

MATHEMATICAL PRACTICES
Attend to Precision

28. Explain why the x-intercept can have a meaning in the context of a situation, such as population growth, but cannot be shown on the graph.

Adding and Subtracting Polynomials

ACTIVITY 24

Polynomials in the Sun

Lesson 24-1 Polynomial Terminology

My Notes

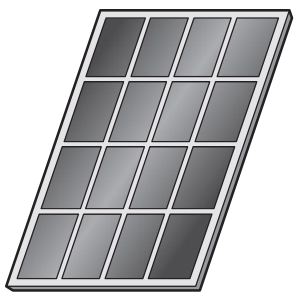
Learning Targets:

- Identify parts of a polynomial.
- Identify the degree of a polynomial.

SUGGESTED LEARNING STRATEGIES: Create Representations, Vocabulary Organizer, Interactive Word Wall, Think-Pair-Share, Close Reading

A solar panel is a device that collects and converts solar energy into electricity or heat. The solar panel consists of interconnected solar cells. The panels can have differing numbers of solar cells and can come in square or rectangular shapes.

1. How many solar cells are in the panel below?



2. Reason abstractly. If a solar panel has four rows as the picture does, but can be extended to have an unknown number of columns, x, write an expression to give the number of solar cells that could be in the panel.

3. Write an expression that would give the total number of cells in the panel for a solar panel having x rows and x columns.

4. If there were 5 panels like those found in Item 3, write an expression to represent the total number of solar cells.

All the answers in Items 1–4 are called *terms*. A **term** is a number, variable, or the product of a number and/or variable(s).

5. Write an expression to represent the sum of your answers from Items 1, 2, and 4.

CONNECT TO SCIENCE

Solar panels, also known as photovoltaic panels, are made of semiconductor materials. A panel has both positive and negative layers of semiconductor material. When sunlight hits the semiconductor, electrons travel across the intersection of the two different layers of materials, creating an electric current.

My Notes

MATH TERMS

A **coefficient** is the numeric factor of a term.

A **constant term** is a term that contains only a number, such as the answer to Item 1. The constant term of a polynomial is a term of degree zero.

Expressions like the answer to Item 5 are called polynomials. A **polynomial** is a single term or the sum of two or more terms with *whole-number powers*.

6. List the terms of the polynomial you wrote in Item 5.

7. What are the **coefficients** of the polynomial in Item 5? What is the **constant term**?

Check Your Understanding

8. Tell whether each expression is a polynomial. Explain your reasoning.

a. $3x^{-2} - 5$ **b.** $6x + 4x^2$ **c.** 15 **d.** $2 + x^{\frac{1}{2}}$

9. For the expressions in Item 8 that are polynomials, identify the terms, coefficients, and constant terms.

The **degree of a term** is the sum of the exponents of the variables contained in the term.

10. Identify the degree and coefficient of each term in the polynomial $4x^5 + 12x^3 + x^2 - x + 5$.

Term	Degree	Coefficient
$4x^5$	5	
$12x^3$		12
x^2		
$-x$		

11. Make use of structure. For the polynomial $2x^3y - 6x^2y^2 + 9xy - 13y^5 + 5x + 15$, list each term and identify its degree and coefficient. Identify the constant term.

Lesson 24-1
Polynomial Terminology

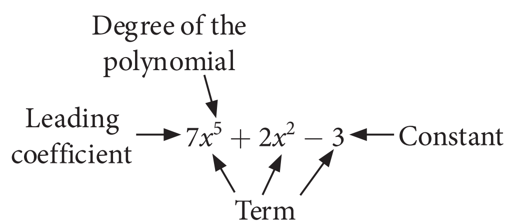
My Notes

The **degree of a polynomial** is the greatest degree of any term in the polynomial.

12. Identify the degree and constant term of each polynomial.

Polynomial	Degree of Polynomial	Constant Term
$2x^2 + 3x + 7$	2	
$-5y^3 + 4y^2 - 8y - 3$		
$36 + 12x + x^2$		36

The **standard form of a polynomial** is a polynomial whose terms are written in **descending order** of degree. The **leading coefficient** is the coefficient of a polynomial's leading term when the polynomial is written in standard form.



MATH TERMS

Descending order of degree means that the term that has the highest degree is written first, the term with the next highest degree is written next, and so on.

A polynomial can be classified by the number of terms it has when it is in simplest form.

Name	Number of Terms n	Examples
monomial	1	8 or $-2x$ or $3x^2$
binomial	2	$3x + 2$ or $4x^2 - 7x$
trinomial	3	$-x^2 - 3x + 9$
polynomial	$n > 3$	$9x^4 - x^3 - 3x^2 + 7x - 2$

READING MATH

The prefixes mono (one), bi (two), tri (three), and poly (many) appear in many math terms such as bisect (cut in half), triangle (three-sided figure), and polygon (many-sided figure).

My Notes

13. Fill in the missing information in the table below.

Polynomial	Number of Terms	Name	Leading Coefficient	Constant Term	Degree
$3x^2 - 5x$					
$-2x^2 + 13x + 6$					
$15x^2$					
$5p^3 + 2p^2 - p - 7$					
$a^2 - 25$					
$0.23x^3 + 0.54x^2 - 0.58x + 0.0218$					
$-9.8t^2 - 20t + 150$					

Check Your Understanding

14. Is the following statement true or false? Explain.
"All polynomials are binomials."

15. Describe your first step for writing $3x - 5x^2 + 7$ in standard form.

LESSON 24-1 PRACTICE

For Items 16–20, use the polynomial $4x^3 + 3x^2 - 9x + 7$.

16. Name the coefficients of the terms in the polynomial that have variables.

17. List the terms, and give the degree of each term.

18. What is the degree of the polynomial?

19. Identify the leading coefficient of the polynomial.

20. Identify the constant term of the polynomial.

Write each polynomial in standard form.

21. $9 + 8x^2 + 2x^3$

22. $y^2 + 1 + 4y^3 - 2x$

23. **Construct viable arguments.** Is the expression $5x^2 + \sqrt{2}x$ a polynomial? Justify your response.

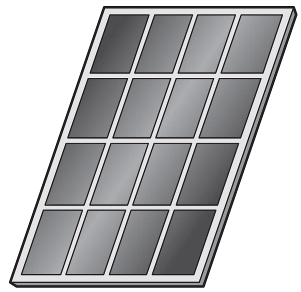
My Notes

Learning Targets:

- Use algebra tiles to add polynomials.
- Add polynomials algebraically.

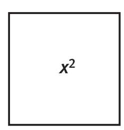
SUGGESTED LEARNING STRATEGIES: Discussion Groups, Use Manipulatives, Create Representations, Close Reading, Note Taking

Notice that in the solar panels at the right, there are 4^2 or 16 cells. Each column has 4 cells.



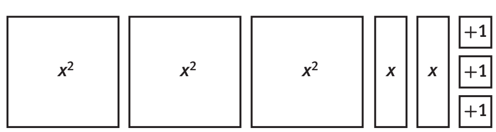
1. If a square solar panel with an unknown number of cells along the edge can be represented by x^2, how many cells would be in one column of the panel?

A square solar panel with x rows and x columns can be represented by the algebra tile:



A column of x cells can be represented by using the tile [x], and a single solar cell can be represented by [+1].

Suppose there were 3 square solar panels that each had x columns and x rows, 2 columns with x cells, and 3 single solar cells. You can represent $3x^2 + 2x + 3$ using algebra tiles.



2. Represent $2x^2 - 3x + 2$ using algebra tiles. Draw a picture of the representation below.

MATH TIP

The additive inverse of the x^2, x, and 1 algebra tiles can be represented with another color, or the flip side of the tile.

My Notes

Adding polynomials using algebra tiles can be done by:

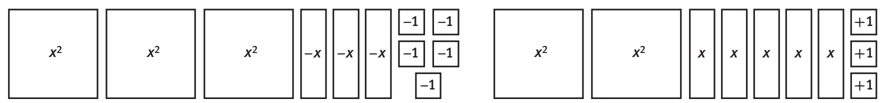
- modeling each polynomial
- identifying and removing zero pairs
- writing the new polynomial

Example A

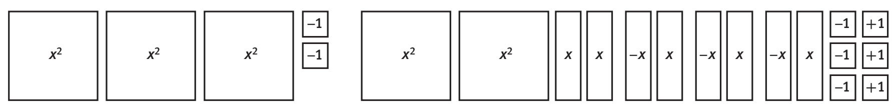
Add $(3x^2 - 3x - 5) + (2x^2 + 5x + 3)$ using algebra tiles.

Step 1: Model the polynomials.

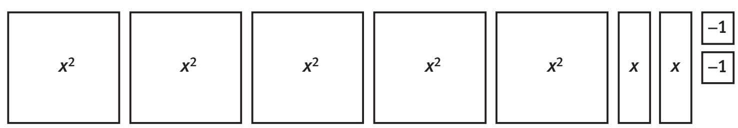
$3x^2 - 3x - 5$ $\qquad$ $2x^2 + 5x + 3$



Step 2: Identify and remove zero pairs.



Step 3: Combine like tiles.



Step 4: Write the polynomial for the model in Step 3.

$5x^2 + 2x - 2$

Solution: $(3x^2 - 3x - 5) + (2x^2 + 5x + 3) = 5x^2 + 2x - 2$

Try These A

Add using algebra tiles.

a. $(x^2 - 2) + (2x^2 + 5)$

b. $(2y^2 + 3y + 6) + (3y^2 - 4)$

c. $(2x^2 + 3x + 9) + (-x^2 - 4x - 6)$

d. $(5 - 3x + x^2) + (2x + 4 - 3x^2)$

My Notes

3. **Use appropriate tools strategically.** Can you use algebra tiles to add $(4x^4 + 3x^2 + 15) + (x^4 + 10x^3 - 4x^2 + 22x - 23)$? If so, model the polynomials and add. If not, explain why.

Like terms in an expression are terms that have the same variable and exponent for that variable. All constants are like terms.

4. State whether the terms are like or unlike terms. Explain.

   a. $2x$; $2x^3$

   b. 5; $5x$

   c. $-3y$; $3y$

   d. x^2y; xy^2

   e. 14; -0.6

5. **Attend to precision.** Using vocabulary from this activity, describe a method that could be used to add polynomials without using algebra tiles.

My Notes

Use the properties of real numbers to add polynomials algebraically.

Example B

Add $(3x^3 + 2x^2 - 5x + 7) + (4x^3 + 2x - 3)$ horizontally and vertically. Write your answer in standard form.

Horizontally

Step 1: Identify like terms. $(3x^3 + 2x^2 - 5x + 7) + (4x^3 + 2x - 3)$

Step 2: Group like terms. $(3x^3 + 4x^3) + (2x^2) + (-5x + 2x) + (7 - 3)$

Step 3: Add the coefficients of like terms. $7x^3 + 2x^2 - 3x + 4$

Solution: $(3x^3 + 2x^2 - 5x + 7) + (4x^3 + 2x - 3) = 7x^3 + 2x^2 - 3x + 4$

Vertically

Step 1: Vertically align like terms.

Step 2: Add the coefficients of like terms.

$$\begin{array}{r} 3x^3 + 2x^2 - 5x + 7 \\ +4x^3 \qquad\quad + 2x - 3 \\ \hline 7x^3 + 2x^2 - 3x + 4 \end{array}$$

Solution: $(3x^3 + 2x^2 - 5x + 7) + (4x^2 + 2x - 3) = 7x^3 + 2x^2 - 3x + 4$

MATH TIP

The Commutative and Associative Properties of Addition allow you to re-order and group like terms.

Try These B

Add. Write your answers in standard form.

a. $(4x^2 + 3) + (x^2 - 3x + 5)$

b. $(10y^2 + 8y + 6) + (17y^2 - 11)$

c. $(9x^2 + 15x + 21) + (-13x^2 - 11x - 26)$

6. Are the answers to Try These B polynomials? Justify your response.

7. Explain why the sum of two polynomials will always be a polynomial.

MATH TIP

Polynomials are *closed* under addition. A set is closed under addition if the sum of any two elements in the set is also an element of the set.

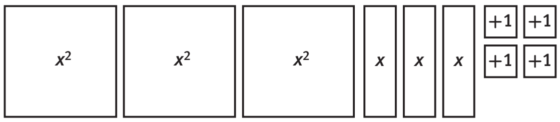
Lesson 24-2
Adding Polynomials

My Notes

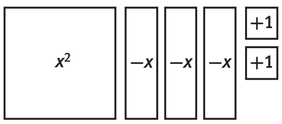
Check Your Understanding

Write a polynomial for each expression represented by algebra tiles.

8.



9.



10. Add the expressions you wrote for Items 8 and 9.

11. What property or properties justify Steps 1 and 2 below?

$(2x^2 + x + 1) + (x^2 + 2x - 1)$

Step 1: $(2x^2 + x^2) + (x + 2x) + (1 - 1)$

Step 2: $(2 - 1)x^2 + (1 + 2)x + (1 - 1)$

Step 3: $x^2 + 3x$

LESSON 24-2 PRACTICE

Add. Write your answers in standard form.

12. $(3x^2 + x + 5) + (2x^2 + x - 5)$

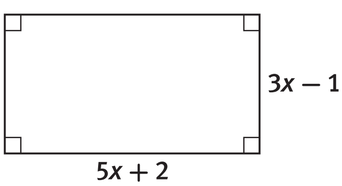
13. $(-4x^2 + 2x - 1) + (x^2 - x + 9)$

14. $(7x^2 - 2x + 3) + (3x^2 + 2x + 7)$

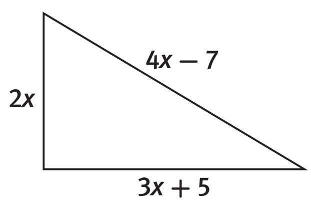
15. $(-x^2 + 5x + 2) + (-3x^2 + x - 9)$

Write the perimeter of each figure as a polynomial in standard form.

16.



17.



18. Critique the reasoning of others. A student added the expressions $x^4 + 5x^2 - 2x + 1$ and $2x^4 + x^3 + 2x - 7$. Identify and correct the student's error.

$$\begin{array}{r} x^4 + 5x^2 - 2x + 1 \\ \underline{2x^4 + x^3 + 2x - 7} \\ 3x^4 + 6x^2 - 6 \end{array}$$

My Notes

MATH TERMS

The **opposite** of a number or a polynomial is its additive inverse.

Learning Target:

- Subtract polynomials algebraically.

SUGGESTED LEARNING STRATEGIES: Note Taking, Close Reading, Think-Pair-Share

To subtract a polynomial you add its **opposite**, or subtract each of its terms.

Example A

Subtract $(2x^3 + 8x^2 + x + 10) - (5x^2 - 4x + 6)$ horizontally and vertically. Write the answer in standard form.

__Horizontally__

Step 1: Distribute the negative. $(2x^3 + 8x^2 + x + 10) - (5x^2 - 4x + 6)$

Step 2: Identify like terms. $2x^3 + 8x^2 + x + 10 - 5x^2 + 4x - 6$

Step 3: Group like terms. $2x^3 + (8x^2 - 5x^2) + (x + 4x) + (10 - 6)$

Step 4: Combine coefficients of like terms. $2x^3 + 3x^2 + 5x + 4$

Solution: $(2x^3 + 8x^2 + x + 10) - (5x^2 - 4x + 6) = 2x^3 + 3x^2 + 5x + 4$

__Vertically__

Step 1: Vertically align like terms.

$$\begin{array}{r} 2x^3 + 8x^2 + x + 10 \\ \underline{-(5x^2 - 4x + 6)} \end{array}$$

Step 2: Distribute the negative.

$$\begin{array}{r} 2x^3 + 8x^2 + x + 10 \\ \underline{-5x^2 + 4x - 6} \end{array}$$

Step 3: Combine coefficients of like terms. $2x^3 + 3x^2 + 5x + 4$

Solution: $(2x^3 + 8x^2 + x + 10) - (5x^2 - 4x + 6) = 2x^3 + 3x^2 + 5x + 4$

Lesson 24-3
Subtracting Polynomials

My Notes

Try These A

Subtract. Write your answers in standard form.

a. $(5x^2 - 5) - (x^2 + 7)$

b. $(2x^2 + 3x + 2) - (-5x^2 - 2x - 9)$

c. $(y^2 + 3y + 8) - (4y^2 - 9)$

d. $(12 + 5x + 14x^2) - (8x + 15 - 7x^2)$

1. Are the answers to Try These A polynomials? Justify your response.

MATH TIP

Polynomials are *closed* under subtraction. A set is closed under subtraction if the difference of any two elements in the set is also an element of the set.

2. Explain why the difference of two polynomials will always be a polynomial.

My Notes

Check Your Understanding

Rewrite each difference as addition of the opposite, or additive inverse, of the second polynomial.

3. $(x^2 + 2x + 3) - (4x^2 - x + 5)$
4. $(5y^2 + y - 2) - (-y^2 - 3y + 4)$
5. **Critique the reasoning of others.** Gil used the vertical method to subtract $(3x^2 - 5x + 2) - (x^2 + 2x + 4)$ as shown below. Identify Gil's error.

$$\begin{array}{r} 3x^2 - 5x + 2 \\ \underline{-\ x^2 + 2x + 4} \\ 2x^2 - 3x + 6 \end{array}$$

LESSON 24-3 PRACTICE

Subtract. Write your answers in standard form.

6. $(2x^2 + 4x + 1) - (7x^2 - 3x - 4)$
7. $(x^2 + 3x - 9) - (x^2 + 2x - 8)$
8. $(9x^2 + x - 12) - (14x^2 - 7x - 2)$
9. $(x^2 + 3x - 6) - (5x - 6)$
10. $(y^4 + y^2 + 2y) - (-y^4 + 3)$
11. Write two polynomials whose difference is $6x + 3$.
12. **Model with mathematics.** A rectangular piece of paper has area $4x^2 + 3x + 2$. A square is cut from the rectangle and the remainder of the rectangle is discarded. The area of the discarded paper is $3x^2 + x + 1$. What is the area of the square?

ACTIVITY 24 PRACTICE

Write your answers on notebook paper.
Show your work.

Lesson 24-1

For Items 1–5, use the polynomial $5x^4 - 2x^2 + 8x - 3$.

1. Identify the coefficients of the variable terms of the polynomial.
2. List the terms, and give the degree of each term.
3. State the degree of the polynomial.
4. Identify the leading coefficient of the polynomial.
5. Identify the constant term of the polynomial.
6. Consider the expressions $3x^2 + 2x - 7$ and $3x^2 + 2x - 7x^0$. Are the expressions equivalent? Explain.

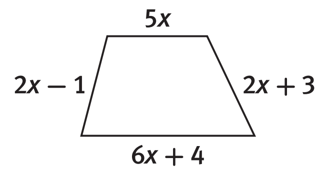
Write each polynomial in standard form.

7. $5x^2 - 2x^3 - 10$
8. $11 + y^2 - 8y$
9. $y^2 - 12 - y^3 + y^4$
10. $9x + 7 - 5x^3$

Lesson 24-2

Add. Write your answers in standard form.

11. $(4x + 9) + (3x - 5)$
12. $(2x^2 + 3x - 1) + (x^2 - 5x + 2)$
13. $(x^3 + 5x^2 + 3) + (2x^3 - 5x^2)$
14. $(7y^3 - 2y^2 + 5) + (4y^3 - 3y)$
15. Which expression represents the perimeter of the trapezoid?



A. $11x + 4$
B. $4x + 2$
C. $15x + 6$
D. $13x + 8$

16. The length of each side of a square is $4y + 5$. Draw and label the square, and write an expression to represent its perimeter.

Lesson 24-3

17. Which expression is equivalent to $10x - (7x - 1)$?
A. $3x - 1$
B. $3x + 1$
C. $17x - 1$
D. $17x + 1$

Subtract. Write your answers in standard form.

18. $(5x - 4) - (3x + 2)$
19. $(3x^2 - 2x + 7) - (2x^2 + 2x - 7)$
20. $(8y^2 - 3y + 6) - (-2y^2 - 3)$
21. $(x^2 - 5x) - (4x - 6)$

Determine the sum or difference. Write your answers in standard form.

22. $(5y^2 + 3y + 7) + (7y - 2)$

23. $(3x^2 + x + 9) - (2x^2 + x + 2)$

24. $(x^3 + 3x^2 + 12) - (5x^3 + 7x^2)$

25. $(8x^3 - 5x + 7) + (4x^3 + 3x^2 - 3x - 4)$

26. $(-4y^2 - 2y + 1) - (7y^3 + y^2 - y + 5)$

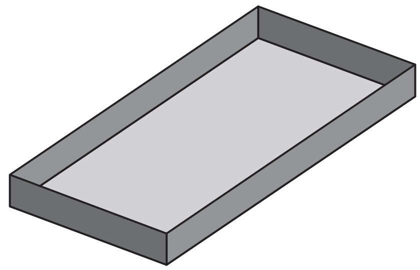
27. $(3x + 7y) + (-4x + 3y)$

28. $(5x^2 + 8xy + y^2) - (-x^2 + 4xy - 5y^2)$

29. A playground has a sidewalk border around a play area.

The total area of the playground, the larger rectangle, is $16x^2 - 5x + 2$. The area of the play area, the smaller rectangle, is $10x^2 + 3x - 1$. Write an expression to represent the area of the sidewalk.

30. To make a box, four corners of a rectangular piece of cardboard are cut out and the box is folded and taped.



The area of the cardboard, after the corners are cut out, is $28x^2 + 12x + 32$. The area of each cut-out corner is $2x^2 + 3$. Write an expression to represent the area of the original piece of cardboard.

MATHEMATICAL PRACTICES
Reason Abstractly and Quantitatively

31. The set of polynomials is closed under the operations of addition and subtraction. This means that when you add or subtract two polynomials, the result is also a polynomial.

a. Are the integers closed under addition and subtraction? In other words, when you add or subtract two integers, is the result always an integer? Justify your response.

b. Give a counterexample to show that the whole numbers are **not** closed under subtraction.

Multiplying Polynomials

Tri-Com Computers

Lesson 25-1 Multiplying Binomials

My Notes

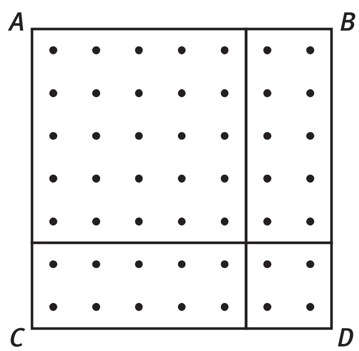
Learning Targets:

- Use a graphic organizer to multiply expressions.
- Use the Distributive Property to multiply expressions.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Look for a Pattern, Discussion Groups, Create Representations, Graphic Organizer

Tri-Com Computers is a company that sets up local area networks in offices. In setting up a network, consultants need to consider not only where to place computers, but also where to place peripheral equipment, such as printers.

Tri-Com typically sets up local area networks of computers and printers in square or rectangular offices. Printers are placed in each corner of the room. The primary printer A serves the first 25 computers and the other three printers, B, C, and D, are assigned to other regions in the room. Below is an example.

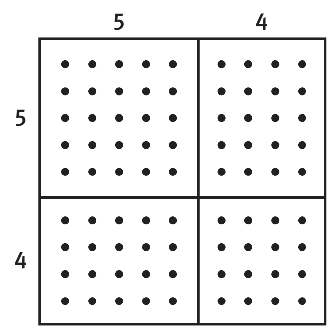


1. If each dot represents a computer, how many computers in this room will be assigned to each of the printers?

2. What is the total number of computers in the room? Describe two ways to find the total.

My Notes

Another example of an office in which Tri-Com installed a network had 9 computers along each wall. The computers are aligned in an array with the number of computers in each region determined by the number of computers along the wall.



3. A technician claimed that since $9 = 5 + 4$, the number of computers in the office could be written as an expression using only the numbers 5 and 4. Is the technician correct? Explain.

4. Show another way to determine the total number of computers in the office.

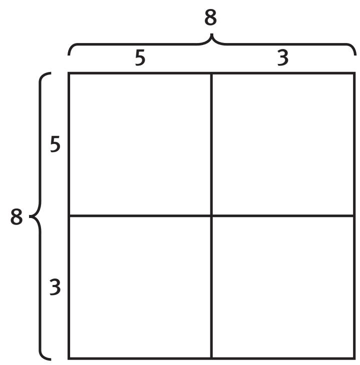
5. Rewrite the expression $(5 + 4)(5 + 4)$ using the Distributive Property.

6. **Make sense of problems.** Explain why $(5 + 4)(5 + 4)$ could be used to determine the total number of computers.

My Notes

7. The office to the right has 8^2 computers. Fill in the number of computers in each section if it is split into a $(5 + 3)^2$ configuration.

8. What is the total number of computers? Describe two ways to find the total.



9. For each possible office configuration below, draw a diagram like the one next to Item 7. Label the number of computers on the edge of each section and determine the total number of computers in the room by adding the number of computers in each section.

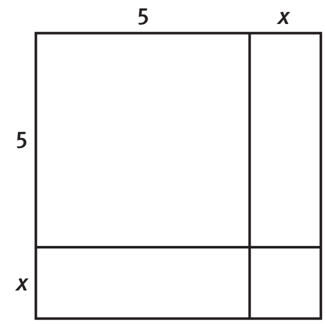
a. $(2 + 3)^2$

b. $(4 + 1)^2$

c. $(3 + 7)^2$

My Notes

Tri-Com has a minimum requirement of 25 computers per installation arranged in a 5 by 5 array. Some rooms are larger than others and can accommodate more than 5 computers along each wall to complete a square array. Use a variable expression to represent the total number of computers needed for any office having x more than the 5 computer minimum along each wall.



10. One technician said that $5^2 + x^2$ would be the correct way to represent the total number of computers in the office space. Use the diagram to explain how the statement is incorrect.

11. **Model with mathematics.** Write an expression for the sum of the number of computers in each region in Item 10.

12. For each of the possible room configurations, determine the total number of computers in the room.

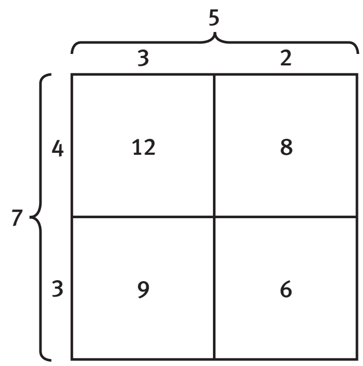
a. $(2 + x)^2$

b. $(x + 3)^2$

c. $(x + 6)^2$

My Notes

The graphic organizer below can be used to help arrange the multiplications of the Distributive Property. It does not need to be related to the number of computers in an office. For example, this graphic organizer shows $5 \cdot 7 = (3 + 2)(4 + 3)$.



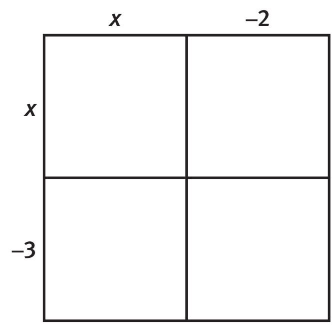
13. Draw a graphic organizer to represent the expression $(5 + 2)(2 + 3)$. Label each inner rectangle and find the sum.

14. Draw a graphic organizer to represent the expression $(6 - 3)(4 - 2)$. Label each inner rectangle and find the sum.

15. Multiply the binomials in Item 14 using the Distributive Property. What do you notice?

My Notes

You can use the same graphic organizer to multiply binomials that contain variables. The following diagram represents $(x - 2)(x - 3)$.



16. Use the graphic organizer above to represent the expression $(x - 2)(x - 3)$. Label each inner rectangle and find the sum.

17. Multiply the binomials in Item 16 using the Distributive Property. What do you notice?

18. Determine the product of the binomials.

a. $(x - 7)(x - 5)$

b. $(x - 7)(x + 5)$

c. $(x + 7)(x + 5)$

d. $(x + 7)(x - 5)$

e. $(4x + 1)(x + 3)$

f. $(2x - 1)(3x + 2)$

19. **Reason abstractly.** Examine the products in Item 18. How can you predict the sign of the last term?

Check Your Understanding

20. Use a graphic organizer to calculate $(6 + 2)^2$. Explain why the product is not $6^2 + 2^2$.

Determine the product of the binomials using a graphic organizer or by using the Distributive Property.

21. $(x + 7)(x + 2)$

22. $(x + 7)(3x - 2)$

23. Compare the use of the graphic organizer and the use of the Distributive Property to find the product of two binomials.

LESSON 25-1 PRACTICE

Determine each product.

24. $(2 + 1)(3 + 5)$

25. $(2 + 3)(2 + 7)$

26. $(x + 9)(x + 3)$

27. $(x + 5)(x + 1)$

28. $(x - 3)(x + 4)$

29. $(x + 1)(x - 5)$

30. $(x + 3)(x - 3)$

31. $(x + 3)(x + 3)$

32. $(2x - 3)(x - 1)$

33. $(x + 7)(3x - 5)$

34. $(4x + 3)(2x + 1)$

35. $(6x - 2)(5x + 1)$

36. **Critique the reasoning of others.** A student determined the product $(x - 2)(x - 4)$. Identify and correct the student's error.

$$(x - 2)(x - 4)$$
$$x(x - 4) - 2(x - 4)$$
$$x^2 - 4x - 2x - 8$$
$$x^2 - 6x - 8$$

My Notes

Learning Targets:

- Multiply binomials.
- Find special products of binomials.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Look for a Pattern

1. Determine each product.

a. $(x + 1)(x - 1)$

b. $(x + 4)(x - 4)$

c. $(x - 7)(x + 7)$

d. $(2x - 3)(2x + 3)$

2. Describe any patterns in the binomials and products in Item 1.

3. **Express regularity in repeated reasoning.** The product of binomials of the form $(a + b)(a - b)$, has a special pattern called a **difference of two squares**. Use the patterns you found in Items 1 and 2 to explain how to find the product $(a + b)(a - b)$.

MATH TERMS

A binomial of the form $a^2 - b^2$ is known as the **difference of two squares**.

My Notes

4. Determine each product.

a. $(x + 1)^2$

b. $(4 + y)^2$

c. $(x + 7)^2$

d. $(2y + 3)^2$

e. $(x - 5)^2$

f. $(4 - x)^2$

g. $(y - 7)^2$

h. $(2x - 3)^2$

5. Describe any patterns in the binomials and products in Item 4.

6. Reason abstractly. The **square of a binomial**, $(a + b)^2$ or $(a - b)^2$, also has a special pattern. Use the pattern you found in Items 4 and 5 to explain how to determine the square of any binomial.

MATH TERMS

A binomial of the form $(a + b)^2$ or $(a - b)^2$ is known as the **square of a binomial.**

My Notes

Check Your Understanding

7. Use the difference of two squares pattern to find the product $(p + k)(p - k)$.

8. Use the square of a binomial pattern to determine $(p + k)^2$.

9. Can you use a special products pattern to determine $(x + 1)(x - 2)$? Explain your reasoning.

LESSON 25-2 PRACTICE

Determine each product.

10. $(x - 4)(x + 4)$

11. $(x + 4)^2$

12. $(y + 10)(y - 10)$

13. $(y - 10)^2$

14. $(2x - 3)^2$

15. $(2x - 3)(2x + 3)$

16. $(5x + 1)^2$

17. $(2y - 1)(2y - 1)$

18. **Construct viable arguments.** Explain why the products of $(x - 3)^2$ and $(x + 3)(x - 3)$ have a different number of terms.