My Notes

Learning Targets:

- Use a graphic organizer to multiply polynomials.
- Use the Distributive Property to multiply polynomials.

SUGGESTED LEARNING STRATEGIES: Graphic Organizer, Create Representations, Think-Pair-Share, Look for a Pattern

A graphic organizer can be used to multiply polynomials that have more than two terms, such as a binomial times a trinomial. The graphic organizer at right can be used to multiply $(x + 2)(x^2 + 2x + 3)$.

	x	2
x^2	x^3	$2x^2$
$2x$	$2x^2$	$4x$
3	$3x$	6

$= x^3 + 4x^2 + 7x + 6$

1. Draw a graphic organizer in the space provided in the *My Notes* section to represent $(x - 3)(x^2 + 5x + 6)$. Label each inner rectangle and find the sum.

2. How many boxes would you need to represent the multiplication of $(x^3 + 5x^2 + 3x - 3)(x^4 - 6x^3 - 7x^2 + 5x + 6)$ using the graphic organizer?

a. Explain how you determined your answer.

b. Use appropriate tools strategically. Would you use the graphic organizer for other multiplications with this many terms? Explain your reasoning.

The Distributive Property can be used to multiply any polynomial by another. Multiply each term in the first polynomial by each term in the second polynomial.

$$(x - 3)(5x^2 - 2x + 1) = 5x^3 - 17x^2 + 7x - 3$$

3. Determine each product.

a. $x(x + 5)$

b. $(x - 3)(x + 6)$

c. $(x + 7)(3x^2 - x - 1)$

d. $(3x - 7)(4x^2 + 4x - 3)$

4. How can you predict the number of terms the product will have before you combine like terms?

My Notes

MATH TIP

Polynomials are *closed* under multiplication. A set is closed under multiplication if the product of any two elements in the set is also an element of the set.

5. Are all of the answers to Item 3 polynomials? Justify your response.

6. Explain why the product of two polynomials will always be a polynomial.

7. You can find the product of more than two polynomials, such as $(x + 3)(2x + 1)(3x - 2)$.

a. To multiply $(x + 3)(2x + 1)(3x - 2)$, first determine the product of the first two polynomials, $(x + 3)(2x + 1)$.

$(x + 3)(2x + 1) =$

b. Multiply your answer to Part (a) by the third polynomial, $(3x - 2)$.

8. Determine each product.

a. $(x - 2)(x + 1)(2x + 2)$

b. $(x + 3)(3x + 1)(2x - 1)$

c. $(x - 1)(3x - 2)(x + 4)$

d. $(2x - 4)(4x + 1)(3x + 3)$

Check Your Understanding

Determine each product.

9. $a(b + c)$

10. $(a + b)(a + c)$

11. $(a + b)(a^2 + b + c)$

12. $(a + b)(a + c)(b + c)$

LESSON 25-3 PRACTICE

Determine each product.

13. $x(x + 7)$

14. $x(2x - 5)$

15. $(y + 3)(y + 6)$

16. $(y + 3)(y - 6)$

17. $x(2x^2 - 5x + 1)$

18. $(x - 1)(2x^2 - 5x + 1)$

19. $(2x - 7)(5x^2 - 1)$

20. $(2x - 7)(5x^2 - 3x - 1)$

21. $(x + 2)(x - 3)(x + 1)$

22. $(x + 2)(2x - 3)(2x + 1)$

23. **Attend to precision.** A binomial of degree 2 and variable x and a trinomial of degree 4 and variable x are multiplied. What will be the degree of the product? Explain your reasoning.

ACTIVITY 25 PRACTICE

Write your answers on notebook paper. Show your work.

Lesson 25-1

Determine each product.

1. $(10 - 3)(10 - 8)$

2. $(x - 3)(x - 8)$

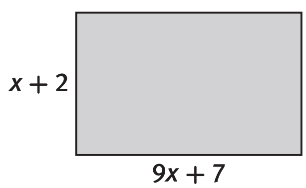
3. $(y - 7)(y + 2)$

4. $(x + 5)(x - 9)$

5. $(2y - 6)(3y - 8)$

6. $(4x + 3)(x - 11)$

7. Which expression represents the area of the rectangle?



A. $10x + 9$
B. $9x^2 + 14$
C. $20x + 18$
D. $9x^2 + 25x + 14$

Lesson 25-2

Determine each product.

8. $(x - 7)(x + 7)$

9. $(y + 6)(y - 6)$

10. $(2x - 5)(2x + 5)$

11. $(3y + 1)(3y - 1)$

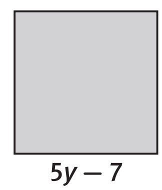
12. $(x - 11)^2$

13. $(x + 8)^2$

14. $(2y - 3)^2$

15. $(3y - 2)^2$

16. Which expression represents the area of the square?



A. $20y^2 + 28$
B. $20y - 28$
C. $25y^2 - 70y + 49$
D. $25y^2 - 49$

Lesson 25-3

Determine each product.

17. $x(x^2 - 7)$

18. $2x(x^2 - 3x + 2)$

19. $(x + 2)(4x^2 - 7x + 5)$

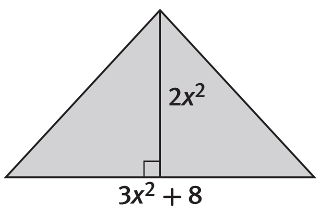
20. $(y - 5)(4y^2 + 5y + 2)$

21. $(5x - 9)^2$

22. $(3x - 4)(3x + 4)$

23. $(2y + 1)(y^2 + 3y - 5)$

24. Which expression represents the area of the triangle? Use the formula $A = \frac{1}{2}bh$.



A. $3x^4 + x^2$
B. $3x^4 + 8x^2$
C. $6x^2 + 8$
D. $6x^4 + 16x$

Determine each product.

25. $(x - 1)(7x - 1)(x + 2)$

26. $(x + 5)(4x - 1)(2x + 3)$

27. $(y + 1)^3$

28. Devise a plan for finding the product of four polynomials.

MATHEMATICAL PRACTICES
Look for and Make Use of Structure

29. Determine each product and describe any patterns you observe.

$(x - 1)(x + 1)$

$(x - 1)(x^2 + x + 1)$

$(x - 1)(x^3 + x^2 + x + 1)$

From the patterns you see, predict the product of $(x - 1)(x^4 + x^3 + x^2 + x + 1)$. Describe the pattern that helps you know the answer without needing to multiply.

Factoring

Factors of Construction

Lesson 26-1 Factoring by Greatest Common Factor (GCF)

ACTIVITY 26

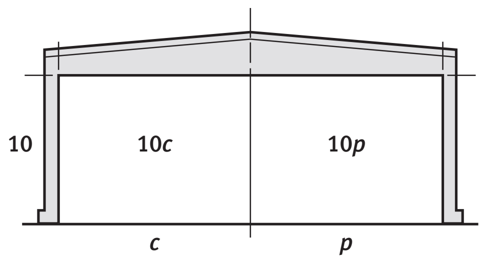
My Notes

Learning Targets:

- Identify the GCF of the terms in a polynomial.
- Factor the GCF from a polynomial.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Think-Pair-Share, Discussion Groups, Note Taking

Factor Steele Buildings is a company that manufactures prefabricated metal buildings that are customizable. All the buildings come in square or rectangular designs. Most office buildings have an entrance area or great room, large offices, and cubicles. The diagram below shows the front face of one of their designs. The distance c represents space available for large offices, and p represents the space available for the great room.



1. To determine how much material is needed to cover the front wall of the building, represent the total area as a product of a **monomial** and a **binomial**.

2. Represent the same area from Item 1 as a sum of two monomials.

3. **Make use of structure.** What property can be used to show that the two quantities in Items 1 and 2 are equal?

4. Factor Steele Buildings inputs the length of the large office space c into an expression that gives the area of an entire space: $6c^2 + 12c - 9$. Determine the **greatest common factor (GCF)** of the terms in this polynomial. Explain your choice.

MATH TERMS

A **monomial** is a number, a variable, or a product of numbers and variables with whole-number exponents. For example, 4, $-9x$, and $5xy^2$ are all monomials. A **binomial** is a sum or difference of two monomials.

MATH TERMS

The **greatest common factor (GCF)** of the terms in a polynomial is the greatest monomial that divides into each term without a remainder.

My Notes

MATH TERMS

A **factor** is any of the numbers or symbols that when multiplied together form a product. For example, 2 and x are factors of $2x$ because 2 and x are multiplied to get $2x$. *Factor* can be used as a noun or a verb.

To **factor** a number or expression means to write the number or expression as a product of its **factors**.

Example A

To Factor a Monomial (the GCF) from a Polynomial	
Steps to Factoring	**Example**
• Determine the GCF of all terms in the polynomial.	$6x^3 + 2x^2 - 8x$ GCF $= 2x$
• Write each term as the product of the GCF and another factor.	$2x(3x^2) + 2x(x) + 2x(-4)$
• Use the Distributive Property to factor out the GCF.	$2x(3x^2 + x - 4)$

Try These A

Find the greatest common factor of the terms in each polynomial. Then write each polynomial with the GCF factored out.

a. $3x^2 - 6x + 12$

b. $4x^5 - 6x^3 + 10x^2$

c. $15t^3 + 10t^2 - 5t$

Check Your Understanding

5. Identify the GCF of the terms in the polynomial $21x^3 + 14x^2 + 35x$.

Factor a monomial (the GCF) from each polynomial.

6. $36x + 9$

7. $6x^4 + 12x^2 - 18x$

8. $125n^6 + 250n^5 + 25n^3$

9. $3x^3 + 9x^2 + 6x$

10. $\frac{2}{3}y^4 + \frac{1}{3}y^3 - \frac{4}{3}y^2$

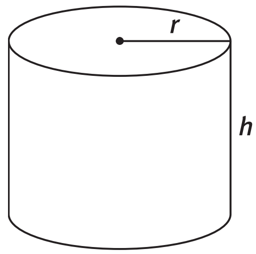
11. $4x^2y^2 + 12xy^2 - 8x^2y - 4xy$

Lesson 26-1
Factoring by Greatest Common Factor (GCF)

My Notes

LESSON 26-1 PRACTICE

Use the cylinder for Items 12–15.



The surface area of a cylinder is given by the formula $SA = 2\pi r^2 + 2\pi rh$, where r is the radius and h is the height.

12. Factor a monomial (the GCF) from the formula.

13. Suppose the radius of the cylinder is y and the height is $y + 2$. Rewrite the formula in this case, using multiplication and exponent rules as needed to simplify the expression.

14. Factor the expression from Item 13 completely.

15. **Construct viable arguments.** Answer each of the following questions and justify your responses.

a. If the radius of a cylinder doubles, what happens to the GCF of its surface area?

b. What happens to the GCF of the cylinder's surface area if its radius is squared?

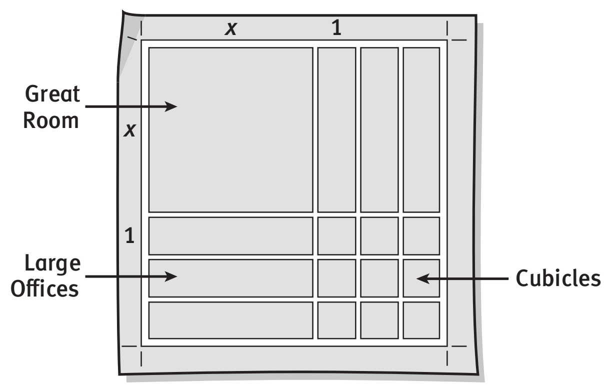
My Notes

Learning Targets:

- Factor a perfect square trinomial.
- Factor a difference of two squares.

SUGGESTED LEARNING STRATEGIES: Create Representations, Discussion Groups, Look for a Pattern, Sharing and Responding, Think-Pair-Share

Factor Steele Buildings can create many floor plans with different size spaces. In the diagram below the great room has a length and width of x units, and each cubicle has a length and width of 1 unit. Use the diagram below for Items 1–3.

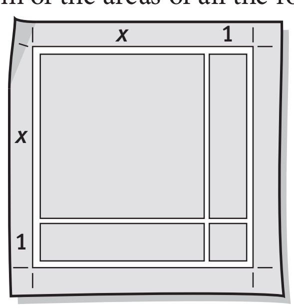


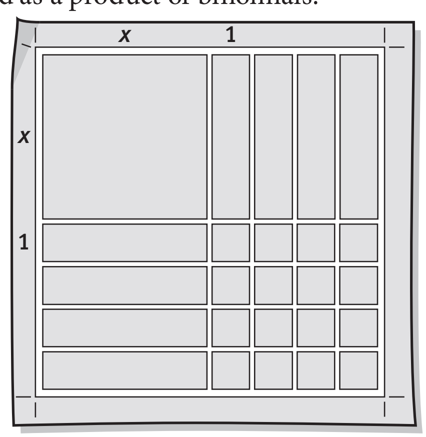
1. **Model with mathematics.** Represent the area of the entire office above as a sum of the areas of all the rooms.

2. Write the area of the entire office as a product of two binomials.

3. What property can you use to show how the answers to Items 1 and 2 are related? Show this relationship.

4. For each of the following floor plans, write the area of the office as a sum of the areas of all the rooms and as a product of binomials.

a.


b.


c. What patterns do you observe?

5. Complete the following table. The first row has been done for you.

Polynomial	1st Factor	2nd Factor	First Term in Each Factor	Second Term in Each Factor
$x^2 + 6x + 9$	$(x + 3)$	$(x + 3)$	x	3
	$(x - 3)$	$(x - 3)$		
	$(x + 4)$	$(x + 4)$		
	$(x - 4)$	$(x - 4)$		
	$(x + 5)$	$(x + 5)$		
	$(x - 5)$	$(x - 5)$		

6. Express regularity in repeated reasoning. Describe any patterns that you observe in the table from Item 5.

7. Explain how to factor polynomials of the form $a^2 + 2ab + b^2$ and $a^2 - 2ab + b^2$.

Polynomials of the form $a^2 + 2ab + b^2$ and $a^2 - 2ab + b^2$ are called **perfect square trinomials**.

Check Your Understanding

Factor each perfect square trinomial.

8. $x^2 - 14x + 49$ **9.** $m^2 + 20m + 100$ **10.** $y^2 - 16y + 64$

11. Complete the table by finding the polynomial product of each pair of binomial factors. The first row has been done for you.

1st Factor	2nd Factor	Polynomial
$(x + 3)$	$(x - 3)$	$x^2 - 9$
$(x + 4)$	$(x - 4)$	
$(x - 5)$	$(x + 5)$	
$(9 - x)$	$(9 + x)$	
$(2x - 7)$	$(2x + 7)$	
$(6x - 2y)$	$(6x + 2y)$	

My Notes

12. Describe any patterns you observe in the table from Item 11.

13. **a.** One factor of $36 - y^2$ is $6 + y$. What is the other factor?

b. One factor of $p^2 - 144$ is $p - 12$. What is the other factor?

c. Describe any patterns you observe.

14. Factor each of the following.

a. $49 - x^2$ **b.** $n^2 - 9$ **c.** $64w^2 - 25$

d. Describe any patterns you observe.

15. Explain how to factor a polynomial of the form $a^2 - b^2$.

A polynomial of the form $a^2 - b^2$ is referred to as the **difference of two squares**.

Check Your Understanding

Factor each difference of two squares.

16. $x^2 - 121$ **17.** $16m^2 - 81$ **18.** $9 - 25p^2$

LESSON 26-2 PRACTICE

Identify each polynomial as a perfect square trinomial, a difference of two squares, or neither. Then factor the polynomial if it is a perfect square trinomial or a difference of two squares.

19. $z^2 + 6z + 12$ **20.** $4x^2 - 121$ **21.** $y^2 - 8y + 16$

22. $y^2 - 8y - 16$ **23.** $n^2 + 25$ **24.** $169 - 9x^2$

25. What factor would you need to multiply by $(4c + 7)$ to get $16c^2 - 49$?

26. What factor would you need to multiply by $(3d + 1)$ to get $9d^2 + 6d + 1$?

Factor completely. (*Hint*: First look for a GCF.)

27. $2x^2 + 8x + 8$ **28.** $3y^2 - 75$ **29.** $12x^2 - 12x + 3$

30. **Use appropriate tools strategically.** Explain how you can use your calculator to check that you have factored a polynomial correctly.

ACTIVITY 26 PRACTICE

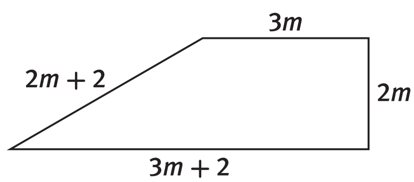
Write your answers on notebook paper.
Show your work.

Lesson 26-1

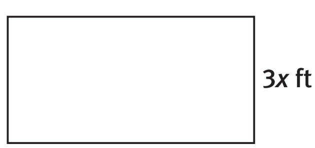
1. What is the greatest common factor of the terms in the polynomial $24x^8 + 6x^5 + 9x^2$?
   A. 3 **B.** $3x^2$
   C. $6x$ **D.** $6x^2$

Factor a monomial (the GCF) from each polynomial.

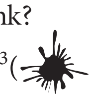
2. $15x^4 + 20x^3 + 35x$
3. $12m^3 - 8m^2 + 16m + 8$
4. $32y^2 + 48y - 16$
5. $x^5 + x^4 + 3x^3 + 3x^2$
6. Which of these polynomials cannot be factored by factoring out the GCF?
   A. $7x^2 + 14x + 21$ **B.** $49x^3 + 21x^2 + x$
   C. $x^2 + 14x + 7$ **D.** $35x^3 + 28x^2 + 7x$
7. The figure shows the dimensions of a garden plot in the shape of a trapezoid. Write and simplify a polynomial for the perimeter of the plot. Then factor the polynomial completely.



8. The area of the rectangle shown below is $6x^2 + 9x$ square feet. The width of the rectangle is given in the figure. What is the length of the rectangle? Justify your answer.



9. Marcus saw the factorization shown below in his textbook, but part of the factorization was covered by a drop of ink. What expression was covered by the drop of ink?

   $-24x^5 - 16x^3 = -8x^3($  $+ 2)$

10. Write a polynomial with four terms that has a GCF of $4x^2$.

Lesson 26-2

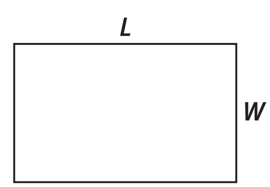
Identify each polynomial as a perfect square trinomial, a difference of two squares, or neither. Then factor the polynomial if it is a perfect square trinomial or a difference of two squares.

11. $9x^2 - 121$
12. $m^2 - 16m + 64$
13. $y^2 + 12y - 36$
14. $16z^2 + 25$
15. $25 - 144p^2$
16. $x^2 + 50x + 625$

Factor completely.

17. $2x^2 - 32$
18. $32 - 8p^2$
19. $3x^3 + 12x^2 + 12x$
20. $4y^3 - 32y^2 + 64y$
21. $5x^4 - 125x^2$
22. What factor would you need to multiply by $(4x - 1)$ to get $16x^2 - 8x + 1$?
    A. $4x - 1$ **B.** $4x + 1$
    C. $4x^2$ **D.** $4x$

Use the rectangle for Items 23–25.



23. The area of a rectangle is $64b^2 - 4$ and $W = 8b - 2$. What is L?
24. The area of another rectangle is $144c^2 - 4$ and $L = 12c + 2$. What is W?

25. Suppose the area of a rectangle is $4x^2 - 4x + 1$ and $L = 2x - 1$.
a. What is W?
b. What must be true about the rectangle in this case? Explain.

26. The area of a square window is given by the expression $m^2 - 16m + 64$. Which expression represents the length of one side of the window?
A. $m - 4$ **B.** $m + 4$
C. $m - 8$ **D.** $m + 8$

27. What value of k makes the polynomial $x^2 + 6x + k$ a perfect square trinomial?
A. 3 **B.** 6
C. 9 **D.** 36

28. Consider the following values of c in the polynomial $36x^2 + c$.
I. $c = -25$
II. $c = 25$
III. $c = -36$

Which value or values of c make it possible to factor the polynomial?
A. I only
B. I and II only
C. I and III only
D. I, II, and III

29. Write a perfect square trinomial that includes the term $9x^2$.

30. The polynomial $x^2 + bx + 25$ is a perfect square trinomial. What is the value of b? Is there more than one possibility? Explain.

31. Sasha and Pedro were asked to factor the polynomial $9x^2 - 9$ completely and explain their process. Their work is shown below. Has either student factored the polynomial completely? Explain. If not, give the complete factorization.

Sasha's Work
$9x^2 - 9 = (3x + 3)(3x - 3)$
I used the fact that $9x^2 - 9$ is a difference of two squares.

Pedro's Work
$9x^2 - 9 = 9(x^2 - 1)$
I factored out the GCF.

32. Which of the following polynomials has $m - 4$ as a factor?
A. $m^2 - 4$ **B.** $m^2 + 16$
C. $m^2 - 8m + 16$ **D.** $m^2 - 8m - 16$

33. Given that $x^2 + \square + 100$ is a perfect square trinomial, which of these could be the missing term?
A. $10x$ **B.** $20x$
C. $50x$ **D.** $100x$

34. Factor $x^4 - 81$ completely. (*Hint*: Use the fact that $x^4 = (x^2)^2$ to factor $x^4 - 81$ as a difference of two squares. Then consider whether any of the resulting factors can be factored again.)

35. Use the method in Item 34 to factor $y^8 - 625$ completely.

MATHEMATICAL PRACTICES
Reason Abstractly and Quantitatively

36. Could a product in the form $(a + b)(a - b)$ ever be equal to $a^2 + b^2$? Justify your answer.

Factoring Trinomials

Deconstructing Floor Plans

Lesson 27-1 Factoring $x^2 + bx + c$

My Notes

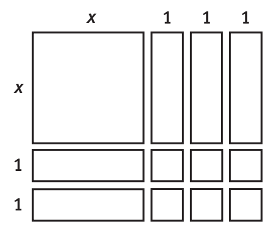
Learning Targets:

- Use algebra tiles to factor trinomials of the form $x^2 + bx + c$.
- Factor trinomials of the form $x^2 + bx + c$.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Create Representations, Think-Pair-Share, Look for a Pattern, Discussion Groups

Recall that Factor Steele Buildings can create many floor plans with different-size spaces. Custom Showrooms has asked Factor Steele Buildings for a floor plan with one great room, five large offices, and six cubicles. Each great room has a length and width equal to x units, each large office has a width of x units and a length of 1 unit, and each cubicle has a length and width of 1 unit.

Factor Steele Buildings proposes the rectangular floor plan shown below.



1. Represent the area of the entire office as a sum of the areas of all the rooms.

2. Write the area of the entire office as a product of two binomials by multiplying the length of the entire office by the width of the entire office.

3. **Make use of structure.** Multiply the binomials in Item 2 to check that their product is the expression you wrote in Item 1. Justify your steps and name any properties you use to multiply the binomials.

My Notes

Items 1 through 3 show how to use algebra tiles to factor a trinomial. However, drawing tiles to factor a trinomial can become time-consuming. Analyzing patterns and using graphic organizers can help factor a trinomial of the form $x^2 + bx + c$ without using tiles.

4. Consider the binomials $(x - 5)$ and $(x + 3)$.

a. Determine their product.

b. How is the coefficient of the trinomial's middle term related to the constant terms of the binomials?

c. How is the constant term of the trinomial related to the constant terms of the binomials?

5. Consider the binomials $(x + 6)$ and $(x + 1)$.

a. Determine their product.

b. How is the coefficient of the trinomial's middle term related to the constant terms of the binomials?

c. How is the constant term of the trinomial related to the constant terms of the binomials?

6. Express regularity in repeated reasoning. Use the patterns you observed in Items 4 and 5 to analyze a trinomial of the form $x^2 + bx + c$. Describe how the numbers in the binomial factors are related to the constant term c, and to b, the coefficient of x.

My Notes

Example A

Factor $x^2 + 12x + 32$.

Step 1: Create a graph organizer as shown. Place the first term in the upper left region. Place the last term in the lower right region.

x^2	
	32

Step 2: Identify the factors of c that add to b. Use a table to help you test factors.

Factors of 32		Sum of the Factors		
32	1	$32 + 1$	$=$	33
16	2	$16 + 2$	$=$	18
8	4	$8 + 4$	$=$	12✓

Step 3: Fill in the missing factors and products in the graphic organizer.

	x	8
x	x^2	$8x$
4	$4x$	32

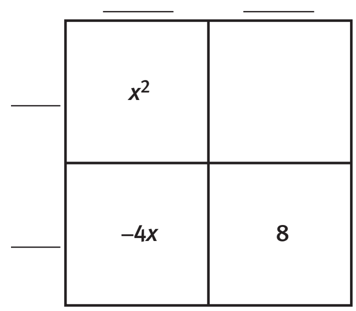
Step 4: Write the original trinomial as the product of two binomials.

$$x^2 + 12x + 32 = (x + 4)(x + 8)$$

My Notes

Try These A

a. Fill in the missing sections of the graphic organizer for the trinomial $x^2 - 6x + 8$. Express the trinomial as a product of two binomials.



b. Make a graphic organizer like the one above for the trinomial $x^2 + 14x + 45$. Express the trinomial as a product of two binomials.

c. Factor $x^2 + 6x - 27$.

d. Factor $x^2 + 10x + 1$.

MATH TIP

If there are no factors of *c* that add to *b*, the trinomial cannot be factored. A polynomial that cannot be factored is called *unfactorable* or a *prime polynomial*.

My Notes

Check Your Understanding

Factor each trinomial. Then multiply your factors to check your work.

7. $x^2 + 15x + 56$

8. $x^2 + 22x + 120$

9. $x^2 + 6x - 27$

10. $x^2 - 14x + 48$

11. $x^2 - x + 1$

LESSON 27-1 PRACTICE

Factor each trinomial.

12. $x^2 + 8x + 15$

13. $x^2 - 5x - 14$

14. $x^2 - 5x + 3$

15. $x^2 - 16x + 48$

16. $24 + 10x + x^2$

17. Custom Showrooms has expanded and now wants Factor Steele Buildings to create a floor plan with one great room, 15 large offices, and 50 cubicles.

a. Write the area of the new floor plan as a trinomial.

b. Factor the trinomial.

c. Multiply the binomials in Part (b) to check your work.

18. **Reason abstractly.** Suppose $x^2 + bx + c$ is a factorable trinomial in which c is a positive prime number.

a. Write an expression to represent the value of b.

b. Write $x^2 + bx + c$ as the product of two factors using only c as an unknown constant.

MATH TIP

A prime number has only itself and 1 as factors. For example, the numbers 3 and 11 are prime numbers.

My Notes

Learning Targets:

- Factor trinomials of the form $ax^2 + bx + c$ when the GCF is 1.
- Factor trinomials of the form $ax^2 + bx + c$ when the GCF is not 1.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Note Taking, Guess and Check, Look for a Pattern, Work Backward

Custom Showrooms now wants Factor Steele Buildings to create a floor plan with more than one great room. Instead, Custom Showrooms wants two great rooms, seven large offices, and six cubicles.

The trinomial $2x^2 + 7x + 6$ can be factored to determine the length and width of the entire office space.

1. **Attend to precision.** How is the trinomial $2x^2 + 7x + 6$ different from the trinomials you factored in Lesson 27-1?

MATH TIP

The factors of c will both have the same sign if $c > 0$. If $b < 0$, both factors will be negative. If $b > 0$, both factors will be positive.

Example A

Factor $2x^2 + 7x + 6$ using a guess and check method.

Possible Binomial Factors	Reasoning
$(2x \quad)(x \quad)$	$a = 2$ can be factored as $2 \cdot 1$.
$(2x + \quad)(x + \quad)$	$c = 6$, so both factors have the same sign. $b = 7$, so both factors are positive. 6 can be factored as $1 \cdot 6$, $6 \cdot 1$, $2 \cdot 3$, or $3 \cdot 2$.
$(2x + 1)(x + 6)$	Product: $2x^2 + 13x + 6$, incorrect
$(2x + 6)(x + 1)$	Product: $2x^2 + 8x + 6$, incorrect
$(2x + 2)(x + 3)$	Product: $2x^2 + 8x + 6$, incorrect
$(2x + 3)(x + 2)$	Product: $2x^2 + 7x + 6$, correct factors

Example B

Factor $3x^2 + 8x - 11$ using a guess and check method.

Possible Binomial Factors	Reasoning
$(3x \quad)(x \quad)$	$a = 3$ can be factored as $3 \cdot 1$.
$(3x + \quad)(x - \quad)$ or $(3x - \quad)(x + \quad)$	$c = -11$, so the factors have different signs. 11 can be factored as $11 \cdot 1$ or $1 \cdot 11$.
$(3x + 11)(x - 1)$	Product: $3x^2 + 8x - 11$, correct factors
$(3x - 11)(x + 1)$	Product: $3x^2 - 8x - 11$, incorrect
$(3x + 1)(x - 11)$	Product: $3x^2 - 32x - 11$, incorrect
$(3x - 1)(x + 11)$	Product: $3x^2 + 32x - 11$, incorrect

My Notes

Try These A–B

Factor the trinomials.

a. $3x^2 + 5x + 2$

b. $2x^2 + 5x - 18$

c. $2x^2 + 6x - 7$

Example C

Factor $4x^2 - 4x - 15$ using a guess and check method.

Possible Binomial Factors	Reasoning
$(4x \quad)(x \quad)$ or $(2x \quad)(2x \quad)$	$a = 4$ can be factored as $4 \cdot 1$ or $2 \cdot 2$.
$(4x - \quad)(x + \quad)$ or $(4x + \quad)(x - \quad)$ or $(2x - \quad)(2x + \quad)$ or $(2x + \quad)(2x - \quad)$	$c = -15$, so the factors have different signs. 15 can be factored as $1 \cdot 15$, $15 \cdot 1$, $3 \cdot 5$, or $5 \cdot 3$.
$(4x - 1)(x + 15)$	Product: $4x^2 + 59x - 15$, incorrect
$(4x + 1)(x - 15)$	Product: $4x^2 - 59x - 15$, incorrect
$(4x - 15)(x + 1)$	Product: $4x^2 - 11x - 15$, incorrect
$(4x + 15)(x - 1)$	Product: $4x^2 + 11x - 15$, incorrect
$(4x - 3)(x + 5)$	Product: $4x^2 + 17x - 15$, incorrect
$(4x + 3)(x - 5)$	Product: $4x^2 - 17x - 15$, incorrect
$(4x - 5)(x + 3)$	Product: $4x^2 + 7x - 15$, incorrect
$(4x + 5)(x - 3)$	Product: $4x^2 - 7x - 15$, incorrect
$(2x - 1)(2x + 15)$	Product: $4x^2 + 28x - 15$, incorrect
$(2x + 1)(2x - 15)$	Product: $4x^2 - 28x - 15$, incorrect
$(2x - 3)(2x + 5)$	Product: $4x^2 + 4x - 15$, incorrect
$(2x + 3)(2x - 5)$	Product: $4x^2 - 4x - 15$, correct factors

Try These C

Factor the trinomials.

a. $6x^2 - 11x - 2$

b. $6x^2 - 13x - 4$

c. $4x^2 - 20x + 21$

Example D

Factor $9x^2 - 24x + 12$.

Step 1: The coefficients 9, −24, and 12 are all divisible by 3. Factor out the GCF.

$$9x^2 - 24x + 12 = 3(3x^2 - 8x + 4)$$

Step 2: Factor $3x^2 - 8x + 4$ using a guess and check method.

Possible Binomial Factors	Reasoning
$(3x\quad)(x\quad)$	$a = 3$ can be factored as $3 \cdot 1$.
$(3x -\quad)(x -\quad)$	$c = 4$, so the factors have the same sign. $b = -8$, so both factors are negative. 4 can be factored as $4 \cdot 1$, $1 \cdot 4$, or $2 \cdot 2$.
$(3x - 4)(x - 1)$	Product: $3x^2 - 7x + 4$, incorrect
$(3x - 1)(x - 4)$	Product: $3x^2 - 13x + 4$, incorrect
$(3x - 2)(x - 2)$	Product: $3x^2 - 8x + 4$, correct factors

Solution: Write the complete factorization, including the GCF from Step 1:
$3(3x - 2)(x - 2)$

Check: Multiply to check your answer.
$3(3x - 2)(x - 2)$
$= 3(3x^2 - 6x - 2x + 4) = 3(3x^2 - 8x + 4) = 9x^2 - 24x + 12$

Try These D

Factor the trinomials completely. Check your work by multiplying the factors.

a. $10x^2 + 19x + 6$ **b.** $8x^2 + 20x - 28$ **c.** $8x^3 - 14x^2 + 6x$

Check Your Understanding

Factor each trinomial completely. Check your work by multiplying the factors.

2. $5x^2 + x - 4$ **3.** $49x^2 - 126x + 56$ **4.** $9x^3 - 39x^2 - 30x$

LESSON 27-2 PRACTICE

Model with mathematics. Factor Steele Buildings has received several floor plan requests. For Items 5–8, factor each floor plan scenario completely to help Factor Steele Buildings determine the space's dimensions.

5. 3 great rooms, 23 large offices, 14 cubicles

6. 10 great rooms, 31 large offices, 15 cubicles

7. 8 great rooms, 92 large offices, 180 cubicles

8. 12 great rooms, 38 large offices, 20 cubicles

9. Suppose $ax^2 + bx + c$ is a factorable trinomial in which both a and c are positive prime numbers. Write an expression to represent the value of b.

ACTIVITY 27 PRACTICE

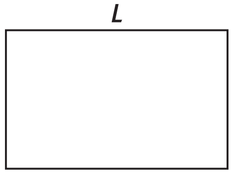
Write your answers on notebook paper. Show your work.

Lesson 27-1

Factor each trinomial.

1. $x^2 + 11x + 30$
2. $x^2 + 22x + 121$
3. $x^2 + x - 30$
4. $x^2 - 7x - 18$
5. $x^2 - 169$
6. $x^2 + 9x - 36$

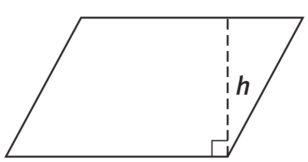
Mrs. Harbrook can choose from two rectangular pool sizes. The pool manufacturer provides her with the area of the pool, but she needs to find the dimensions in order to determine if the pool will fit in her yard. Use the rectangle for Items 7 and 8.



7. If the area of the pool is $x^2 - 17x + 72$, what are possible expressions to represent the length L and the width W?

8. a. If the area of the pool is $x^2 + 24x + 144$, what are possible expressions to represent the length L and the width W?
   b. What do these dimensions tell you about the shape of the pool?

The area of a parallelogram is given by the formula $A = bh$, where b is the base and h is the height. Use this information for Items 9 and 10.



9. If the area of the parallelogram is $x^2 + x - 42$, what are possible expressions to represent the base b and the height h?

10. If the area of the parallelogram is $x^2 + 4x - 117$, what are possible expressions to represent the base b and the height h?

11. Which of the following trinomials cannot be factored?
    A. $x^2 + 3x + 2$
    B. $x^2 + 3x - 2$
    C. $x^2 - 3x + 2$
    D. $x^2 + 2x - 3$

12. Which of the following binomials is a factor of the trinomial $y^2 - y - 20$?
    A. $y - 4$
    B. $y + 4$
    C. $y - 10$
    D. $y + 10$

For Items 13–15, consider the trinomial $x^2 + 2x + c$. Determine whether each statement is always, sometimes, or never true.

13. If c is a prime number, then the trinomial cannot be factored.
14. If c is an even number, then the GCF of the terms in the trinomial is 2.
15. If $c < 0$, then the trinomial can be factored.
16. Write a trinomial that can be factored such that one of the binomial factors is $x - 5$. Explain how you found the trinomial.

Lesson 27-2

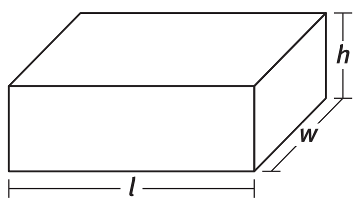
Factor each trinomial completely.

17. $3x^2 + 8x - 11$
18. $5x^2 - 7x + 2$
19. $2x^2 - 9x - 5$
20. $3x^2 + 17x - 28$
21. $7x^2 + 9x + 2$
22. $6x^2 - 11x - 7$
23. $12x^2 - 11x + 2$
24. $8x^2 + 16x + 6$

25. Which of the following is **not** a factor of the trinomial $24x^3 - 6x^2 - 9x$?
A. $3x$ **B.** $4x - 3$
C. $2x + 1$ **D.** $2x - 1$

26. Which binomial is a factor of $4x^2 + 12x + 5$?
A. $2x + 5$ **B.** $2x - 5$
C. $4x + 1$ **D.** $4x - 1$

The volume of a rectangular prism is found using the formula $V = lwh$, where l is the length, w is the width, and h is the height. Use the rectangular prism for Items 27–29.



27. If the volume of a rectangular prism is $6x^3 + 3x^2 - 18x$, what are possible expressions to represent the length, width, and height?

28. If the volume of a rectangular prism is $10x^2 - 55x + 60$, what are possible expressions to represent the length, width, and height?

29. If the volume of a rectangular prism is $12x^2 + 22x + 6$, what are possible expressions to represent the length, width, and height?

30. For which value of k is it possible to factor the trinomial $2x^2 + 3x + k$?
A. -1 **B.** 1
C. 2 **D.** 3

31. Which of the following trinomials has the binomial $x + 1$ as a factor?
A. $2x^2 - x - 1$
B. $2x^2 - 3x + 1$
C. $3x^2 - 5x + 2$
D. $3x^2 + x - 2$

32. Mayumi was asked to completely factor the trinomial $4x^2 + 10x + 4$. Her work is shown below. Is her solution correct? Justify your response.

> 4 can be factored as $4 \cdot 1$ or $2 \cdot 2$.
> Try $(2x + \quad)(2x + \quad)$.
> $(2x + 2)(2x + 2) = 4x^2 + 8x + 4$; incorrect
> $(2x + 4)(2x + 1) = 4x^2 + 10x + 4$; correct!
> The factorization is $(2x + 4)(2x + 1)$.

33. Given that the trinomial $5x^2 + bx + 10$ can be factored, which of the following statements must be true?
A. The value of b must be positive.
B. The value of b must be negative.
C. The value of b cannot be 3.
D. The value of b cannot be -27.

34. What is the factorization of the trinomial $p^2x^2 - 2pqx + q^2$?

35. Write a trinomial of the form $ax^2 + bx + c$ (with $a \neq 1$) that cannot be factored into binomial factors. Explain how you know the trinomial cannot be factored.

36. The area of a rectangular carpet is $6x^2 - 11x + 4$ square yards. The length of the carpet is $3x - 4$ yards. Which of the following is the width?
A. $2x - 1$ yards **B.** $2x + 1$ yards
C. $3x - 1$ yards **D.** $3x + 1$ yards

MATHEMATICAL PRACTICES
Construct Viable Arguments and Critique the Reasoning of Others

37. Guillaume is asked to factor a trinomial of the form $x^2 + bx - 8$. He says that because the constant term is negative, both binomial factors of the trinomial will involve subtraction. Is he correct? Explain.

Simplifying Rational Expressions

ACTIVITY 28

Totally Rational

Lesson 28-1 Simplifying Rational Expressions

My Notes

Learning Targets:

- Simplify a rational expression by dividing a polynomial by a monomial.
- Simplify a rational expression by dividing out common factors.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Note Taking, Identify a Subtask

A field trips costs \$800 for the charter bus plus \$10 per student for x students. The cost per student is represented by the expression $\frac{10x + 800}{x}$.

The cost-per-student expression is a rational expression. A **rational expression** is the ratio of two polynomials.

MATH TERMS

rational expression

Like fractions, rational expressions can be simplified and combined using the operations of addition, subtraction, multiplication, and division.

When a rational expression has a polynomial in the numerator and a monomial in the denominator, it may be possible to simplify the expression by dividing each term of the polynomial by the monomial.

Example A

Simplify by dividing: $\frac{12x^5 + 6x^4 - 9x^3}{3x^2}$

Step 1: Rewrite the rational expression to indicate each term of the numerator divided by the denominator.

$$\frac{12x^5}{3x^2} + \frac{6x^4}{3x^2} - \frac{9x^3}{3x^2}$$

Step 2: Divide. Use the Quotient of Powers Property.

$$\frac{12x^5}{3x^2} + \frac{6x^4}{3x^2} - \frac{9x^3}{3x^2}$$

$$4x^{5-2} + 2x^{4-2} - 3x^{3-2}$$

$$4x^3 + 2x^2 - 3x^1$$

Solution: $4x^3 + 2x^2 - 3x$

Try These A

Simplify by dividing.

a. $\frac{5y^4 - 10y^3 - 5y^2}{5y^2}$

b. $\frac{32n^6 - 24n^4 + 16n^2}{-8n^2}$

My Notes

To simplify a rational expression, first factor the numerator and denominator. Remember that factors can be monomials, binomials, or even polynomials. Then, divide out the common factors.

Example B

Simplify $\frac{12x^2}{6x^3}$.

Step 1: Factor the numerator and denominator.

$$\frac{2 \cdot 6 \cdot x \cdot x}{6 \cdot x \cdot x \cdot x}$$

Step 2: Divide out the common factors.

$$\frac{2 \cdot \cancel{6} \cdot \cancel{x} \cdot \cancel{x}}{\cancel{6} \cdot x \cdot \cancel{x} \cdot \cancel{x}}$$

Solution: $\frac{2}{x}$

Example C

Simplify $\frac{2x^2 - 8}{x^2 - 2x - 8}$.

Step 1: Factor the numerator and denominator.

$$\frac{2(x+2)(x-2)}{(x+2)(x-4)}$$

Step 2: Divide out the common factors.

$$\frac{2\cancel{(x+2)}(x-2)}{\cancel{(x+2)}(x-4)}$$

Solution: $\frac{2(x-2)}{x-4}$

MATH TIP

If a, b, and c are polynomials, and b and c do not equal 0, then $\frac{ac}{bc} = \frac{a}{b}$, because $\frac{c}{c} = 1$.

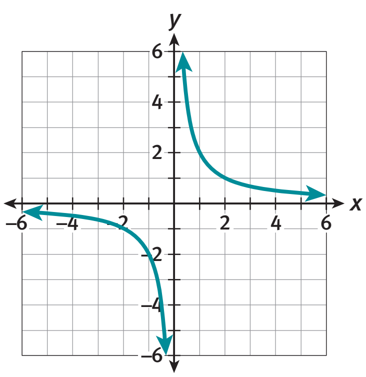
Try These B–C

Simplify each rational expression.

a. $\frac{6x^4y}{15xy^3}$ **b.** $\frac{x^2 + 3x - 4}{x^2 - 16}$ **c.** $\frac{15x^2 - 3x}{25x^2 - 1}$

MATH TIP

The graph of $y = \frac{2}{x}$ will never cross the x-axis since x cannot equal 0.



The value of the denominator in a rational expression cannot be zero because division by zero is undefined.

- In Example B, x cannot equal 0 because $6 \cdot (0)^3 = 0$.
- To find the excluded values of x in Example C, first factor the denominator. This shows that $x \neq -2$ because that would make the factor $x + 2 = 0$. Also, $x \neq 4$ because that would make the factor $x - 4 = 0$. Therefore, in Example C, x cannot equal -2 or 4.

Example D

Divide $\frac{1-x}{x-1}$. Simplify your answer if possible.

Step 1: Factor the numerator. $-1(x-1)$

Step 2: Divide out the common factor. $\frac{-1(\cancel{x-1})}{\cancel{x-1}}$

Solution: -1

Try These D

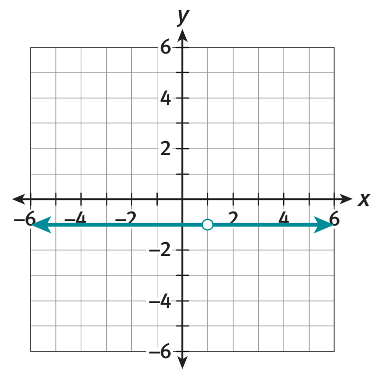
Divide. Simplify your answer if possible. Identify any excluded values of the variable.

a. $\frac{x-5}{5-x}$ **b.** $\frac{3x-3}{1-x}$

My Notes

MATH TIP

The graph of the rational function $f(x) = \frac{1-x}{x-1}$ looks like:



It looks like the graph of $y = -1$.

Check Your Understanding

Attend to precision. Describe the steps you would take to simplify each rational expression. Identify any excluded values of the variable.

1. $\frac{x^2-36}{6-x}$ **2.** $\frac{x^2-10x+24}{4x-16}$

LESSON 28-1 PRACTICE

Simplify by dividing.

3. $\frac{16x^5-8x^3+4x^2}{4x^2}$

4. $\frac{15x^6-20x^4}{-5x^3}$

5. $\frac{24x^6+18x^5-15x^3+12x^2}{3x^2}$

Simplify.

6. $\frac{3x^2yz}{12xyz^3}$

7. $\frac{25x^4y^3z^4}{-5x^5y^2z^3}$

8. $\frac{x^2-2x+1}{x^2+3x-4}$

9. $\frac{2x^2}{4x^3-16x}$

10. $\frac{x+1}{-4-4x}$

11. $\frac{x^2+6x+9}{x^2-9}$

12. Model with mathematics. The four algebra classes at Sanchez School are going on a field trip to a museum. Each class contains s students. The museum charges \$8 per student for admission. There is also a flat fee of \$200 for the buses.

a. Write an expression for the total cost of the buses and the museum admission fees for all four classes.

b. Write a rational expression for the cost per student. Simplify the expression as much as possible.

c. Use the expression you wrote in Part (b) to find the cost per student if each class has 20 students.

My Notes

Learning Targets:

- Divide a polynomial of degree one or two by a polynomial of degree one or two.
- Express the remainder of polynomial division as a rational expression.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Identify a Subtask, Close Reading, Note Taking, Discussion Groups

Division of polynomials is similar to long division of real numbers.

Example A

Divide $\frac{525}{25}$ using long division.

Step 1: Divide 52 by 25.

$$\begin{array}{r} 2 \\ 25\overline{)525} \\ -\underline{50} \\ 2 \end{array}$$

Step 2: Bring down 5.

$$\begin{array}{r} 2 \\ 25\overline{)525} \\ -\underline{50}\downarrow \\ 25 \end{array}$$

Step 3: Divide 25 by 25.

$$\begin{array}{r} 21 \\ 25\overline{)525} \\ -\underline{50}\downarrow \\ 25 \\ -\underline{25} \\ 0 \end{array}$$

Solution: The quotient is 21.

Lesson 28-2
Dividing Polynomials

My Notes

Division with polynomials can be done in the same way as long division with whole numbers.

Example B

Simplify using long division: $\dfrac{12x^5 + 6x^4 - 9x^3}{3x^2}$

Step 1: Divide $12x^5$ by $3x^2$.

$$\begin{array}{r} 4x^3 \qquad\qquad\quad \\ 3x^2\overline{)\,12x^5 + 6x^4 - 9x^3} \\ -\underline{12x^5} \qquad\qquad\quad \\ 0 \qquad\qquad\quad \end{array}$$

Step 2: Bring down $6x^4$.

$$\begin{array}{r} 4x^3 \qquad\qquad\quad \\ 3x^2\overline{)\,12x^5 + 6x^4 - 9x^3} \\ -\underline{12x^5} \quad\downarrow \qquad \\ 6x^4 \qquad \end{array}$$

Step 3: Divide $6x^4$ by $3x^2$.

$$\begin{array}{r} 4x^3 + 2x^2 \qquad \\ 3x^2\overline{)\,12x^5 + 6x^4 - 9x^3} \\ -\underline{12x^5} \qquad\qquad\quad \\ 6x^4 \qquad \\ -\underline{6x^4} \qquad \\ 0 \qquad \end{array}$$

Step 4: Bring down $-9x^3$.

$$\begin{array}{r} 4x^3 + 2x^2 \qquad \\ 3x^2\overline{)\,12x^5 + 6x^4 - 9x^3} \\ -\underline{12x^5} \qquad\qquad\quad \\ 6x^4 \quad\;\; \big\downarrow \\ -\underline{6x^4} \qquad \\ -9x^3 \end{array}$$

Step 5: Divide $-9x^3$ by $3x^2$.

$$\begin{array}{r} 4x^3 + 2x^2 - 3x \\ 3x^2\overline{)\,12x^5 + 6x^4 - 9x^3} \\ -\underline{12x^5} \qquad\qquad\quad \\ 6x^4 \qquad \\ -\underline{6x^4} \qquad \\ -9x^3 \\ -\underline{(-9x^3)} \\ 0 \end{array}$$

Solution: The quotient is $4x^3 + 2x^2 - 3x$.

My Notes

Example C

Simplify using long division: $\frac{x^2+9x+14}{x+7}$.

Step 1: Divide x^2 by x.

$$\begin{array}{r} x \\ x+7\overline{)\,x^2+9x+14} \\ -\underline{(x^2+7x)} \end{array}$$

Step 2: Distribute the negative and subtract x^2+7x from x^2+9x.

$$\begin{array}{r} x \\ x+7\overline{)\,x^2+9x+14} \\ -\underline{x^2-7x} \\ 2x \end{array}$$

Step 3: Bring down the next term, 14.

$$\begin{array}{r} x \\ x+7\overline{)\,x^2+9x+14} \\ -\underline{x^2-7x} \downarrow \\ 2x+14 \end{array}$$

Step 4: Divide $2x$ by x.

$$\begin{array}{r} x+2 \\ x+7\overline{)\,x^2+9x+14} \\ -\underline{x^2-7x} \\ 2x+14 \\ -\underline{(2x+14)} \end{array}$$

Step 5: Distribute the negative and subtract $2x+14$ from $2x+14$.

$$\begin{array}{r} x+2 \\ x+7\overline{)\,x^2+9x+14} \\ -\underline{x^2-7x} \\ 2x+14 \\ -\underline{2x-14} \\ 0 \end{array}$$

Solution: The quotient is $x+2$.

Try These A–B–C

Simplify using long division.

a. $\frac{24x^5-8x^4+12x^3-4x^2}{4x^2}$

b. $\frac{x^2-x-12}{x-4}$

MATH TIP

You can check the quotient in a division problem by using multiplication. Multiply the quotient, $x+2$, by the divisor, $x+7$. If you have divided correctly, the product will be the dividend, $x^2+9x+14$.

$(x+2)(x+7)=$
$x^2+7x+2x+14=$
$x^2+9x+14$

Sometimes there are remainders when dividing integers. In a similar way, sometimes there are remainders when dividing polynomials.

MATH TIP

When dividing with integers, the remainder is often written as a fraction whose denominator is the divisor.

$$\begin{array}{r} 76\frac{1}{2} \\ 2\overline{)\,153} \\ \underline{-14} \\ 13 \\ \underline{-12} \\ 1 \end{array}$$

Example D

Simplify using long division: $\dfrac{2x^3 - 6x + 15}{x + 1}$.

Step 1: Divide. Add the term $0x^2$ to the dividend as a placeholder.

$$\begin{array}{r} 2x^2 - 2x - 4 \\ x+1\overline{)\,2x^3 + 0x^2 - 6x + 15} \\ \underline{-(2x^3 + 2x^2)} \\ -2x^2 - 6x \\ \underline{-(-2x^2 - 2x)} \\ -4x + 15 \\ \underline{-(-4x - 4)} \\ 19 \end{array}$$

Step 2: Write the remainder as $\dfrac{19}{x+1}$.

$$\begin{array}{r} 2x^2 - 2x - 4 + \frac{19}{x+1} \\ x+1\overline{)\,2x^3 + 0x^2 - 6x + 15} \\ \underline{-(2x^3 + 2x^2)} \\ -2x^2 - 6x \\ \underline{-(-2x^2 - 2x)} \\ -4x + 15 \\ \underline{-(-4x - 4)} \\ 19 \end{array}$$

Solution: The quotient is $2x^2 - 2x - 4 + \dfrac{19}{x+1}$.

Try These D

Simplify using long division.

a. $(3x^2 + 6x + 1) \div (3x)$

b. $(6x^2 + 5x - 20) \div (3x + 4)$

c. $(3x^3 + 5x - 10) \div (x - 4)$

My Notes

1. **Make sense of problems.** Consider the following polynomial division problem: $\dfrac{3x^2 - 8x + 15}{x^2 + 3x - 4}$.

   a. How does this division problem differ from those in the examples?

   b. Use long division to perform the division.

   c. Write the remainder as a rational expression.

   d. What is the quotient?

Check Your Understanding

2. **Make use of structure.** Describe how dividing polynomials using long division is similar to dividing whole numbers using long division.
3. Explain how to check a division problem involving whole numbers that has a remainder.
4. Explain how to check a division problem involving polynomials that has a remainder. To demonstrate, use $(3x^2 + x - 2) \div (x^2 + 2x + 3) = 3 + \dfrac{-5x - 11}{x^2 + 2x + 3}$.

LESSON 28-2 PRACTICE

Simplify using long division.

5. $\dfrac{4x^2 + 6x}{2x}$
6. $\dfrac{3x^4 - 9x^3 + 6x^2}{3x^2}$
7. $\dfrac{12x^5 + 24x^4 - 16x^3 - 12x^2}{-4x^2}$
8. $\dfrac{3x^2 - 6x - 24}{3x - 6}$
9. $\dfrac{5x^2 - 21x + 4}{5x - 1}$
10. $\dfrac{12x^2 - 15}{x + 5}$
11. $\dfrac{3x^2 + 6x - 9}{x + 1}$
12. $\dfrac{25x^2 + 20x - 15}{5x^2 + 5x + 5}$
13. **Reason abstractly.** The area of a rectangular swimming pool is $2x^2 + 11x + 4$ square feet. The width of the pool is $x - 2$ feet.

    a. Write a rational expression that represents the length of the pool. Simplify the expression using long division.

    b. What are the length, width, and area of the pool when $x = 19$?

My Notes

Learning Targets:

- Multiply rational expressions.
- Divide rational expressions.

SUGGESTED LEARNING STRATEGIES: Note Taking, Close Reading

To multiply rational expressions, first factor the numerator and denominator of each expression. Next, divide out any common factors. Then simplify, if possible.

Example A

Multiply $\frac{2x-4}{x^2-1} \cdot \frac{3x+3}{x^2-2x}$. Simplify your answer if possible.

Step 1: Factor the numerators and denominators.

$$\frac{2(x-2)}{(x+1)(x-1)} \cdot \frac{3(x+1)}{x(x-2)}$$

Step 2: Divide out common factors.

$$\frac{2(\cancel{x-2}) \cdot 3(\cancel{x+1})}{(\cancel{x+1})(x-1)(x)(\cancel{x-2})}$$

Solution: $\frac{6}{x(x-1)}$

Try These A

Multiply. Simplify your answer.

a. $\frac{y^2+5y+6}{y+2} \cdot \frac{y}{2y+6}$

b. $\frac{2x+2}{x^2-16} \cdot \frac{x^2-5x+4}{4x^2-4}$

To divide rational expressions, use the same process as dividing fractions. Write the division as multiplication of the reciprocal. Then simplify.

Example B

Divide: $\frac{x^2-5x+6}{x^2-9} \div \frac{2x-4}{x^2+2x-3}$. Simplify your answer.

Step 1: Rewrite the division as multiplication by the reciprocal.

$$\frac{x^2-5x+6}{x^2-9} \cdot \frac{x^2+2x-3}{2x-4}$$

Step 2: Factor the numerators and the denominators.

$$\frac{(x-2)(x-3)}{(x+3)(x-3)} \cdot \frac{(x+3)(x-1)}{2(x-2)}$$

Step 3: Divide out common factors.

$$\frac{(\cancel{x-2})(\cancel{x-3})(\cancel{x+3})(x-1)}{(\cancel{x+3})(\cancel{x-3})(2)(\cancel{x-2})}$$

Solution: $\frac{x-1}{2}$

MATH TIP

When dividing fractions, write the division as multiplication by the reciprocal.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

If *a*, *b*, *c*, and *d* have any common factors, you can divide them out before you multiply.

$$\frac{4}{15} \div \frac{8}{3} = \frac{4}{15} \cdot \frac{3}{8}$$

$$= \frac{\cancel{4}}{\cancel{3} \cdot 5} \cdot \frac{\cancel{3}}{2 \cdot \cancel{4}} = \frac{1}{10}$$

My Notes

Try These B

Divide. Simplify your answer.

a. $\frac{w^2 - 2w - 3}{w^2 - 6w + 9} \div \frac{5}{w - 3}$

b. $\frac{3xy}{3x^2 - 12} \div \frac{xy + y}{x^2 + 3x + 2}$

Check Your Understanding

1. **Critique the reasoning of others.** A student was asked to divide the rational expressions shown below. Examine the student's solution, and then identify and correct the error.

$$\frac{a^2 - 9}{3a} \div \frac{a + 3}{a - 3} = \frac{3a}{a^2 - 9} \cdot \frac{a + 3}{a - 3}$$

$$= \frac{3a}{(a + 3)(a - 3)} \cdot \frac{(a + 3)}{a - 3} = \frac{3a}{(a - 3)^2}$$

2. What is the quotient when $\frac{2x + 6}{x + 5}$ is divided by $\frac{2}{x + 5}$?

LESSON 28-3 PRACTICE

Multiply or divide.

3. $\frac{x^2 - 5x - 6}{x^2 - 4} \cdot \frac{x + 2}{x^2 - 12x + 36}$

4. $\frac{x^3}{x^2 - 1} \cdot \frac{2x + 2}{4x}$

5. $\frac{x^2 - y^2}{12} \cdot \frac{36}{x + y}$

6. $(b^2 + 12b + 11) \cdot \frac{b + 9}{b^2 + 20b + 99}$

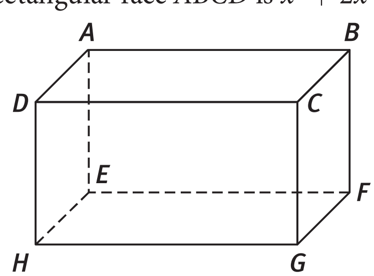
7. $\frac{x^2 + 4x + 4}{2x + 4} \div \frac{x^2 - 4}{x^2 - 6x + 5}$

8. $\frac{1}{x - 1} \div \frac{x}{x - 1}$

9. $\frac{2x + 4}{x^2 + 11x + 18} \div \frac{x + 1}{x^2 + 14x + 45}$

10. $\frac{m^2 + m - 6}{m^2 + 8m + 15} \div \frac{m^2 - m - 2}{m^2 + 9m + 20}$

11. **Make sense of problems.** The figure shows a rectangular prism. The area of the rectangular face *ABCD* is $x^2 + 2x - 15$.



a. The length of edge $\overline{DC}$ is $x + 1$. Write a rational expression that represents the length of edge $\overline{BC}$.

b. The length of edge $\overline{BF}$ is $\frac{x^2 + 2x + 1}{x + 5}$. Write and simplify a product to find the area of face *BFGC*.

My Notes

Learning Targets:

- Identify the least common multiple (LCM) of algebraic expressions.
- Add and subtract rational expressions.

SUGGESTED LEARNING STRATEGIES: Note Taking, Close Reading, Sharing and Responding, Identify a Subtask

To add or subtract rational expressions with the same denominator, add or subtract the numerators and then simplify if possible.

Example A

Simplify $\frac{10}{x} - \frac{5}{x}$.

Step 1: Subtract the numerators. $\frac{10}{x} - \frac{5}{x} = \frac{10-5}{x}$

Solution: $\frac{5}{x}$

Example B

Simplify $\frac{2x}{x+1} + \frac{2}{x+1}$.

Step 1: Add the numerators. $\frac{2x}{x+1} + \frac{2}{x+1} = \frac{2x+2}{x+1}$

Step 2: Factor. $= \frac{2(x+1)}{x+1}$

Step 3: Divide out common factors. $= \frac{2(\cancel{x+1})}{\cancel{x+1}}$

Solution: 2

Try These A–B

Add or subtract. Simplify your answer.

a. $\frac{3}{x^2} - \frac{x}{x^2}$

b. $\frac{2}{x+3} - \frac{6}{x+3} + \frac{x}{x+3}$

c. $\frac{x}{x^2-x} + \frac{4x}{x^2-x}$

To add or subtract rational expressions with unlike denominators, first identify a common denominator. The **least common multiple** (LCM) of the denominators is used for the common denominator.

One way to determine the LCM is to factor each expression. The LCM is the product of each factor common to the expressions as well as any non-common factors.

MATH TERMS

The **least common multiple** is the smallest multiple that two or more numbers or expressions have in common.

The numbers 10 and 25 have many common multiples. The number 50 is the least common multiple.

My Notes

Example C

Determine the LCM of $x^2 - 4$ and $2x + 4$.

Step 1: Factor each expression.

$$x^2 - 4 = (x + 2)(x - 2)$$
$$2x + 4 = 2(x + 2)$$

Step 2: Identify the factors.
Common Factor: $(x + 2)$
Factors Not in Common: 2 and $(x - 2)$

Step 3: The LCM is the product of all of the factors in Step 2.

Solution: The LCM is $2(x + 2)(x - 2)$.

Try These C

a. Determine the LCM of $2x + 2$ and $x^2 + x$. Use the steps below.
Factor each expression:

Common Factor(s):

Factors Not in Common:

LCM:

b. Determine the LCM of $x^2 - 2x - 15$ and $3x + 9$.

Now add and subtract rational expressions with different denominators. First, determine the LCM of the denominators. Next, write each fraction with the LCM as the denominator. Then, add or subtract. Simplify if possible.

Example D

Subtract $\frac{2}{x} - \frac{3}{x^2 - 2x}$. Simplify your answer if possible.

Step 1: Determine the LCM.
Factor the denominators: x and $x(x - 2)$
The LCM is $x(x - 2)$.

Step 2: Multiply the numerator and denominator of the first term by $(x - 2)$. The denominator of the second term is the LCM.

$$\frac{2}{x} \cdot \frac{(x-2)}{(x-2)} - \frac{3}{x(x-2)}$$

Step 3: Use the Distributive Property in the numerator.

$$\frac{2x - 4}{x(x-2)} - \frac{3}{x(x-2)}$$

Step 4: Subtract the numerators. $\frac{2x - 7}{x(x-2)}$

Solution: $\frac{2x - 7}{x(x-2)}$

MATH TIP

Multiplying a fraction by a form of 1 gives an equivalent fraction.

$\frac{1}{3} = \frac{1 \cdot 2}{3 \cdot 2} = \frac{2}{6}$, because $\frac{2}{2} = 1$.

The same is true for rational expressions. Multiplying by $\frac{(x-2)}{(x-2)}$ gives an equivalent expression because $\frac{(x-2)}{(x-2)} = 1$.

My Notes

Example E

Add $\frac{-4}{5-p} + \frac{3}{p-5}$. Simplify your answer if possible.

Step 1: Determine a common denominator. $p-5$

Step 2: Multiply the numerator and denominator of the first term by -1.

$$\frac{-4}{5-p} \cdot \frac{-1}{-1} + \frac{3}{p-5}$$

Step 3: Multiply. $\frac{4}{p-5} + \frac{3}{p-5}$

Step 4: Add. $\frac{7}{p-5}$

Solution: $\frac{7}{p-5}$

MATH TIP

If you multiply $(5 - p)$ by -1, the product is $p - 5$.

Try These D–E

a. Add $\frac{1}{x^2-1} + \frac{2}{x+1}$. Use the steps below.

Factor each denominator:

Common Factors:

Factors Not in Common:

LCM:

Factor the denominator of the first term. Multiply the numerator and denominator of the second term by ____________________ :

Add the numerators:

Use the Distributive Property:

Combine like terms:

Solution:

Add or subtract. Simplify your answer.

b. $\frac{3}{x+1} - \frac{x}{x-1}$ **c.** $\frac{2}{x} - \frac{3}{x^2-3x}$ **d.** $\frac{2}{x^2-4} + \frac{x}{x^2+4x+4}$

My Notes

Check Your Understanding

1. **Make use of structure.** Sometimes the denominator of one fraction or one rational expression works as a common denominator for all fractions or rational expressions in a set.
   a. Write two fractions (rational numbers) in which the denominator of one of the fractions is a common denominator.
   b. Write two rational expressions in which the denominator of one of the expressions is a common denominator.
   c. Show how to add the two rational expressions you wrote in Part (b).
2. List the steps you usually use to add or subtract rational expressions with unlike denominators.

LESSON 28-4 PRACTICE

Determine the least common multiple of each set of expressions.

3. $2x + 4$ and $x^2 - 4$
4. $2x - 8$ and $x - 4$
5. $x - 3$ and $x + 3$
6. $x + 6$, $x + 7$, and $x^2 + 7x + 6$
7. $x + 3$, $x^2 + 6x + 9$, and $x^2 - 7x - 30$

Perform the indicated operation.

8. $\frac{x}{x+1} - \frac{2}{x+3}$
9. $\frac{2}{3x-3} - \frac{x}{x^2-1}$
10. $\frac{x}{x+5} - \frac{2}{x+3}$
11. $\frac{3}{x-3} - \frac{x}{x+4}$
12. $\frac{x}{3x-2} + \frac{2}{x-5}$
13. $\frac{x-2}{x^2+4x+4} + \frac{x-2}{x+2}$
14. **Model with mathematics.** In the past week, Emilio jogged for a total of 7 miles and biked for a total of 7 miles. He biked at a rate that was twice as fast as his jogging rate.
    a. Suppose Emilio jogs at a rate of r miles per hour. Write an expression that represents the amount of time he jogged last week and an expression that represents the amount of time he biked last week. $\left(\textit{Hint:} \text{distance} = \text{rate} \times \text{time, so time} = \frac{\text{distance}}{\text{rate}}.\right)$
    b. Write and simplify an expression for the total amount of time Emilio jogged and biked last week.
    c. Emilio jogged at a rate of 5 miles per hour. What was the total amount of time Emilio jogged and biked last week?

ACTIVITY 28 PRACTICE

Write your answers on notebook paper. Show your work.

Lesson 28-1

1. Allison correctly simplified the rational expression shown below by dividing.

$$\frac{35x^7 + 15x^5 - 10x^3}{5x^3}$$

Which of these is a term in the resulting expression?

A. $3x^8$ **B.** $3x^4$
C. $-2x$ **D.** -2

For Items 2–5, simplify each expression.

2. $\frac{56x^2y}{70x^3y}$

3. $\frac{28x^2}{49xy}$

4. $\frac{x^2 - 25}{5x + 25}$

5. $\frac{x + 5}{x^2 + x - 20}$

6. Which of the following expressions is equivalent to a negative integer?

A. $\frac{5y + 5}{5y - 5}$ **B.** $\frac{6y - 6}{3 - 3y}$
C. $\frac{2 - 2y}{4y - 4}$ **D.** $\frac{8y - 8}{4y - 4}$

7. A rental car costs \$24 plus \$3 per mile.

a. Write an expression that represents the total cost of the rental if you drive the car m miles.
b. Write and simplify an expression that represents the cost per day if you keep the car for 3 days.
c. What is the cost per day if you drive 50 miles?

8. The expression $\frac{x^2 + 8x + c}{x + 4}$ can be simplified to $x + 4$. What is the value of c?

A. -16 **B.** 0
C. 16 **D.** 64

Lesson 28-2

For Items 9–14, determine each quotient by using long division.

9. $(3x^2 + 6x + 2) \div 3x$

10. $(3x^2 - 7x - 6) \div (3x + 2)$

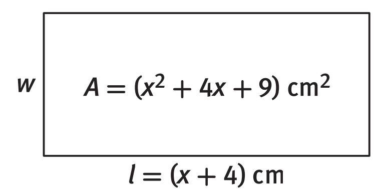
11. $\frac{2x^2 - 7x - 16}{2x + 3}$

12. $\frac{x^2 - 19x + 9}{x - 4}$

13. $\frac{4x^2 + 17x - 1}{4x + 1}$

14. $\frac{5x^3 + x - 2}{x - 1}$

15. The area A and length ℓ of a rectangle are shown below. Write a rational expression that represents the width w of the rectangle. Then simplify the expression using long division.



16. Greg was asked to simplify each expression below using long division. For which expression should he have a remainder?

A. $\frac{6x^2 + 9x + 3}{3x}$ **B.** $\frac{8x^4 + 12x^3 + 16x^2}{4x^2}$
C. $\frac{15x^2 + 5x + 25}{5}$ **D.** $\frac{6x^4 + 12x^3 + 6x^2}{6x}$

17. A student performed the long division shown below. Is the student's work correct? Justify your response.

$$\begin{array}{r} 4 \\ x^2 + 2x - 5 \overline{)\, 4x^2 - 6x + 11} \\ \underline{4x^2 + 8x - 20} \\ 2x \quad - 9 \end{array}$$

The quotient is $4 + \frac{2x - 9}{x^2 + 2x - 5}$.

Lesson 28-3

Multiply or divide. Simplify your answer if possible.

18. $\frac{x+4}{3x} \cdot \frac{4x^2}{x^2+9x+20}$

19. $\frac{3x+9}{x} \cdot \frac{x^2}{x^2-9}$

20. $\frac{x^2-x-6}{x^2-9} \cdot \frac{x^2+7x+12}{x^2+4x+4}$

21. $\frac{x^2-25}{x^2-10x+25} \div \frac{x^2+10x+25}{2x-10}$

22. $\frac{n^2-4n-5}{n^2+2n+1} \div \frac{n^2-6n+5}{n^2-1}$

In the expression $\frac{1}{(x+5)^2} \div \frac{k}{(x+5)^2}$, k is a real number with $k \neq 0$. For Items 23–25, determine whether each statement is always, sometimes, or never true.

23. The expression may be simplified so that the variable x does not appear.

24. The value of the expression is a real number less than 1.

25. When $k > 0$, the value of the expression is also greater than 0.

26. A student was asked to divide the rational expressions shown below. Examine the student's solution, then identify and correct the error.

$$\frac{x^2-6x+9}{5x} \div \frac{x-3}{x+3} = \frac{(x+3)(x-3)}{5x} \cdot \frac{x+3}{x-3}$$
$$= \frac{(x+3)\cancel{(x-3)}}{5x} \cdot \frac{x+3}{\cancel{x-3}}$$
$$= \frac{(x+3)^2}{5x}$$

27. Which expression is equivalent to $\frac{3x+3}{x^2} \cdot \frac{x^2-x}{x^2-1}$?

A. 3

B. $\frac{3}{x}$

C. $3-x$

D. $\frac{3x+3}{x}$

Lesson 28-4

For Items 28–31, determine the least common multiple of each set of expressions.

28. x^2-25 and $x+5$

29. $y+3$, y, and y^2

30. x^2+5x+6 and $x^2+7x+12$

31. x^2-4x+4, $x-2$, and $(x-2)^3$

32. Which pair of expressions has a least common multiple that is the product of the expressions?

A. $x+7$ and $x^2+14x+49$

B. $x+7$ and $x-7$

C. $x-3$ and x^2-9

D. $x-3$ and $(x-3)^2$

Add or subtract. Express in simplest form.

33. $\frac{4}{x} + \frac{3}{x}$

34. $\frac{x}{2} + \frac{x}{2}$

35. $\frac{x}{x+1} + \frac{1}{x+1}$

36. $\frac{x}{x^2-4x} - \frac{5x}{x-4}$

37. $\frac{3x+2}{3x-6} + \frac{x+2}{x^2-4}$

38. $\frac{-18}{3-x} + \frac{7}{x-3}$

MATHEMATICAL PRACTICES
Reason Abstractly and Quantitatively

39. Justine lives one mile from the grocery store. While she was driving to the store, there was a lot of traffic. On her way home, there was no traffic at all, and her average rate (speed) was twice the average rate of her trip to the store.

a. Let r represent Justine's average rate on her way to the store. Write an expression for the time it took her to get to the store. $\left(\textit{Hint:}\right.$ distance = rate × time, so time = $\left.\frac{\text{distance}}{\text{rate}}.\right)$

b. Write an expression for the time it took Justine to drive home.

c. Write and simplify an expression for the total time of the round trip to and from the store.

d. If Justine drove at 30 miles per hour to the store, what was the total time for the round trip? Write your answer in minutes.