My Notes

Learning Targets:

- Derive the quadratic formula.
- Solve quadratic equations using the quadratic formula.

SUGGESTED LEARNING STRATEGIES: Close Reading, Note Taking, Identify a Subtask

Generalizing a solution method into a formula provides an efficient way to perform complicated procedures. You can complete the square on the general form of a quadratic equation $ax^2 + bx + c = 0$ to find a formula for solving all quadratic equations.

$$ax^2 + bx + c = 0$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$x^2 + \frac{b}{a}x + \square = -\frac{c}{a} + \square$$

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$

$$\left(x + \frac{b}{2a}\right)^2 = -\frac{4ac}{4a^2} + \frac{b^2}{4a^2}$$

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Quadratic Formula
When $a \neq 0$, the solutions of $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

My Notes

To apply the quadratic formula, make sure the equation is in standard form $ax^2 + bx + c = 0$. Identify the values of a, b, and c in the equation and then substitute these values into the quadratic formula. If the expression under the radical sign is not a perfect square, write the solutions in simplest radical form or use a calculator to approximate the solutions.

Example A

Solve $x^2 + 3 = 6x$ using the quadratic formula.

Step 1: Write the equation in standard form.

$$x^2 + 3 = 6x$$
$$x^2 - 6x + 3 = 0$$

Step 2: Identify a, b, and c.

$$a = 1, b = -6, c = 3$$

Step 3: Substitute these values into the quadratic formula.

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(3)}}{2(1)}$$

Step 4: Simplify using the order of operations.

$$x = \frac{6 \pm \sqrt{36 - 12}}{2} = \frac{6 \pm \sqrt{24}}{2}$$

Step 5: Write as two solutions.

$$x = \frac{6 + \sqrt{24}}{2} \text{ or } x = \frac{6 - \sqrt{24}}{2}$$

Solution: Use a calculator to approximate the two solutions.
$x \approx 5.45$ or $x \approx 0.55$

If you do not have a calculator, write your solution in simplest radical form. To write the solution in simplest form, simplify the radicand and then divide out any common factors.

$$x = \frac{6 \pm \sqrt{24}}{2} = \frac{6 \pm \sqrt{4 \cdot 6}}{2} = \frac{6 \pm 2\sqrt{6}}{2} = \frac{2(3 \pm \sqrt{6})}{2} = 3 \pm \sqrt{6}$$

Try These A

Solve using the quadratic formula.

a. $3x^2 = 4x + 3$

b. $x^2 + 4x = -2$

My Notes

Check Your Understanding

Solve using the quadratic formula.

1. $3x^2 - 5x + 1 = 0$

2. $x^2 + 6 = -8x + 12$

LESSON 32-3 PRACTICE

Solve using the quadratic formula.

3. $x^2 + 5x - 1 = 0$

4. $-2x^2 - x + 4 = 0$

5. $4x^2 - 5x - 2 = 1$

6. $x^2 + 3x = -x + 1$

7. $3x^2 = -6x + 4$

8. A baseball player tosses a ball straight up into the air. The function $y = -16x^2 + 30x + 5$ models the motion of the ball, where x is the time in seconds and y is the height of the ball, in feet.

a. Write an equation you can solve to find out when the ball is at a height of 15 feet.

b. Use the quadratic formula to solve the equation. Round to the nearest tenth.

c. How many solutions did you find for Part (b)? Explain why this makes sense.

9. **Critique the reasoning of others.** José and Marta each solved $x^2 + 4x = -3$ using two different methods. Who is correct and what is the error in the other student's work?

José

$$x^2 + 4x = -3$$
$$x^2 + 4x - 3 = 0$$
$$a = 1, b = 4, c = -3$$
$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-3)}}{2(1)}$$
$$= \frac{-4 \pm \sqrt{16 + 12}}{2}$$
$$= \frac{-4 \pm \sqrt{28}}{2}$$
$$= \frac{-4 \pm 2\sqrt{7}}{2} = -2 \pm \sqrt{7}$$

Marta

$$x^2 + 4x = -3$$
$$x^2 + 4x + \square = -3 + \square$$
$$x^2 + 4x + 4 = -3 + 4$$
$$(x + 2)^2 = 1$$
$$x + 2 = \pm\sqrt{1}$$
$$x + 2 = 1 \text{ or } x + 2 = -1$$
$$x = -1 \text{ or } x = -3$$

Lesson 32-4
Choosing a Method and Using the Discriminant

My Notes

Learning Targets:

- Choose a method to solve a quadratic equation.
- Use the discriminant to determine the number of real solutions of a quadratic equation.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Graphic Organizer, Look for a Pattern, Create a Plan, Quickwrite

There are several methods for solving a quadratic equation. They include factoring, using square roots, completing the square, and using the quadratic formula. Each of these techniques has different advantages and disadvantages. Learning how and why to use each method is an important skill.

1. Solve each equation below using a different method. State the method used.

a. $x^2 + 5x - 24 = 0$

b. $x^2 - 6x + 2 = 0$

c. $2x^2 + 3x - 5 = 0$

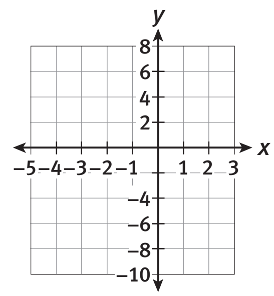
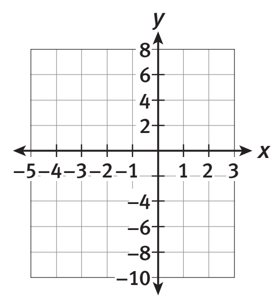
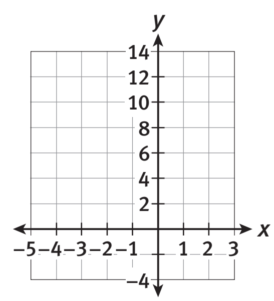
d. $x^2 - 100 = 0$

2. How did you decide which method to use for each equation in Item 1?

My Notes

The expression $\sqrt{b^2 - 4ac}$ in the quadratic formula helps you understand the nature of the quadratic equation. The **discriminant**, $b^2 - 4ac$, of a quadratic equation gives information about the number of real solutions, as well as the number of *x*-intercepts of the related quadratic function.

3. Solve each equation using any appropriate solution method. Then complete the rest of the table.

<table>
<tr><th>Equation</th><th>Discriminant</th><th>Solutions</th><th>Number of Real Solutions</th><th>Number of x-Intercepts</th><th>Graph of Related Quadratic Function</th></tr>
<tr><td>$x^2 + 2x - 8 = 0$</td><td></td><td></td><td></td><td></td><td>
</td></tr>
<tr><td>$x^2 + 2x + 1 = 0$</td><td></td><td></td><td></td><td></td><td>
</td></tr>
<tr><td>$x^2 + 2x + 5 = 0$</td><td></td><td></td><td></td><td></td><td>
</td></tr>
</table>

Lesson 32-4
Choosing a Method and Using the Discriminant

My Notes

4. **Express regularity in repeated reasoning.** Complete each statement below using the information from the table in Item 3.

   - If $b^2 - 4ac > 0$, the equation has ______ real solution(s) and the graph of the related function has ______ x-intercept(s).
   - If $b^2 - 4ac = 0$, the equation has ______ real solution(s) and the graph of the related function has ______ x-intercept(s).
   - If $b^2 - 4ac < 0$, the equation has ______ real solution(s) and the graph of the related function has ______ x-intercept(s).

Check Your Understanding

Use the discriminant to determine the number of real solutions.

5. $4x^2 + 2x - 12 = 0$
6. $x^2 - 7x + 14 = 0$
7. $x^2 - 10x + 25 = 0$

LESSON 32-4 PRACTICE

For each equation, use the discriminant to determine the number of real solutions. Then solve the equation.

8. $x^2 - 1 = 0$
9. $x^2 - 4x + 4 = 0$
10. $-4x^2 + 3x = -2$
11. $x^2 - 2 = 12x$
12. $x^2 + 5x - 1 = 0$
13. $-x^2 - 2x - 10 = 0$
14. **Model with mathematics.** Lin launches a model rocket that follows a path given by the function $y = -0.4t^2 + 3t + 0.5$, where y is the height in meters and t is the time in seconds.
    a. Explain how you can write an equation and then use the discriminant to determine whether Lin's rocket ever reaches a height of 5 meters.
    b. If Lin's rocket reaches a height of 5 meters, at approximately what time(s) does it do so? If not, what is the rocket's maximum height?

My Notes

Learning Targets:

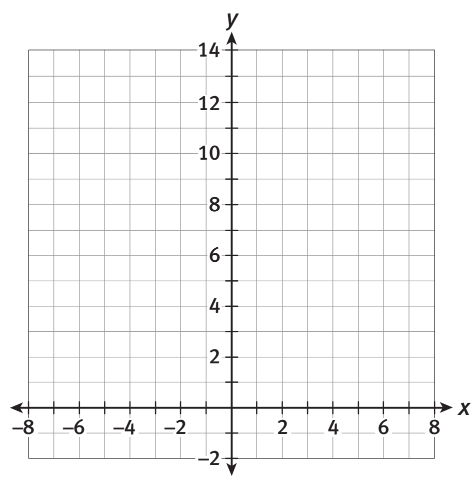
- Use the imaginary unit i to write complex numbers.
- Solve a quadratic equation that has complex solutions.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Close Reading, Note Taking, Construct an Argument, Identify a Subtask

When solving quadratic equations, there are always one, two, or no real solutions. Graphically, the number of x-intercepts is helpful for determining the number of real solutions.

- When there is one real solution, the graph of the related quadratic function touches the x-axis once, and the vertex of the parabola is on the x-axis.
- When there are two real solutions, the graph crosses the x-axis twice.
- When there are no real solutions, the graph never crosses the x-axis.

1. Graph the function $y = x^2 - 6x + 13$. Use the graph to determine the number of real solutions to the equation $x^2 - 6x + 13 = 0$.



2. Construct viable arguments. What does the number of real solutions to the equation in Item 1 indicate about the value of the discriminant of the equation? Explain.

When the value of the discriminant is less than zero, there are no *real* solutions. This is different from stating there are *no* solutions. In cases where the discriminant is negative, there are two solutions that are *not* real numbers. **Imaginary numbers** offer a way to determine these non-real solutions. The **imaginary unit**, i, equals $\sqrt{-1}$. Imaginary numbers are used to represent square roots of negative numbers, such as $\sqrt{-4}$.

My Notes

Example A

Simplify $\sqrt{-4}$.

Step 1: Write the radical as a product involving $\sqrt{-1}$. $\qquad \sqrt{-4} = \sqrt{-1} \cdot \sqrt{-4}$

Step 2: Replace $\sqrt{-1}$ with the imaginary unit, *i*. $\qquad = i\sqrt{4}$

Step 3: Simplify the radical. The principal square root of 4 is 2. $\qquad = 2i$

Solution: $\sqrt{-4} = 2i$

Try These A

Simplify.

a. $\sqrt{-16}$ **b.** $-\sqrt{-9}$ **c.** $\sqrt{-8}$

WRITING MATH

Notice that the solution to Example A is written 2*i* and not *i*2. However, when a value includes a radical and *i*, *i* is written in front of the radical, as in $i\sqrt{3}$.

Problems involving imaginary numbers can also result in **complex numbers**, $a + bi$, where *a* and *b* are real numbers. In this form, *a* is the real part and *b* is the imaginary part.

Example B

Solve $x^2 - 6x + 12 = 0$.

Step 1: Identify *a*, *b*, and *c*. $\qquad a = 1, b = -6, c = 12$

Step 2: Substitute these values into the quadratic formula. $\qquad x = \dfrac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(12)}}{2(1)}$

Step 3: Simplify using the order of operations. $\qquad x = \dfrac{6 \pm \sqrt{36 - 48}}{2} = \dfrac{6 \pm \sqrt{-12}}{2}$

Step 4: Simplify using the imaginary unit, *i*. $\qquad x = \dfrac{6 \pm \sqrt{-1}\sqrt{12}}{2} = \dfrac{6 \pm i\sqrt{12}}{2}$

Step 5: Simplify the radical and the fraction. $\qquad x = \dfrac{6 \pm 2i\sqrt{3}}{2} = 3 \pm i\sqrt{3}$

Solution: $x = 3 \pm i\sqrt{3}$

Try These B

Solve each equation.

a. $x^2 + 100 = 0$ **b.** $x^2 - 4x = -11$

My Notes

Check Your Understanding

3. Simplify $\sqrt{-27}$.

4. Solve $x^2 + 4x + 6 = 0$.

LESSON 32-5 PRACTICE

Simplify.

5. $-\sqrt{-11}$

6. $\sqrt{-42}$

7. $\pm\sqrt{-81}$

Solve.

8. $5x^2 - 2x + 3 = 0$

9. $-x^2 - 6 = 0$

10. $(x - 1)^2 + 3 = 0$

11. **Make use of structure.** Consider the quadratic function $y = x^2 + 2x + c$, where c is a real number.

a. Write and simplify an expression for the discriminant.

b. Explain how you can use your result from Part (a) to write and solve an inequality that tells you when the function will have two zeros that are complex numbers.

c. Use your results to describe the zeros of the function $y = x^2 + 2x + 3$.

ACTIVITY 32 PRACTICE

Write your answers on notebook paper. Show your work.

Lesson 32-1

Solve each equation using square roots.

1. $x^2 + 7 = 43$
2. $(x - 5)^2 + 2 = 11$
3. $x^2 - 8x + 16 = 3$
4. Antonio drops a rock from a cliff that is 400 feet high. The function $y = -16t^2 + 400$ gives the height of the rock in feet after t seconds. Write and solve an equation to determine how long it takes the rock to land at the base of the cliff. (*Hint*: At the base of the cliff, the height y is 0.)
5. Maya wants to use square roots to solve the equation $x^2 - 6x + 9 = k$, where k is a positive real number. Which of these is the best representation of the solution?
   A. $x = 3 \pm \sqrt{k}$ **B.** $x = -3 \pm \sqrt{k}$
   C. $x = \pm\sqrt{k+3}$ **D.** $x = \pm\sqrt{k-3}$

Lesson 32-2

6. Given the equation $x^2 - 8x = 3$, what number should be added to both sides to complete the square?
   A. -4 **B.** 8
   C. 16 **D.** 64

Write each of the following equations in the form $y = a(x - h)^2 + k$. Then identify the direction of opening, vertex, maximum or minimum value, and x-intercepts.

7. $y = x^2 - 4x + 11$
8. $y = -x^2 - 6x - 8$
9. $y = x^2 + 2x - 8$
10. A golfer stands on a platform 16 feet above a driving range. Once the golf ball is hit, the function $y = -16t^2 + 64t + 16$ represents the height of the ball in feet after t seconds.
    a. Write an equation you can solve to determine the number of seconds it takes for the ball to land on the driving range.
    b. Solve the equation by completing the square. Leave your answer in radical form.
    c. Use a calculator to find the number of seconds, to the nearest tenth, that it takes the ball to land on the driving range.
11. Which of the following is a true statement about the graph of the quadratic function $y = x^2 - 2x + 3$?
    A. The vertex of the graph is $(-1, 2)$.
    B. The graph intersects the x-axis at $x = 1$.
    C. The graph is a parabola that opens upward.
    D. There is exactly one x-intercept.

Solve by completing the square.

12. $x^2 - 4x = 12$
13. $x^2 + 10x + 21 = 0$
14. $2x^2 - 4x - 4 = 0$
15. $x^2 + 6x = -10$
16. A climbing structure at a playground is represented by the function $y = -x^2 + 4x + 1$, where y is the height of the structure in feet and x is the distance in feet from a wall. What is the maximum height of the structure?
    A. 1 foot **B.** 2 feet
    C. 4 feet **D.** 5 feet

Lesson 32-3

Solve using the quadratic formula.

17. $4x^2 - 4x = 3$

18. $5x^2 - 9x - 2 = 0$

19. $x^2 = 2x + 4$

20. A football player kicks a ball. The function $y = -16t^2 + 32t + 3$ models the motion of the ball, where t is the time in seconds and y is the height of the ball in feet.

a. Write an equation you can solve to find out when the ball is at a height of 11 feet.

b. Use the quadratic formula to solve the equation. Round to the nearest tenth.

21. Kyla was asked to solve the equation $2x^2 + 6x - 1 = 0$. Her work is shown below. Is her solution correct? If not, describe the error and give the correct solution.

$2x^2 + 6x - 1 = 0$

$a = 2, b = 6, c = -1$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(2)(-1)}}{2(2)}$$

$$= \frac{-6 \pm \sqrt{36 - 8}}{4}$$

$$= \frac{-6 \pm \sqrt{28}}{4}$$

$$= \frac{-6 \pm 2\sqrt{7}}{4}$$

$$= \frac{-3 \pm \sqrt{7}}{2}$$

Lesson 32-4

Use the discriminant to determine the number of real solutions.

22. $x^2 + 3x + 5 = 0$

23. $4x^2 - 4x + 1 = 0$

24. The discriminant of a quadratic equation is -1. Which of the following must be a true statement about the graph of the related quadratic function?

A. The graph intersects the x-axis in exactly two points.

B. The graph lies entirely above the x-axis.

C. The graph intersects the x-axis at $x = -1$.

D. The graph has no x-intercepts.

25. A dolphin jumps straight up from the water. The quadratic function $y = -16t^2 + 20t$ models the motion of the dolphin, where t is the time in seconds and y is the height of the dolphin, in feet. Use the discriminant to explain why the dolphin does not reach a height of 7 feet.

Lesson 32-5

Simplify.

26. $\pm\sqrt{-2}$

27. $-\sqrt{-25}$

28. $\sqrt{-8}$

29. $-\sqrt{-121}$

30. $12 - \sqrt{-144}$

31. $\pm\sqrt{-32}$

Solve.

32. $2x^2 - 5x + 5 = 0$

33. $x^2 + x + 3 = 0$

34. $-3x^2 - 3x - 1 = 0$

35. $-x^2 - x - 2 = 0$

MATHEMATICAL PRACTICES
Construct Viable Arguments and Critique the Reasoning of Others

36. For what values of p does the quadratic function $y = x^2 + 4x + p$ have two real zeros? Justify your answer.

Applying Quadratic Equations

ACTIVITY 33

Rockets in Flight

Lesson 33-1 Fitting Data with a Quadratic Function

My Notes

Learning Targets:

- Write a quadratic function to fit data.
- Use a quadratic model to solve problems.

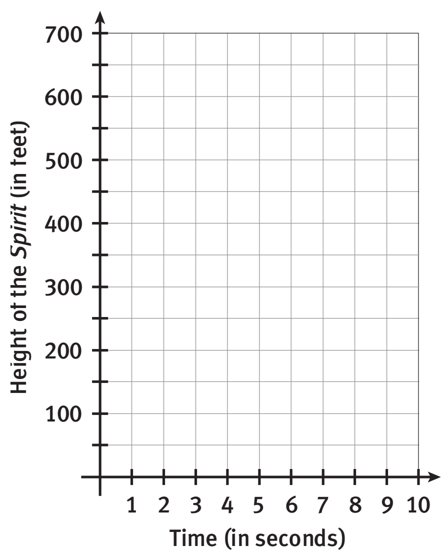
SUGGESTED LEARNING STRATEGIES: Create Representations, Look for a Pattern, Summarizing, Predict and Confirm, Discussion Groups

Cooper is a model rocket fan. Cooper's model rockets have single engines and, when launched, can rise as high as 1000 ft depending upon the engine size. After the engine is ignited, it burns for 3–5 seconds, and the rocket accelerates upward. The rocket has a parachute that will open as the rocket begins to fall back to Earth.

Cooper wanted to investigate the flight of a rocket from the time the engine burns out until the rocket lands. He set a device in a rocket, named *Spirit*, to begin collecting data the moment the engine shut off. Unfortunately, the parachute failed to open. When the rocket began to descend, it was in *free fall*.

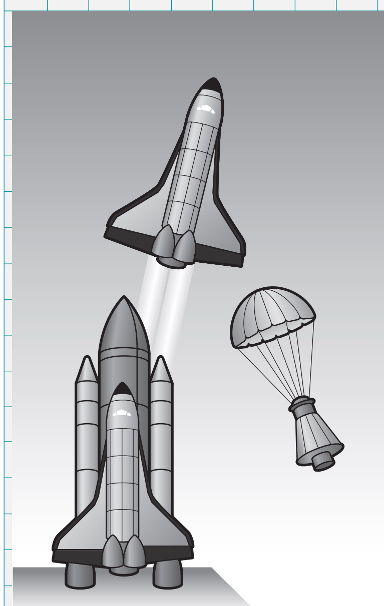
The table shows the data that was collected.

The *Spirit*	
Time Since the Engine Burned Out (s)	Height (ft)
0	512
1	560
2	576
3	560
4	512
5	432
6	320



CONNECT TO SCIENCE

A *free falling* object is an object that is falling under the sole influence of gravity. The approximate value of acceleration due to gravity for an object in free fall on Earth is 32 ft/s^2 or 9.8 m/s^2.



1. a. Use the table to determine whether the height of the *Spirit* can be modeled by a linear function.

b. Graph the data for the height of the *Spirit* as a function of time on the grid.

My Notes

2. Use the table and graph from Item 1.

a. How high was the *Spirit* when the engine burned out?

b. How long did it take the rocket to reach its maximum height after the engine cut out?

c. Estimate the time the rocket was in free fall before it reached the earth.

CONNECT TO TECHNOLOGY

For Item 3a, enter the data from the table above Item 1 into a graphing calculator. Use the calculator's quadratic regression feature to find a representative function.

3. Use the table and graph from Item 1.

a. **Use appropriate tools strategically.** Use a graphing calculator to determine a quadratic function $h(t)$ for the data.

b. Sketch the graph of the function on the grid in Item 1.

c. **Attend to precision.** Give a reasonable domain and range for $h(t)$ within the context of the problem. Be sure to include units.

4. Use the function found in Item 3 to verify the height of the *Spirit* when the engine burned out.

5. **Construct viable arguments.** Use the graph of $h(t)$ to approximate the time interval in which the *Spirit* was in free fall. Explain how you determined your answer.

My Notes

Check Your Understanding

Use the data in the table for Items 6–8.

x	0	1	2	3
$f(x)$	5	4.5	3	0.5

6. Graph the data in the table. Do the data appear to be linear? Explain.

7. Use a graphing calculator to write a function to model $f(x)$.

8. Use the function to determine the value of $f(x)$ when $x = 1.5$.

LESSON 33-1 PRACTICE

9. Model with mathematics. Cooper wanted to track another one of his rockets, the *Eagle*, so that he could investigate its time and height while in flight. He installed a device into the nose of the *Eagle* to measure the time and height of the rocket as it fell back to Earth. The device started measuring when the parachute opened. The data for one flight of the *Eagle* is shown in the table below.

The *Eagle*										
Time Since Parachute Opened (s)	0	1	2	3	4	5	6	7	8	9
Height(ft)	625	618	597	562	513	450	373	282	177	58

a. Graph the data from the table.

b. Use a graphing calculator to determine a quadratic function $h(t)$ that models the data.

10. Use the function from Item 9b to find the time when the rocket's height is 450 ft. Verify that your result is the same as the data in the table.

11. After the parachute opened, how long did it take for the rocket to hit Earth?

Lesson 33-2
Interpreting Solutions of Quadratic Equations

My Notes

Learning Targets:

- Solve quadratic equations.
- Interpret the solutions of a quadratic equation in a real-world context.

SUGGESTED LEARNING STRATEGIES: Create Representations, Predict and Confirm, Think-Pair-Share, Discussion Groups

1. The total time that the *Spirit* was in the air after the engine burned out is determined by finding the values of t that make $h(t) = 0$.

a. Rewrite the equation identified in Item 3a in Lesson 33-1 and set it equal to 0.

b. Completely factor the equation.

c. **Make use of structure.** Identify and use the appropriate property to find the time that the *Spirit* took to strike Earth after the engine burned out.

2. Draw a horizontal line on the graph in Item 1 in Lesson 33-1 to indicate a height of 544 ft above Earth. Estimate the approximate time(s) that the *Spirit* was 544 ft above Earth.

My Notes

3. The time(s) that the *Spirit* was 544 ft above Earth can be determined exactly by finding the values of t that make $h(t) = 544$.

a. Rewrite the equation from Item 3a in Lesson 33-1 and set it equal to 544.

b. Is the method of factoring effective for solving this equation? Justify your response.

c. Is the quadratic formula effective for solving this problem? Justify your response.

d. Determine the time(s) that the rocket was 544 ft above Earth. Round your answer to the nearest thousandth of a second. Verify that this solution is reasonable compared to the estimated times from the graph in Item 2.

CONNECT TO AP

In AP Calculus, calculations are approximated to the nearest thousandth.

e. Attend to precision. Explain why it is more appropriate in this context to round to the thousandths place rather than to use the exact answer or an approximation to the nearest whole number.

4. Cooper could not see the *Spirit* when it was higher than 528 ft above Earth.

a. Calculate the values of t for which $h(t) = 528$.

b. Reason quantitatively. Write an inequality to represent the values of t for which the rocket was not within Cooper's sight.

My Notes

Check Your Understanding

The path of a rocket is modeled by $h(t) = -16t^2 + 45t + 220$, where h is the height in feet and t is the time in seconds.

5. Determine the time t when the rocket was on the ground. Round to the nearest thousandth.

6. Identify the times, t, when the height $h(t)$ was greater than 220 ft.

LESSON 33-2 PRACTICE

Solve the quadratic equations. Round to the nearest thousandth.

7. $-16t^2 + 100t + 316 = 0$

8. $-16t^2 + 100t + 316 = 100$

Make sense of problems. For Items 9–12, use the function $h(t) = -16t^2 + 8t + 30$, which represents the height of a diver above the surface of a swimming pool t seconds after she dives.

9. The diver begins her dive on a platform. What is the height of the platform above the surface of the swimming pool? How do you know?

10. At what time does the diver reach her maximum height? What is the maximum height?

11. How long does it take the diver to reach the water? Round to the nearest thousandth.

12. Determine the times at which the diver is at a height greater than 20 ft. Explain how you arrived at your solution.

ACTIVITY 33 PRACTICE

Write your answers on notebook paper. Show your work.

Lesson 33-1

A model rocket is launched from the ground. Its height at different times after the launch is recorded in the table below. Use the table for Items 1–7.

Time Since Launch of Rocket (s)	Height of Rocket (ft)
1	144
2	256
3	336
4	384
5	400
6	384
7	336

1. Are the data in the table linear? Justify your response.
2. Use a graphing calculator to determine a quadratic function $h(t)$ for the data.
3. Identify a reasonable domain for the function within this context.
4. How high is the rocket after 8 seconds?
5. After how many seconds does the rocket come back to the ground?
6. What is the maximum height the rocket reached?
7. **a.** At what time(s) t will the height of the rocket be equal to 300 ft?
   b. How many times did you find in Part (a)? Explain why this makes sense in the context of the problem.

8. Which of the following functions best models the data in the table below?

x	$f(x)$
1	29
2	66
3	101
4	134
5	165
6	194

A. $f(x) = x + 28$
B. $f(x) = -x^2 + 40x - 10$
C. $f(x) = -16x^2 + 10x + 100$
D. $f(x) = -|x + 3| + 42$

9. As part of a fireworks display, a pyrotechnics team launches a fireworks shell from a platform and collects the following data about the shell's height.

Time Since Launch of Shell (s)	Height of Shell (ft)
1	68
2	100
3	100
4	68
5	4

Which of the following is a true statement about this situation?
A. The launch platform is 68 ft above the ground.
B. The maximum height of the shell is 100 ft.
C. The shell hits the ground after 6 seconds.
D. The shell starts to descend 2.5 seconds after it is launched.

Lesson 33-2

A soccer player passes the ball to a teammate, and the teammate kicks the ball. The function $h(t) = -16t^2 + 14t + 4$ represents the height of the ball, in feet, t seconds after it is kicked. Use this information for Items 10–16.

10. What is the height of the ball at the moment it is kicked? Justify your answer.

11. Graph the function.

12. Calculate the time at which the ball reaches its maximum height.

13. What is the maximum height of the ball?

14. Assuming no one touches the ball after it is kicked, determine the time when the ball falls to the ground.

15. Identify a reasonable domain and range for the function.

16. Determine the times when the ball is higher than 6 ft. Explain how you arrived at your solution.

Casey is standing on the roof of a building. She tosses a ball into the air so that her friend Leon, who is standing on the sidewalk, can catch it. The function $y = -16x^2 + 32x + 80$ models the height of the ball in feet, where x is the time in seconds. Use this information for Items 17–20.

17. Leon lets the ball hit the sidewalk. Determine the total time the ball is in the air until it hits the sidewalk.

18. Is the ball ever at a height of 100 ft? Justify your answer.

19. At how many times is the ball at a height of exactly 92 ft?

A. never　　**B.** one time
C. two times　　**D.** three times

20. Casey solves the equation shown below. What does the solution of the equation represent?

$$10 = -16x^2 + 32x + 80$$

A. The height of the ball after 10 seconds
B. The time when the ball is at a height of 10 ft
C. The time when the ball has traveled a total distance of 10 ft
D. The time it takes the ball to rise vertically 10 ft from the rooftop

The function $h(t) = -16t^2 + 50t + k$, where $k > 0$, gives the height, in feet, of a marble t seconds after it is shot into the air from a slingshot. Determine whether each statement is always, sometimes, or never true.

21. The initial height of the marble is k feet.

22. There is some value of t for which the height of the marble is 0.

23. The graph of the function is a straight line.

24. The marble reaches a height of 50 ft.

25. The marble reaches a height of 65 ft.

26. The maximum height of the marble occurs at $t = 1$ second.

MATHEMATICAL PRACTICES
Reason Abstractly and Quantitatively

27. Why do you think quadratic functions are used to model free-fall motion instead of linear functions?

Modeling with Functions

ACTIVITY 34

Photo App

Lesson 34-1 Constructing Models

My Notes

Learning Targets:

- Construct linear, quadratic, and exponential models for data.
- Graph and interpret linear, quadratic, and exponential functions.

SUGGESTED LEARNING STRATEGIES: Discussion Groups, Look for a Pattern, Create Representations, Sharing and Responding, Construct an Argument

Jenna, Cheyenne, and Kim all use a photo app on their smartphones to share their photos online and to track how many people follow their photo stories. The people who choose to follow the girls' photo stories can also stop following or "unsubscribe" at any time. After the first eight months, the data for each of the girl's monthly number of followers was compiled and is shown in the table below.

Months Since Account Was Opened	Jenna's Total Followers	Cheyenne's Total Followers	Kim's Total Followers
1	37	2	16
2	42	5	29
3	46	9	38
4	52	16	42
5	56	33	43
6	63	65	39
7	66	128	31
8	72	251	20

1. Describe any patterns you observe in the table.

My Notes

2. **Reason quantitatively.** Use the terms *linear*, *quadratic*, or *exponential* to identify the type of function that would best model each girl's total number of followers during the first eight months.

a. Jenna:

b. Cheyenne:

c. Kim:

3. Use the regression function of a graphing calculator to write a function that could be used to model each girl's total number of followers over the first eight months.

a. Jenna:

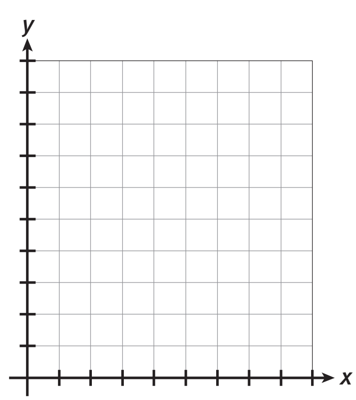
b. Cheyenne:

c. Kim:

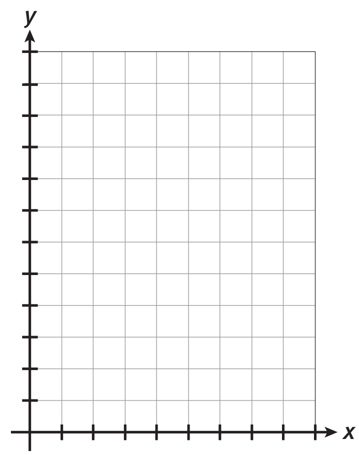
4. Approximately how many followers does Jenna gain each month? Justify your response.

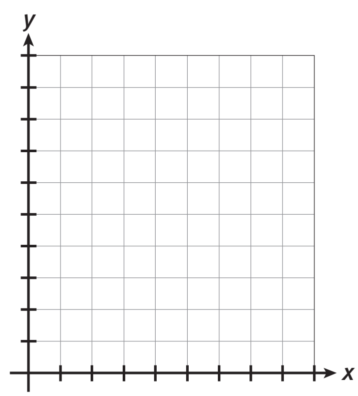
5. **Critique the reasoning of others.** Cheyenne tells the other girls she thinks her following is almost doubling each month. Is she correct? Justify your response using Cheyenne's function or the data in the table.

6. **Model with mathematics.** Use the functions from Item 3 to create graphs to represent each of the girl's total number of followers over the first eight months. Label at least three points on each graph.



My Notes





a. Describe the similarities and differences between the reasonable domains and ranges of each of the functions represented by the graphs.

b. Identify the maximum values, if they exist, of each of the functions represented by the graphs.

My Notes

Check Your Understanding

7. The total number of songs that Ping has downloaded since he joined an online music club is shown in the table below. Use the regression function of a graphing calculator to write a function that best models the data.

Days Since Joining	Total Songs Downloaded
1	1
2	3
3	9
4	28
5	81

8. Use your function from Item 7 to describe how the total number of songs that Ping has downloaded is changing each day.

My Notes

LESSON 34-1 PRACTICE

Alice and Ahmad have started an in-home pet grooming business. They each recorded the number of clients they visited each month for the first seven months. Their data are shown in the table below.

Months in Business	Number of Alice's Clients	Number of Ahmad's Clients
1	5	0
2	7	1
3	10	2
4	11	4
5	13	8
6	15	15
7	18	32

9. Examine the data in the table to determine the type of function that would best model the number of each groomer's clients.

10. Use the regression function of a graphing calculator to find a function to represent the number of Alice's clients.

11. Use the regression function of a graphing calculator to find a function to represent the number of Ahmad's clients.

12. Graph the functions from Items 10 and 11 on separate graphs.

13. **Attend to precision.** Describe the similarities and differences between the two functions. Compare and contrast the following features:

a. domain and range
b. maximum and minimum values
c. rate of increase per day
d. groomer with the most clients during different months

My Notes

Learning Targets:

- Identify characteristics of linear, quadratic, and exponential functions.
- Compare linear, quadratic, and exponential functions.

SUGGESTED LEARNING STRATEGIES: Visualization, Discussion Groups, Construct an Argument, Create Representations

1. Rewrite the functions you found for Jenna, Kim, and Cheyenne in Lesson 34-1.

Jenna:

Cheyenne:

Kim:

2. Which of the three girls—Jenna, Kim, or Cheyenne—has a constant rate of change in her number of followers per month? Explain.

3. Which girl had the greatest number of followers initially? Justify your response using both the functions and the graphs.

4. Did any of the girls experience followers who "unsubscribed" from, or stopped following, their photo story? Explain how you know.

5. Which girl's photo story gained the most followers over the eight months? Justify your response using the functions or the graphs.

6. **Critique the reasoning of others.** Kim states that even if the number of Jenna's followers had grown twice as quickly as it did, Cheyenne's followers would still eventually outnumber Jenna's followers. Is this assumption reasonable? Justify your response.

My Notes

7. Write a new function that describes the total number of Jenna's and Kim's followers combined.

8. Determine the reasonable domain and range, as well as any maximum or minimum values, of the function you wrote in Item 7.

9. **Construct viable arguments.** Will the number of Cheyenne's followers ever exceed the total number of Jenna and Kim's followers combined? Justify your response.

Check Your Understanding

Write the letter of the description that matches the given function.

10. $f(x) = -x^2 - 4x - 3$

11. $f(x) = 4x - 3$

12. $f(x) = 3(4)^x$

A. has a constant rate of change

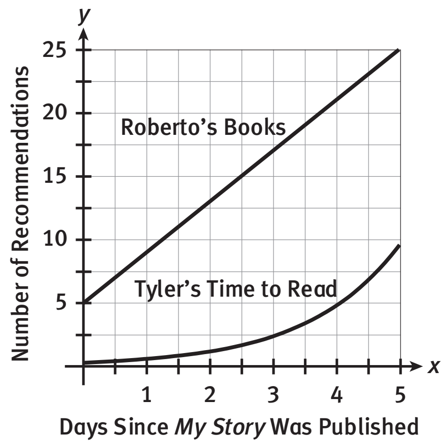
B. has a maximum value

C. increases very quickly at an ever increasing rate

My Notes

LESSON 34-2 PRACTICE

The graphs show the number of times two online retailers—Roberto's Books and Tyler's Time to Read—recommended the bestselling book *My Story* to their customers after x days. Use the graphs for Items 13–16.



13. Who had given more recommendations of *My Story* after three days?

14. Who recommended *My Story* the same number of times each day? How many times was it recommended each day?

15. If this model continues, Roberto's Books will have recommended *My Story* approximately 1465 times after one year. Is this a reasonable amount? Justify your response.

16. Will the number of times Tyler's Time to Read recommends *My Story* ever exceed the number of times Roberto's Books recommends *My Story*? Explain your reasoning.

17. Make use of structure. The total number of times that Roberto's Books recommended the bestseller *A Fatal Memory* after x days is modeled by the function $f(x) = 3x + 2$. The total number of times that Penny's Place recommended the same book after x days is modeled by the function $g(x) = -4x^2 + 12$. Write a function to model the combined number of times that Roberto's Books and Penny's Place recommended *A Fatal Memory*.

My Notes

Learning Targets:

- Compare piecewise-defined, linear, quadratic, and exponential functions.
- Write a verbal description that matches a given graph.

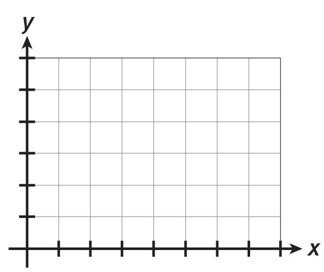
SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Create Representations, Critique Reasoning, Think-Pair-Share, Marking the Text

Rosa begins using the photo app to create a photo story and track her number of followers. The table below shows her results.

Months Since Account Was Started	Rosa's Total Followers
1	25
2	25
3	25
4	25
5	50
6	50
7	50
8	50

1. Describe any patterns you observe in the table.

2. Graph the data in the table.



3. Write a function to model Rosa's number of followers for the first four months.

4. Write a function to model Rosa's number of followers for months five through eight.

My Notes

MATH TERMS

Piecewise-defined functions are defined differently for different elements of the domain. For example, for the piecewise function $f(x) = \begin{cases} x \text{ when } x < 0 \\ 2 \text{ when } x \geq 0 \end{cases}$, $f(x) = x$ when $x < 0$ and $f(x) = 2$ when $x \geq 0$.

5. Write a **piecewise-defined function** to represent the number of followers Rosa has in any given month.

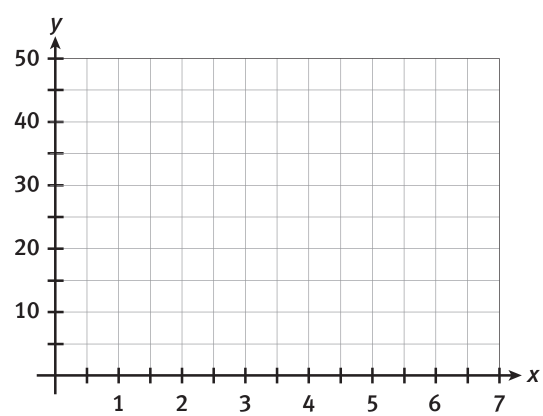
6. Critique the reasoning of others. Rosa says that her total number of followers is changing at a constant rate of 25 followers per month for the first four months. Is Rosa's statement correct? Explain your reasoning.

Juanita has recorded the number of followers for her latest photo story over the last seven days. She finds that the function $f(x) = -5|x - 4| + 45$ represents the number of followers after x days.

7. Complete the table for the number of followers each day.

Days Since Photo Story Was Posted	Number of Followers
1	
2	
3	
4	
5	
6	
7	

8. Graph the function that models Juanita's data.



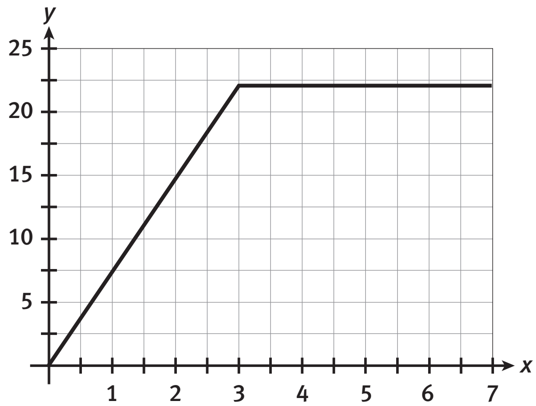
My Notes

9. Determine the reasonable domain and range, as well as the maximum and minimum values of the function within the context of the problem.

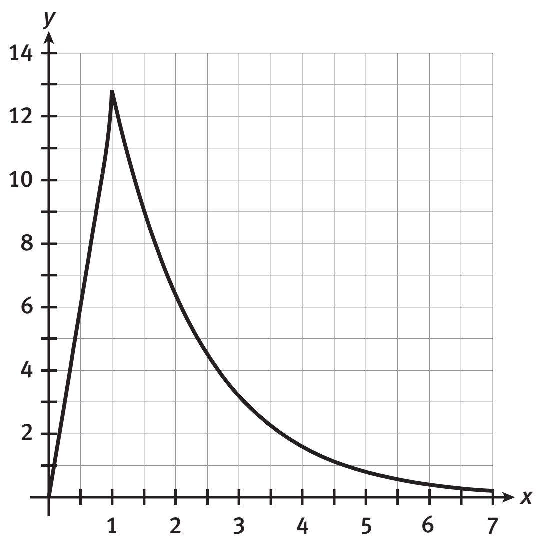
10. Reason abstractly. Describe the similarities and differences between the function $f(x) = -5|x - 4| + 45$ and a quadratic function.

11. The graphs below show the number of followers for two photo stories over one week. Describe how the number of followers changed over time.

a.



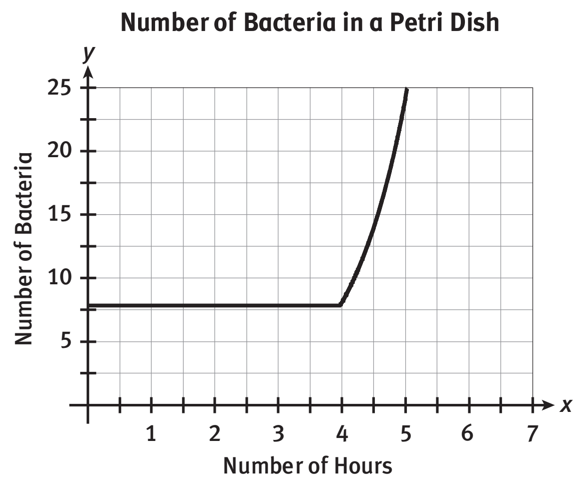
b.



My Notes

Check Your Understanding

Use the graph for Items 12 and 13.

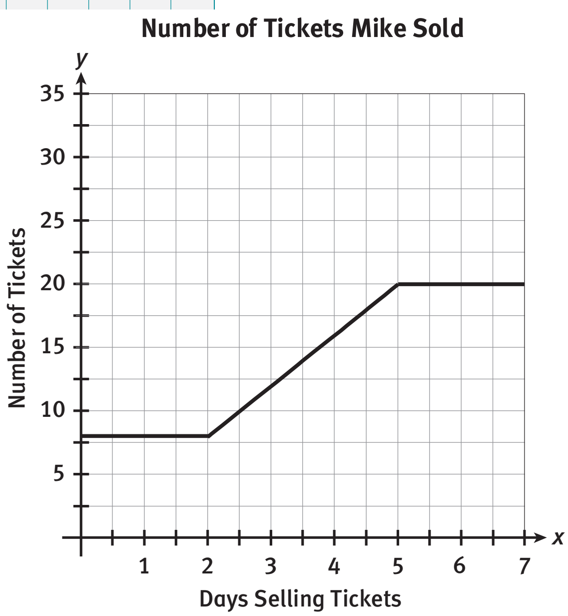


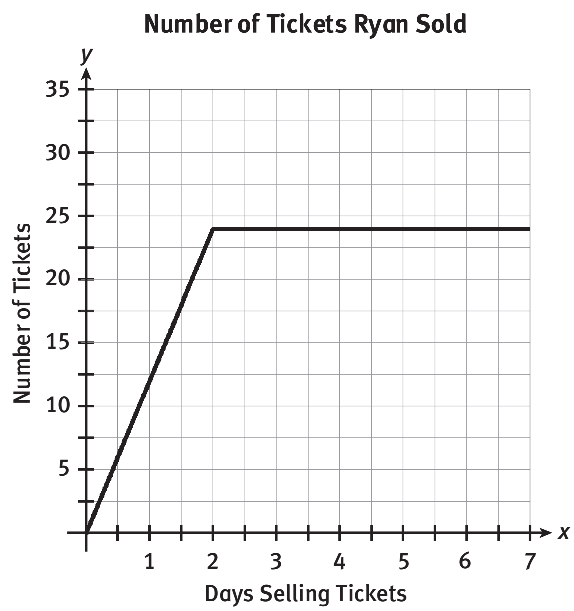
12. During which time period was the number of bacteria constant?

13. Describe the type of change in the number of bacteria for hours four through seven.

LESSON 34-3 PRACTICE

Make sense of problems. The graphs show the number of raffle tickets Mike and Ryan each sold to raise money for a school group.





14. For which days did the number of raffle tickets that Mike sold increase at a constant rate?

15. For which days did the number of raffle tickets that Ryan sold not increase?

16. Describe any patterns you see in the number of tickets that Mike sold over the seven days.

17. Compare and contrast any patterns you observe in the number of tickets that Mike sold and the number of tickets Ryan sold over the seven days.

ACTIVITY 34 PRACTICE

Write your answers on notebook paper.
Show your work.

Li and Alfonso have both opened accounts with an online music store. The data in the table below show the total number of songs each of them has downloaded since opening his account. Use the table for Items 1–12.

Days Since Account Opened	Total Number of Songs Downloaded by Li	Total Number of Songs Downloaded by Alfonso
1	1	5
2	2	15
3	5	26
4	8	34
5	16	45
6	33	56
7	65	67

Lesson 34-1

1. What type of function would best model the number of songs that Li has downloaded after x days?
A. linear
B. quadratic
C. exponential
D. absolute value

2. What type of function would best model the number of songs that Alfonso has downloaded after x days?
A. linear
B. quadratic
C. exponential
D. absolute value

3. Use the regression function of a graphing calculator to write a function that models the number of songs that Li has downloaded after x days.

4. Use the regression function of a graphing calculator to write a function that models the number of songs that Alfonso has downloaded after x days.

5. Determine the reasonable domain and range for each function.

Lesson 34-2

6. How would the number of songs downloaded by Alfonso change if the rate of change of Alfonso's downloads remained the same, but he had not downloaded any songs on day 1?

7. Describe the similarities and differences between the rates of change in the number of songs downloaded by the two boys.

8. If the models continue to represent the number of songs downloaded, who do you predict will have downloaded more songs after 30 days? Explain your reasoning.

9. If the rate of change of the number of songs downloaded by Alfonso tripled, how many songs will he have downloaded after 30 days? Is this a reasonable number?

10. How many songs will Li have downloaded after 30 days if his model continues? Is this a reasonable number?

Lesson 34-3

Caily opened an account at the same time with the same online music store. The following piecewise function represents the total number of songs she has downloaded over the first x days.

$$f(x) = \begin{cases} 10 \text{ when } 1 \le x < 15 \\ 30 \text{ when } 15 \le x < 25 \\ 45 \text{ when } 25 \le x \le 30 \end{cases}$$

Use this function for Items 11 and 12.

11. Describe the number of songs Caily has downloaded during this time.

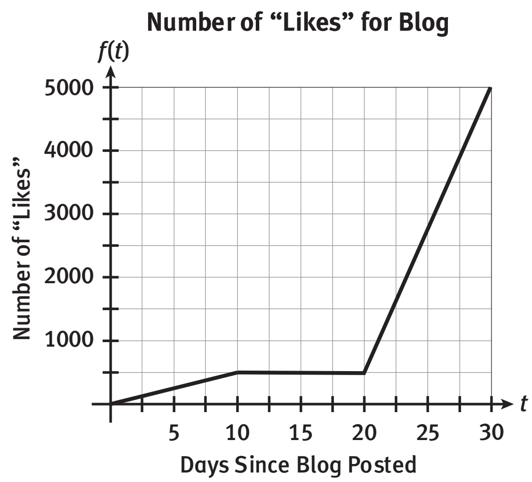
12. Describe the difference between the number of songs downloaded by Caily and by Alfonso over the first 30 days.

13. The functions $g(x) = 10x + 2$ and $h(x) = -0.5x^2 + 4x + 30$ represent the total numbers of two different types of fish in a pond over x weeks. Write a function $k(x)$ that represents the combined number of fish during the same period of time.

14. Graph the function $k(x)$ from Item 13. What is the maximum value of the function?

MATHEMATICAL PRACTICES
Construct Viable Arguments and Critique the Reasoning of Others

15. The graph shows the number of "likes" a blogger received for a blog post t days after it was posted.



The blogger believes that there may have been something wrong with how the "likes" were recorded between days 10 and 20. Why might she believe this?

Systems of Equations

Population Explosion

Lesson 35-1 Solving a System Graphically

ACTIVITY 35

My Notes

Learning Targets:

- Write a function to model a real-world situation.
- Solve a system of equations by graphing.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Create Representations, Sharing and Responding, Discussion Groups, Visualization

Professor Hearst is studying different types of bacteria in order to determine new ways to prevent their population overgrowth. Each bacterium in the first culture that she examines divides to produce another bacterium once each minute. In the second culture, she observes that the number of bacteria increases by 10 bacteria each second.

CONNECT TO BIOLOGY

Bacteria can be harmful or helpful to other organisms, including humans. Bacteria are commonly used for food fermenting, waste processing, and pest control.

1. Each population began with 10 bacteria. Complete the table for the population of each bacteria sample.

Elapsed Time (minutes)	Population of Sample A	Population of Sample B
0	10	10
1		
2		
3		
4		
5		

2. Reason quantitatively. Describe the type of function that would best model each population.

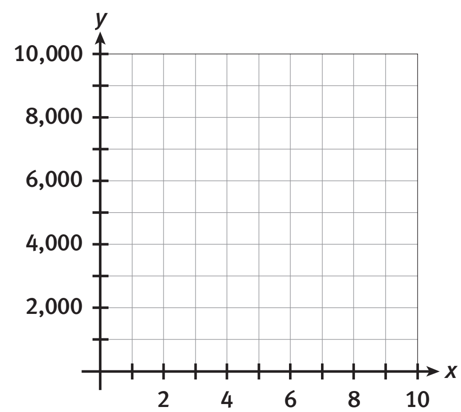
My Notes

3. Write a function $A(t)$ to model the number of bacteria present in Sample A after t minutes.

4. Write a function $B(t)$ to model the number of bacteria present in Sample B after t minutes.

5. **Use appropriate tools strategically.** Use a graphing calculator to graph $A(t)$ and $B(t)$ on the same coordinate plane.

a. Sketch the graph below and label several points on each graph.



b. Determine the points of intersection of the two graphs. Round non-integer values to the nearest tenth.

c. **Make use of structure.** What do the points of intersection indicate about the two graphs? Explain.

d. Interpret the meaning of the points of intersection within the context of the bacteria samples.

6. Which bacteria population contains more bacteria? Explain.

CONNECT TO TECHNOLOGY

You can use a graphing calculator to determine the point of intersection in several ways. You may choose to graph the functions and determine the point of intersection. You also may use the table feature to identify the value of x when y_1 and y_2 are equal.

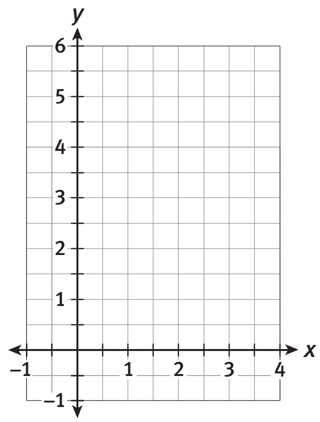
Lesson 35-1
Solving a System Graphically

The solutions you found in Item 5b are solutions to the **nonlinear system of equations** $A(t) = 10(2)^t$ and $B(t) = 600t + 10$.

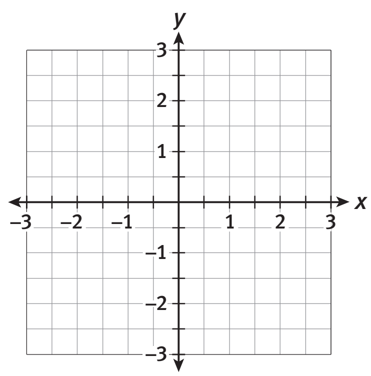
Just as with linear systems, you can solve nonlinear systems by graphing each equation and determining the intersection point(s).

7. Solve each system of equations by graphing.

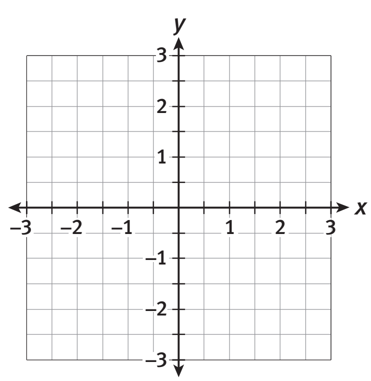
a. $y = 2x + 1$
$y = x^2 + 1$



b. $y = x - 1$
$y = 3^x - 2$



c. $y = 2(5)^x$
$y = -x^2 + 2x - 2$



My Notes

MATH TERMS

A **nonlinear system of equations** is a system in which at least one of the equations is not linear.

My Notes

Check Your Understanding

8. Examine the graphs in Item 7. How many solutions are possible for a nonlinear system of linear, quadratic, and/or exponential equations? Describe how this is different from the number of possible solutions for a linear system.

A population of bacteria is given by $f(t) = t^2 + 2t + 30$, where t is in minutes. Another population begins with 10 bacteria and doubles every minute.

9. Write a function $g(t)$ to model the population of the second sample of bacteria.

10. Use a graph to determine at what time the two populations are equal.

LESSON 35-1 PRACTICE

For Items 11–13, solve each system of equations by graphing.

11. $y = -2x + 4$
$y = -x^2 + 3$

12. $y = x^2 + 5x - 3$
$y = -x^2 + 5x + 1$

13. $y = -4x^2 - 1$
$y = 4(0.5)^x$

14. A nonlinear system contains one linear equation and one quadratic equation. The system has no solutions. Sketch a possible graph of this system.

15. Is it possible for a system with one linear equation and one exponential equation to have two solutions? If so, sketch a graph that could represent such a system. If not, explain why not.

16. **Critique the reasoning of others.** A population of 200 bacteria begins increasing at a constant rate of 100 bacteria per minute. Francis writes the function $P(t) = 200(100)^t$, where t represents the time in minutes, to model this population. Fred disagrees. He writes the function $P(t) = 200 + 100t$ to model this population. Who is correct? Justify your response.

My Notes

Learning Targets:

- Write a system of equations to model a real-world situation.
- Solve a system of equations algebraically.

SUGGESTED LEARNING STRATEGIES: Create a Plan, Identify a Subtask, Construct an Argument, Close Reading, Note Taking, Visualization, Think-Pair-Share, Discussion Groups

Just as with linear systems, nonlinear systems of equations can be solved algebraically.

Example A

Solve the system of equations algebraically.

$$y = -x + 3$$
$$y = x^2 - 2x - 3$$

Step 1: The first equation is solved for y. Substitute the expression for y into the second equation.
$$y = x^2 - 2x - 3$$
$$(-x + 3) = x^2 - 2x - 3$$

Step 2: Solve the resulting equation using any method. In this case, solve by factoring.
$$0 = x^2 - x - 6$$
$$0 = (x + 2)(x - 3)$$

Step 3: Apply the Zero Product Property. $x + 2 = 0$ or $x - 3 = 0$

Step 4: Solve each equation for x. $x = -2$ or $x = 3$

Step 5: Calculate the corresponding y-values by substituting the x-values from Step 4 into one of the original equations.
When $x = -2$, $y = -(-2) + 3 = 5$
When $x = 3$, $y = -(3) + 3 = 0$

Step 6: Check the solution by substituting into the other original equation.
When $x = -2$, $y = (-2)^2 - 2(-2) - 3 = 5$
When $x = 3$, $y = (3)^2 - 2(3) - 3 = 0$

Solution: $(-2, 5)$ and $(3, 0)$

Try These A

Solve each system algebraically.

a. $y = -x + 2$
$y = x^2 - x + 2$

b. $y = 2x^2 - 7$
$y = 7x - 3$

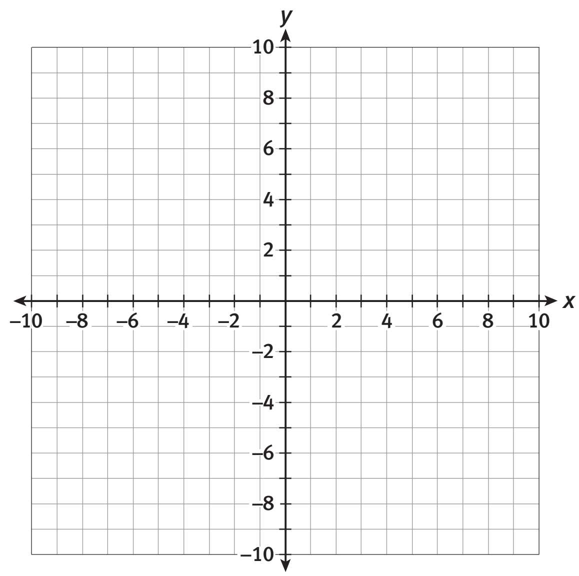
c. $y = -x + 3$
$y = x^2 - 2x - 4$

My Notes

1. Lauren solved the following system of equations algebraically and found two solutions.

$$y = -x + 3$$
$$y = x^2 - 3x - 4$$

Will solved the system by graphing and said that there is only one solution. Who is correct? Justify your response both algebraically and graphically.



2. Model with mathematics. Deshawn drops a ball from the 520-foot-high observation deck of a tower. The height of the ball in feet after t seconds is given by $f(t) = -16t^2 + 520$. At the moment the ball is dropped, Zoe begins traveling up the tower in an elevator that starts at the ground floor. The elevator travels at a rate of 12 feet per second. At what time will Zoe and the ball pass by each other?

a. Write a function $g(t)$ to model Zoe's height above the ground after t seconds.

b. Write a system of equations using the function modeling the height of the ball and the function you wrote in Part (a).

c. Solve the system that you wrote in Part (b) algebraically. Round to the nearest hundredth, if necessary.

My Notes

d. Interpret the meaning of the solution in the context of the problem. Does the solution you found in Part (c) make sense? Explain.

e. Determine the height at which Zoe and the ball pass by each other. Explain how you found your answer.

3. At the same moment that Deshawn drops the ball, Joey begins traveling down the tower in another elevator that starts at the observation deck. This elevator also travels at a rate of 12 feet per second.

a. Write a function $h(t)$ to model Joey's height above the ground after t seconds. Explain any similarities or differences between this function and the function in Item 2a.

b. Solve the system of equations algebraically.

c. Interpret the solution in the context of the problem.

d. **Construct viable arguments.** Determine whether Joey or the ball reaches the ground first. Justify your response.

My Notes

Check Your Understanding

4. How many solutions could the following system of equations have?

$$y = x + 8$$
$$y = x^2 - 10x + 36$$

5. Solve the system of equations in Item 4 using any appropriate algebraic method.

LESSON 35-2 PRACTICE

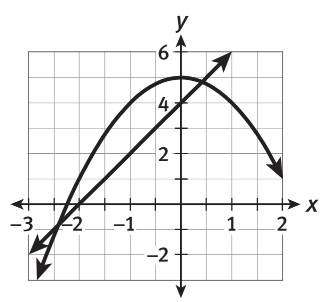
Solve each system of equations algebraically.

6. $y = 16x - 13$
$y = 4x^2 + 3$

7. $y = 5$
$y = -x^2 - x + 1$

8. $y = x$
$y = x^2 + 2x - 4$

9. Jessica has decided to solve a system of equations by graphing. Her graph is shown. Why might she prefer to solve this system algebraically?



10. **Make sense of problems.** A competitive diver dives from a 33-foot high diving board. The height of the diver in feet after t seconds is given by $u(t) = -16t^2 + 4t + 33$. At the moment the diver begins her dive, another diver begins climbing the diving board ladder at a rate of 2 feet per second. At what height above the pool deck do the two divers pass each other?

ACTIVITY 35 PRACTICE

Write your answers on notebook paper.
Show your work.

Lesson 35-1

1. Which function models the size of a neighborhood that begins with one home and doubles in size every year?
   A. $P(t) = t + 2$
   B. $P(t) = 2t + 1$
   C. $P(t) = 2(1)^t$
   D. $P(t) = (2)^t$

2. Which function models the size of a neighborhood that begins with 4 homes and increases by 6 homes every year?
   A. $P(t) = 4t + 6$
   B. $P(t) = 6t + 4$
   C. $P(t) = 6^t + 4$
   D. $P(t) = 4(6)^t$

3. When will the number of homes in Items 1 and 2 be equal?

For Items 4 and 5, sketch the graph of a system that matches the description. If no such system exists, write *not possible*.

4. The system contains a linear equation and an exponential equation. There are no solutions.

5. The system contains two exponential equations. There is one solution.

For Items 6–8, solve each system of equations by graphing.

6. $y = -x^2 + 3$
   $y = x^2 + 4$

7. $y = 0.5x^2 + x - 2$
   $y = x - 2$

8. $y = 2x^2 + 5$
   $y = -2x + 5$

9. Twin sisters Tamara and Sandra each receive \$50 as a birthday present.

   Tamara puts her money into an account that pays 3% interest annually. The amount of money in Tamara's account after x years is given by the function $t(x) = 50(1.03)^x$.

   Sandra puts her money into a checking account that does not pay interest, but she plans to deposit \$50 per year into the account. The amount of money in Sandra's account after x years is given by the function $s(x) = 50x + 50$.

   Use a graphing calculator to graph this system of equations. When will Tamara and Sandra have an equal amount of money in their accounts?

10. Josie and Jamal sold granola bars as a fund raiser, and they each started with 128 granola bars to sell.

    Josie sold 30 granola bars every day. The number of granola bars that Josie sold after x days is given by the function $y = -30x + 128$.

    Every day, Jamal sold half the number of the granola bars than he sold the day before. The number of granola bars that Jamal sold after x days is given by the function $y = 128(0.5)^x$.

    Use a graphing calculator to graph this system of equations. After how many days did Josie and Jamal have the same number of unsold granola bars?

For Items 11 and 12, write a system of equations to model the situation. Then solve the system by graphing.

11. A sample of bacteria starts with 2 bacteria and doubles every minute. Another sample starts with 4 bacteria and increases at a constant rate of 2 bacteria every minute. When will the populations be equal?

12. Jennie and James plan to save money by raking leaves. Jennie already has 1 penny. With each bag of leaves she rakes, she doubles the amount of money she has. James earns 10 cents per bag. When will Jennie have more money than James?

Lesson 35-2

For Items 13–17, solve each system of equations algebraically.

13. $y = 3x^2 - x - 2$
$y = 2x + 3$

14. $y = x^2 - 81$
$y = 18x - 161$

15. $y = x^2 + 4$
$y = 4x$

16. $y = 3x + 3$
$y = x^2 + 3x + 2$

17. $y = 3x$
$y = 2x^2$

For Items 18 and 19, write a system of equations to model the scenarios. Then solve the system of equations algebraically.

18. Simone is driving at a rate of 60 mi/h on the highway. She passes Jethro just as he begins accelerating onto the highway from a complete stop. The distance that Jethro has traveled in feet after t seconds is given by the function $f(t) = 5.5t^2$. When will Jethro catch up to Simone? (*Hint:* Use the fact that 60 mi/h is equivalent to 88 ft/s to write a function that gives the distance Simone has traveled.)

19. Jermaine is playing soccer next to his apartment building. He kicks the ball such that the height of the ball in feet after t seconds is given by the function $g(t) = -16t^2 + 48t + 2$. At the same moment that he kicks the ball, Jermaine's father begins descending in the elevator from his apartment at a rate of 5 ft/s. The apartment is 30 feet above the ground floor. At what time(s) are Jermaine's father and the soccer ball at the same height?

MATHEMATICAL PRACTICES
Construct Viable Arguments and Critique the Reasoning of Others

20. Rachel is solving systems of equations and has concluded that the quadratic formula is always an appropriate solution method when solving a nonlinear system algebraically. Do you agree with Rachel's conclusion? Use examples to support your reasoning.