Proof, Parallel and Perpendicular Lines

1

Unit Overview

In this unit you will begin the study of an axiomatic system, Geometry. You will investigate the concept of proof and discover the importance of proof in mathematics. You will extend your knowledge of the characteristics of angles and parallel and perpendicular lines and explore practical applications involving angles and lines.

Key Terms

As you study this unit, add these and other terms to your math notebook. Include in your notes your prior knowledge of each word, as well as your experiences in using the word in different mathematical examples. If needed, ask for help in pronouncing new words and add information on pronunciation to your math notebook. It is important that you learn new terms and use them correctly in your class discussions and in your problem solutions.

Academic Vocabulary

- compare and contrast
- justify
- argument
- interchange
- negate
- format
- confirm

Math Terms

- inductive reasoning
- conjecture
- deductive reasoning
- proof
- theorem
- axiomatic system
- undefined terms
- two-column proof
- conditional statement
- hypothesis
- conclusion
- counterexample
- converse
- inverse
- contrapositive
- truth value
- biconditional statement
- postulates
- midpoint
- congruent
- bisect
- bisector of an angle
- parallel
- transversal
- same-side interior angles
- alternate interior angles
- corresponding angles
- perpendicular
- perpendicular bisector

ESSENTIAL QUESTIONS

- Why are properties, postulates, and theorems important in mathematics?
- How are angles and parallel and perpendicular lines used in real-world settings?

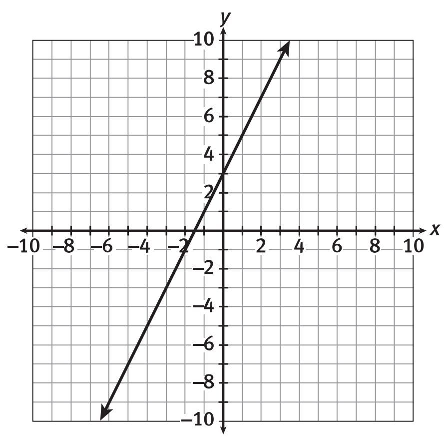
EMBEDDED ASSESSMENTS

These assessments, following Activities 3, 5, and 8 will give you an opportunity to demonstrate what you have learned about reasoning, proof, and some basic geometric figures.

Getting Ready

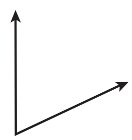
Write your answers on notebook paper.
Show your work.

1. Solve each equation.
   a. $5x - 2 = 8$
   b. $4x - 3 = 2x + 9$
   c. $6x + 3 = 2x + 8$
2. Graph $y = 3x - 3$ and label the x- and y-intercepts.
3. Tell the slope of a line that contains the points $(5, -3)$ and $(7, 3)$.
4. Write the equation of a line that has slope $\frac{1}{3}$ and y-intercept 4.
5. Write the equation of the line graphed below.

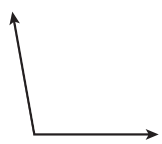


6. Describe a pattern shown in this sequence, and use the pattern to find the next two terms.
   7, 15, 23, 31, 39, _?_, _?_, . . .
7. Draw a right triangle and label the hypotenuse and legs.
8. Use a protractor to find the measure of each angle.

a.



b.



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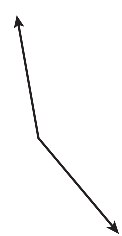


Geometric Figures

What's My Name?

Lesson 1-1 Basic Geometric Figures

My Notes

Learning Targets:

- Identify, describe, and name points, lines, line segments, rays, and planes using correct notation.
- Identify and name angles.

SUGGESTED LEARNING STRATEGIES: Activating Prior Knowledge, Think-Pair-Share, Group Presentation, Interactive Word Wall, Think Aloud, Debriefing, Self Revision/Peer Revision

Below are some types of figures you have seen in earlier mathematics courses. Describe each figure using your own words. If you can recall the mathematical terms that identify the figures, you can use them in your descriptions.

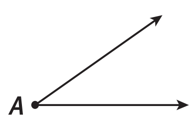
1.

2.

3.

4.

5.



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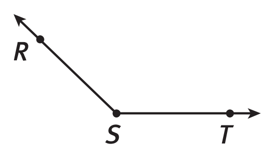

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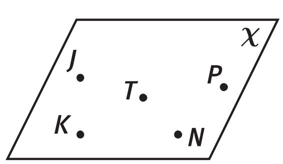
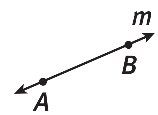
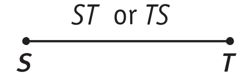
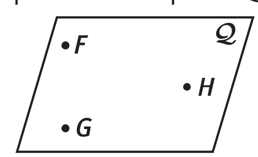


My Notes

READING MATH

$\overleftrightarrow{AB}$ is read, "line *AB*." $\overline{ST}$ is read, "line segment *ST*" or "segment *ST*." $\overrightarrow{MN}$ is read, "ray *MN*."

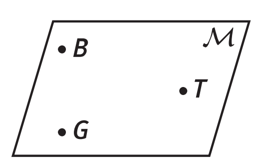
Naming Geometric Figures

Geometric Figure	Naming	Example
point	Named with a capital letter	point *P* 
line	Named using any two points on the line, in any order, with a line symbol drawn above OR Named using a lowercase letter	$\overleftrightarrow{AB}$, $\overleftrightarrow{BA}$, or line *m* 
line segment	Named using the two endpoints, in any order, with a segment symbol drawn above	$\overline{ST}$ or $\overline{TS}$ 
ray	Named using the endpoint and one other point, with a ray symbol drawn above; the endpoint is always listed first	$\overrightarrow{MN}$ 
plane	Named using any three points in the plane that are not on the same line, in any order OR Named using a capital cursive letter	plane *FGH* or plane $\mathcal{Q}$ 

8. Identify each geometric figure. Then give all possible names for the figure.

a.


b.

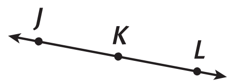

c.


d.


My Notes

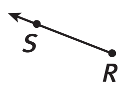
9. Draw $\overrightarrow{FE}$. Explain where the points F and E lie on the ray.

10. Critique the reasoning of others. Caleb says that the figure below can be named $\overleftrightarrow{KJ}$. Jen says the figure can be named $\overleftrightarrow{JL}$. Who is correct? Explain.



Check Your Understanding

11. Is $\overrightarrow{SR}$ a possible name for the figure at the right? Explain.



12. Graham draws a point. Describe how he could label and name the point.

There are three different ways to name an angle.

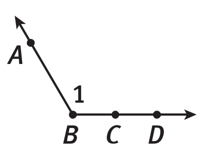
- Use the angle symbol and a number.
- Use the angle symbol and the vertex of the angle.
- Use the angle symbol and three points on the angle. The first point is on one side of the angle, the second point is the vertex, and the third point is on the other side of the angle.

MATH TIP

The *vertex* of an angle is the point where the sides of the angle meet, or intersect.

Example A

Give all possible names for the angle.



Use a number: $\angle 1$.

Use the vertex: $\angle B$.

Use three points. The second point should be the vertex. Be sure the first and third points are not on the same side.

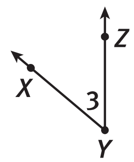
$\angle ABC, \angle ABD, \angle CBA, \angle DBA$

My Notes

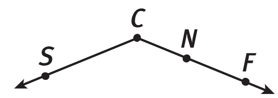
Try These A

Give all possible names for each angle.

a.



b.



TECHNOLOGY TIP

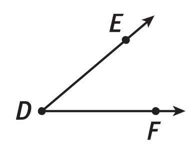
You can make your drawing for Item 13 by using a pencil and a straightedge or by using geometry software.

Check Your Understanding

13. Draw a figure that could be named $\angle LMN$.

14. Is $\angle FDE$ a possible name for the figure at the right? Explain.

15. How many different line segments does the figure at the right include? Name them.

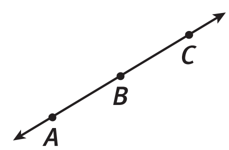


LESSON 1-1 PRACTICE

16. Identify each geometric figure. Then give all possible names for the figure.

a.

b.



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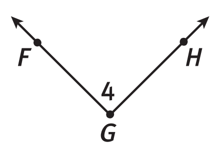
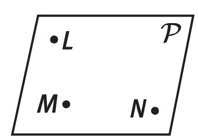
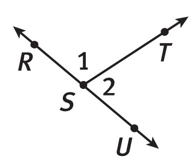


d.



The diagram below includes $\overleftrightarrow{RU}$. Use the figure for Items 17–19.



17. How many different rays does the figure include? Name them.

18. **Reason abstractly.** Explain why $\angle S$ is not an appropriate name for $\angle 1$.

19. Give the other possible appropriate names for $\angle 2$.

20. Draw $\overleftrightarrow{LM}$. Then draw $\overrightarrow{NP}$ so that point N lies on $\overleftrightarrow{LM}$.

My Notes

Learning Targets:

- Describe angles and angle pairs.
- Identify and name parts of circles.

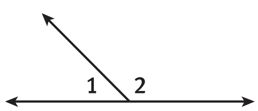
SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Self Revision/Peer Revision, Discussion Groups, Create Representations

As you share your ideas, be sure to use mathematical terms and academic vocabulary precisely. Make notes to help you remember the meaning of new words and how they are used to describe mathematical ideas.

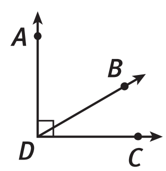
1. Draw four angles with different characteristics. Describe each angle. Name the angles using numbers and letters.

2. ***Compare and contrast*** each pair of angles.

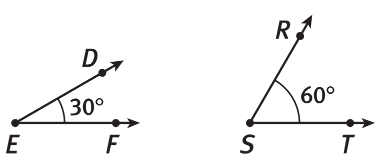
a.



b.



c.



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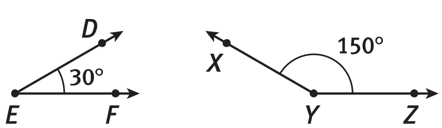


ACADEMIC VOCABULARY

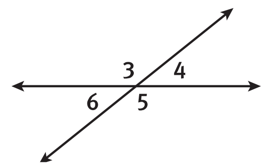
When you ***compare and contrast*** two figures, you describe how they are alike or different.

My Notes

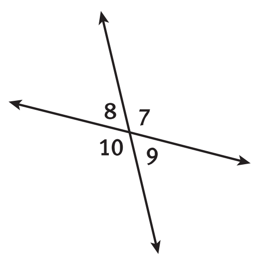
Recall that the sum of the measures of *complementary angles* is 90° and the sum of the measures of *supplementary angles* is 180°.

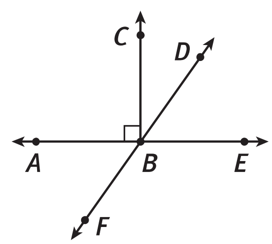
3. a. The figure below shows two intersecting lines. Name two angles that are supplementary to ∠4.

b. Reason quantitatively. Explain why the angles you named in part a must have the same measure.



4. Complete the chart by naming all the listed angle types in each figure.





Acute angles		
Obtuse angles		
Angles with the same measure		
Supplementary angles		
Complementary angles		

MATH TIP

Angles can be classified by their measures.

- An *acute angle* measures greater than 0° and less than 90°.
- A *right angle* measures 90°.
- An *obtuse angle* measures greater than 90° and less than 180°.
- A *straight angle* measures 180°.

Lesson 1-2
More Geometric Figures

My Notes

A *chord* of a circle is a segment with both endpoints on the circle.

A *diameter* is a chord that passes through the center of a circle.

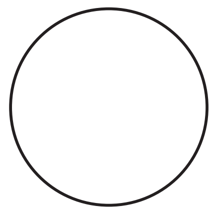
A *radius* is a segment with one endpoint on the circle and one endpoint at the center of the circle.

5. In the circle below, draw and label each geometric term.

a. radius *OA*

b. chord *BA*

c. diameter *CA*

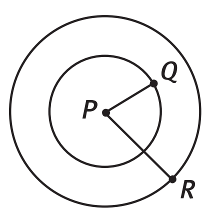


6. Refer to your drawings in the circle above. What is the geometric term for point *O*?

7. The circles below are concentric, meaning that they have the same center. The center of both circles is point *P*.

a. Construct viable arguments. Explain why circle *P* is not an appropriate name for the smaller circle.

b. Propose an alternate name for the smaller circle that would be appropriate. ***Justify*** your choice.



ACADEMIC VOCABULARY

When you ***justify*** a choice, you provide evidence that shows that your choice is correct or reasonable.

My Notes

MATH TIP

You can use a compass to draw a circle. You can also use geometry software.

Check Your Understanding

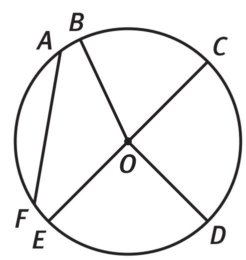
8. Compare and contrast the terms *acute angle*, *obtuse angle*, *right angle*, and *straight angle*.

9. The measure of $\angle A$ is 42°.
  a. What is the measure of an angle that is complementary to $\angle A$?
  b. What is the measure of an angle that is supplementary to $\angle A$?

10. Draw a circle P.
  a. Draw a segment that has one endpoint on the circle but is not a chord.
  b. Draw a segment that intersects the circle in two points and contains the center but is not a radius, diameter, or chord.

LESSON 1-2 PRACTICE

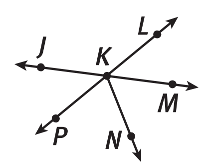
11. Classify each segment in circle O. Use all terms that apply.
  a. $\overline{AF}$
  b. $\overline{BO}$
  c. $\overline{CO}$
  d. $\overline{DO}$
  e. $\overline{EO}$
  f. $\overline{CE}$



The figure below includes $\overleftrightarrow{PL}$ and $\overleftrightarrow{JM}$. Use the figure for Items 12–13.

12. Name three pairs of supplementary angles.

13. Name two angles that appear to be obtuse.



14. **Make sense of problems.** Can two obtuse angles be complementary to each other? Explain.

15. **Model with mathematics.** Is it possible for a pair of nonadjacent angles to share vertex A and arm $\overrightarrow{AB}$? If it is possible, draw an example. If it is not possible, explain your answer.

ACTIVITY 1 PRACTICE

Write your answers on notebook paper.
Show your work.

Lesson 1-1

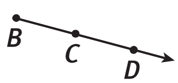
1. Which is a correct name for this line?

A. $\vec{G}$ **B.** $\overline{GM}$

C. $\overleftrightarrow{MG}$ **D.** $\overleftrightarrow{ME}$

2. Draw each figure.
a. point L
b. $\overleftrightarrow{MN}$
c. $\overline{PQ}$
d. plane $\mathcal{R}$
e. $\overrightarrow{ST}$
f. $\angle U$

3. Identify each geometric figure. Then give all possible names for the figure.

a.


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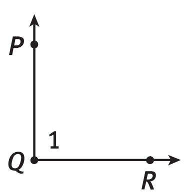
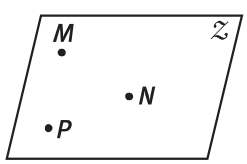
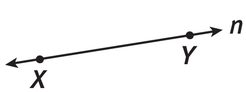


c.


d.

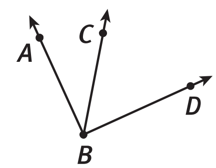

e.


4. Describe all of the acceptable ways to name a plane.

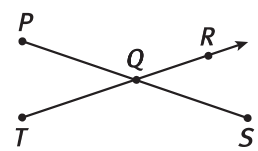
5. What are some acceptable ways to name an angle?

6. Describe all of the acceptable ways to name a line.

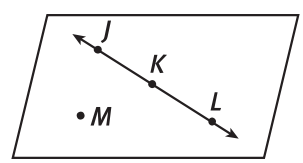
7. a. How many rays are in the figure below? Name them.
b. How many different angles are in the figure? Name them.



8. Describe the diagram below using correct geometric names.



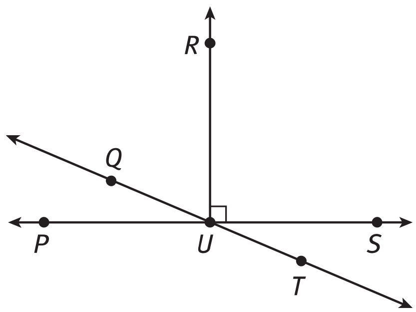
9. a. Explain why plane JKL is not an appropriate name for the plane below.
b. Give three names for the plane that would be appropriate.



10. How can you use the figures in this activity to describe real-world objects and situations? Give examples.

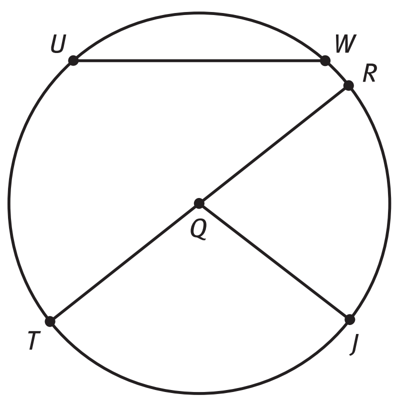
Lesson 1-2

11. In this diagram, $m\angle SUT = 25°$.



a. Name another angle that measures 25°.
b. Name a pair of complementary angles.
c. Name a pair of supplementary angles.

Use circle *Q* for Items 12–15.



12. Name the radii of circle *Q*.

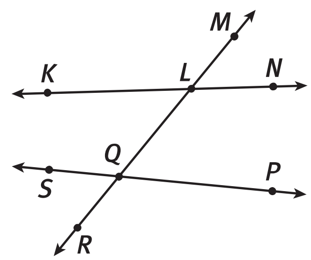
13. Name the diameter(s) in circle *Q*.

14. Name the chord(s) in circle *Q*.

15. Which statement below must be true about circle *Q*?
A. The distance from *U* to *W* is the same as the distance from *R* to *T*.
B. The distance from *U* to *W* is the same as the distance from *Q* to *J*.
C. The distance from *R* to *T* is half the distance from *Q* to *R*.
D. The distance from *R* to *T* is twice the distance from *Q* to *J*.

16. Two angles have the same measure. The angles are also supplementary. Are the angles acute, right, or obtuse? How do you know?

17. This diagram includes $\overleftrightarrow{KN}$, $\overleftrightarrow{SP}$, and $\overleftrightarrow{MR}$.



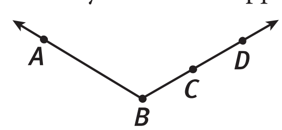
a. Name three angles that appear to be acute.
b. Name three angles that appear to be obtuse.
c. Name three straight angles.

18. $\angle F$ and $\angle G$ are complementary. The measure of $\angle F$ is four times the measure of $\angle G$. What is the measure of each angle?

19. Compare and contrast a chord and a diameter of a circle.

MATHEMATICAL PRACTICES
Construct Viable Arguments and Critique the Reasoning of Others

20. a. Lucinda describes the angle below as $\angle BAC$. Ahmad describes the angle as $\angle CBD$. State whether each of these names is appropriate for the angle, and explain why or why not.
b. Give another name for the angle, and explain why the name you chose is appropriate.



Logical Reasoning

ACTIVITY 2

Riddle Me This

Lesson 2-1 Inductive Reasoning

My Notes

Learning Targets:

- Make conjectures by applying inductive reasoning.
- Recognize the limits of inductive reasoning.

SUGGESTED LEARNING STRATEGIES: Interactive Word Wall, Vocabulary Organizer, Think-Pair-Share, Look for a Pattern, Visualization, Discussion Groups, Self Revision/Peer Revision

The ability to recognize patterns is an important aspect of mathematics. But when is an *observed* pattern actually a *real* pattern that continues beyond just the observed cases? In this activity, you will explore patterns and check to see if the patterns hold true beyond the observed cases.

Inductive reasoning is the process of observing data, recognizing patterns, and making a generalization. This generalization is a ***conjecture***.

1. Five students attended a party and ate a variety of foods. Something caused some of them to become ill. JT ate a hamburger, pasta salad, and coleslaw. She became ill. Guy ate coleslaw and pasta salad but not a hamburger. He became ill. Dean ate only a hamburger and felt fine. Judy didn't eat anything and also felt fine. Cheryl ate a hamburger and pasta salad but no coleslaw, and she became ill. Use inductive reasoning to make a conjecture about which food probably caused the illness.

2. **Reason quantitatively.** Use inductive reasoning to make a conjecture about the next two terms in each sequence. Explain the pattern you used to determine the terms.
   a. A, 4, C, 8, E, 12, G, 16, ______, ______

   b. 3, 9, 27, 81, 243, ______, ______

   c. 3, 8, 15, 24, 35, 48, ______, ______

   d. 1, 1, 2, 3, 5, 8, 13, ______, ______

DISCUSSION GROUP TIPS

If you do not understand something in group discussions, ask for help or raise your hand for help. Describe your questions as clearly as possible, using synonyms or other words when you do not know the precise words to use.

CONNECT TO HISTORY

The sequence of numbers in Item 2d is named after Leonardo of Pisa, who was known as Fibonacci. Fibonacci's 1202 book *Liber Abaci* introduced the sequence to Western European mathematics.

My Notes

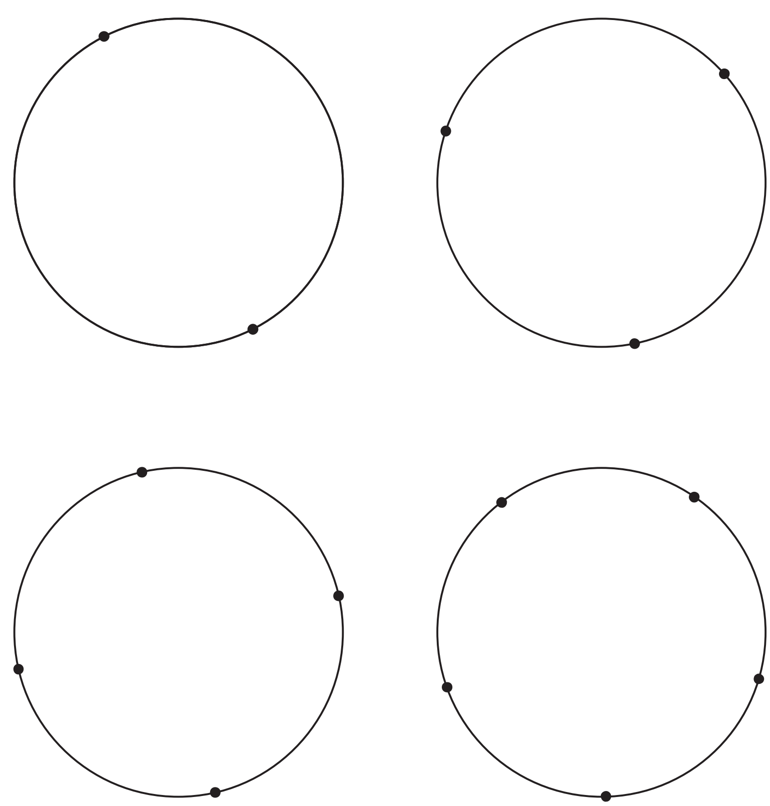
3. Use the four circles below. From each of the given points, draw all possible chords. These chords will form a number of nonoverlapping regions in the interior of each circle. For each circle, count the number of these regions. Then enter this number in the appropriate place in the table below.

Number of Points on the Circle	Number of Nonoverlapping Regions Formed
2	
3	
4	
5	

4. Look for a pattern in the table above.

a. Describe, in words, any patterns you see for the numbers in the column labeled Number of Nonoverlapping Regions Formed.

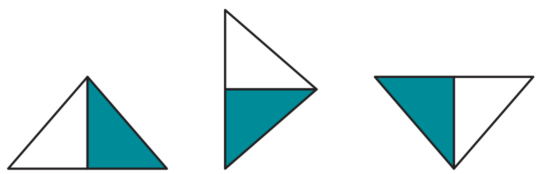
b. Use the pattern that you described to predict the number of nonoverlapping regions that will be formed if you draw all possible line segments that connect six points on a circle.



My Notes

Check Your Understanding

5. Critique the reasoning of others. The diagram shows the first three figures in a pattern. Mario conjectures that the fourth figure in the pattern will be the third figure rotated 90° clockwise. Is Mario's conjecture reasonable? Explain.



6. Anya is training for a 10K race. The table shows the distance she ran during the first 3 weeks of training.

a. Make a conjecture about the distance Anya will run during each practice of the fourth week.

b. How could you test your conjecture?

Week	Distance per Practice (mi)
1	0.5
2	1.0
3	2.0

7. Use the circle on the next page. Draw all possible chords connecting any two of the six points.

a. What is the number of nonoverlapping regions formed by chords connecting the points on this circle?

b. Is the number you obtained above the same number you predicted from the pattern in the table in Item 4b?

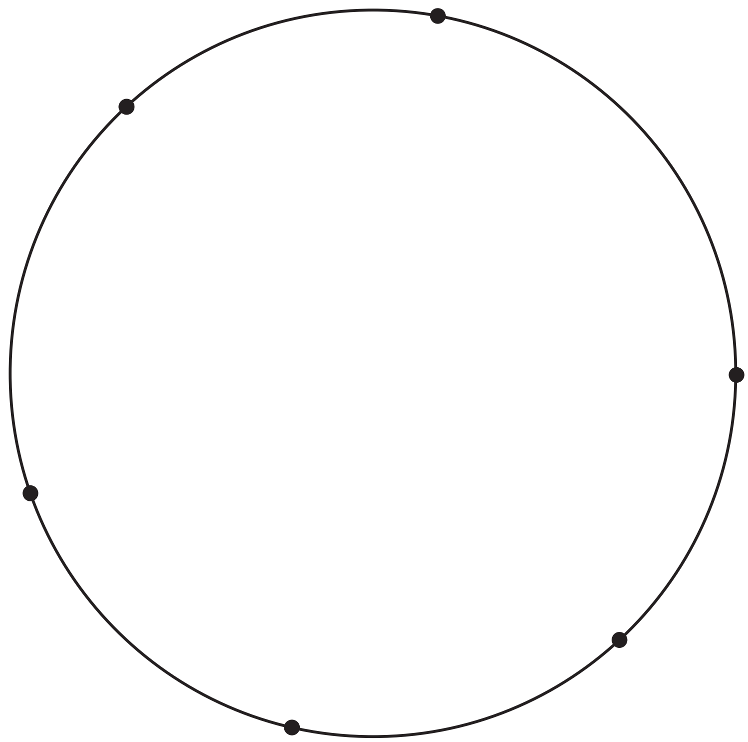
c. Describe what you would do to further investigate the pattern in the number of regions formed by chords joining n points on a circle, where n represents any number of points placed on a circle.

MATH TIP

Number the nonoverlapping regions as you count them. That way, you'll be sure not to skip any or count any more than once.

My Notes

d. **Make use of structure.** Try to find a new pattern for predicting the number of regions formed by chords joining points on a circle when two, three, four, five, and six points are placed on a circle. Describe the pattern in algebraic terms or in words.



My Notes

Check Your Understanding

8. Zack writes a conjecture that is true for the first few terms of a sequence. Is the conjecture necessarily true for *all* terms of the sequence? Explain.

9. The table below represents the function $f(x) = 2x + 5$. Based on the table, Trent conjectures that the value of $f(x)$ is always positive. Is Trent's conjecture true? Explain how you know.

x	-2	-1	0	1	2
$f(x)$	1	3	5	7	9

10. Monique tosses a coin three times. She gets heads each time. Based on this pattern, she makes a conjecture that she will always get heads when she tosses the coin. Do you think Monique's conjecture is reasonable? Why or why not?

MATH TIP

Recall that a *sequence* is an ordered list of numbers or other items. For example, 2, 4, 6, 8, 10, . . . is a sequence.

LESSON 2-1 PRACTICE

11. Use inductive reasoning to determine the next two terms in each sequence.

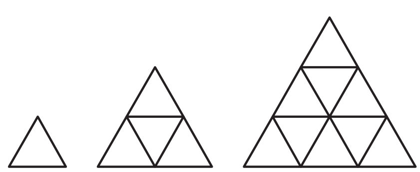
a. $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$

b. A, B, D, G, K, . . .

12. Write the first five terms of two different sequences that have 10 as the second term. Describe the pattern in each of your sequences.

13. Make sense of problems. Generate a sequence using this description: The first term in the sequence is 4, and each other term is three more than twice the previous term.

14. The diagram shows the first three figures in a pattern. Each figure is made of small triangles. How many small triangles will be in the sixth figure of the pattern? Support your answer.



15. Critique the reasoning of others. The first two terms of a number pattern are 2 and 4. Alicia conjectures that the next term will be 6. Mario conjectures that the next term will be 8. Whose conjecture is reasonable? Explain.

My Notes

Learning Targets:

- Use deductive reasoning to prove that a conjecture is true.
- Develop geometric and algebraic arguments based on deductive reasoning.

SUGGESTED LEARNING STRATEGIES: Interactive Word Wall, Vocabulary Organizer, Marking the Text, Think-Pair-Share, Look for a Pattern, Use Manipulatives, Discussion Groups, Summarizing, Paraphrasing, Create Representations, Group Presentation

In this activity, you have described patterns and made conjectures about how these patterns would extend beyond your observed cases, based on collected data. Some of your conjectures were probably shown to be untrue when additional data were collected. Other conjectures took on a greater sense of certainty as more confirming data were collected.

In mathematics, there are certain methods and rules of ***argument*** that mathematicians use to convince someone that a conjecture is true, even for cases that extend beyond the observed data set. These rules are called rules of logical reasoning or rules of ***deductive reasoning***. An argument that follows such rules is called a ***proof***. A statement or conjecture that has been proven, that is, established as true without a doubt, is called a ***theorem***. A proof transforms a conjecture into a theorem.

ACADEMIC VOCABULARY

In this context, an argument is not a disagreement. Instead, an ***argument*** is a series of reasons or facts that support a given statement.

Below are some definitions from arithmetic.

Even integer: An integer that has a remainder of 0 when it is divided by 2.

Odd integer: An integer that has a remainder of 1 when it is divided by 2.

Express regularity in repeated reasoning. In the following items, you will make some conjectures about the sums of even and odd integers.

1. Calculate the sum of some pairs of even integers. Show the examples you use and make a conjecture about the sum of two even integers.

2. Calculate the sum of some pairs of odd integers. Show the examples you use and make a conjecture about the sum of two odd integers.

3. Calculate the sum of pairs of integers consisting of one even integer and one odd integer. Show the examples you use and make a conjecture about the sum of an even integer and an odd integer.

Lesson 2-2
Deductive Reasoning

My Notes

The following items will help you write a convincing argument (a proof) that supports each of the conjectures you made in Items 1–3.

4. Figures A, B, C, and D are puzzle pieces. Each figure represents an integer determined by counting the square pieces in the figure. Use these figures to answer the following questions.

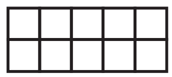


Figure A

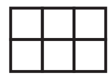


Figure B

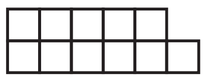


Figure C

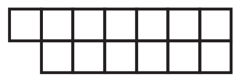


Figure D

a. Which of the figures can be used to model an even integer?

b. Which of the figures can be used to model an odd integer?

c. Compare and contrast the models of even and odd integers.

5. Model with mathematics. Which pairs of puzzle pieces can fit together to form rectangles? Make sketches to show how they fit.

My Notes

6. Explain how the figures (when used as puzzle pieces) can be used to show that each of the conjectures in Items 1 through 3 is true.

In Items 5 and 6, you proved the conjectures in Items 1 through 3 geometrically. In Items 7 through 9, you will prove the same conjectures algebraically.

This is an algebraic definition of even integer: An integer is *even* if and only if it can be written in the form $2p$, where p is an integer. (You can use other variables, such as $2m$, to represent an even integer, where m is an integer.)

7. Reason abstractly. Use the expressions $2p$ and $2m$, where p and m are integers, to confirm the conjecture that the sum of two even integers is an even integer.

MATH TIP

- A set has **closure** under an operation if the result of the operation on members of the set is also in the same set.
- In symbols, the distributive property of multiplication over addition can be written as $a(b + c) = a(b) + a(c)$.

This is an algebraic definition of odd integer: An integer is *odd* if and only if it can be written in the form $2t + 1$, where t is an integer. (Again, you do not have to use t as the variable.)

8. Use expressions for odd integers to confirm the conjecture that the sum of two odd integers is an even integer.

My Notes

9. Use expressions for even and odd integers to confirm the conjecture that the sum of an even integer and an odd integer is an odd integer.

In Items 5 and 6, you developed a geometric puzzle-piece argument to confirm conjectures about the sums of even and odd integers. In Items 7–9, you developed an algebraic argument to confirm these same conjectures. Even though one method is considered geometric and the other algebraic, they are often seen as the same basic argument.

10. Construct viable arguments. Explain the link between the geometric and the algebraic methods of the proof.

11. State three theorems that you have proved about the sums of even and odd integers.

My Notes

Check Your Understanding

12. Compare and contrast a theorem and a conjecture.

13. Todd knows that 3, 5, 7, 11, and 13 are prime numbers. Based on this evidence, he concludes that all prime numbers are odd.

a. Is this an example of inductive or deductive reasoning? Explain.

b. Is Todd's conclusion correct? Support your answer.

14. Shayla knows that all rectangles have four right angles. She also knows that figure *ABCD* is a rectangle. She concludes that figure *ABCD* has four right angles.

a. Is this an example of inductive or deductive reasoning? Explain.

b. **Critique the reasoning of others.** Is Shayla's conclusion correct? Support your answer.

LESSON 2-2 PRACTICE

15. Use expressions for even and odd integers to confirm the conjecture that the product of an even integer and an odd integer is an even integer.

16. **Reason abstractly.** Prove this conjecture geometrically: Any odd integer can be expressed as the sum of an even integer and an odd integer.

17. Use deductive reasoning to prove that the solution of the equation $x - 5 = -2$ is $x = 3$. Be sure to justify each step in your proof.

18. **Reason quantitatively.** Based solely on the pattern in the table, Andre states that the number of sides of a polygon is equal to its number of angles. Is Andre's statement a conjecture or a theorem? Explain.

Polygon	Sides	Angles
Triangle	3	3
Quadrilateral	4	4
Pentagon	5	5

19. Hairs found at a crime scene are consistent with those of a suspect. Based on this evidence, an investigator concludes that the suspect was at the crime scene. Is this an example of inductive or deductive reasoning? Explain.

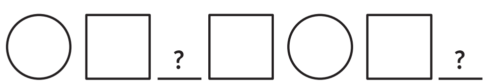
ACTIVITY 2 PRACTICE

Write your answers on notebook paper.
Show your work.

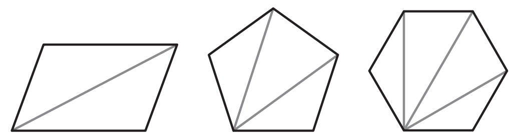
Lesson 2-1

1. Use inductive reasoning to determine the next two terms in the sequence.
   a. 1, 3, 7, 15, 31, …
   b. 3, −6, 12, −24, 48, …
2. Write the first five terms of two different sequences for which 24 is the third term.
3. Generate a sequence using this description: The first term in the sequence is 2, and the terms increase by consecutive odd numbers beginning with 3.
4. Use this picture pattern.

   a. Draw the next shape in the pattern.
   b. Write a sequence of numbers that could be used to express the pattern.
   c. Verbally describe the pattern of the sequence.
5. Two shapes are missing from the sequence below.



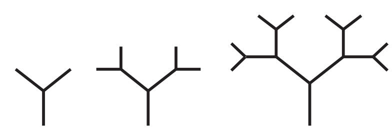
   a. Based on the given shapes, make a conjecture about the missing shapes. Explain the pattern you used to make your conjecture.
   b. Now suppose that you learn that the first missing shape is a triangle. Would this additional information change your conjecture about the other missing shape? Explain.
6. Describe a situation from your everyday life in which you applied inductive reasoning.
7. Kristen divides each convex polygon below into triangles by drawing all diagonals from a single vertex. She conjectures that the number of triangles is always two less than the number of sides of the convex polygon.



   a. Is Kristen's conjecture reasonable? Explain.
   b. Provide an additional example that supports your answer to Item 7a.
8. Which rule describes how to find the next term of the following sequence?

   64, 40, 28, 22, 19, . . .

   A. Subtract 24 from the previous term.
   B. Divide the previous term by 2 and add 8.
   C. Divide the previous term by 4 and add 16.
   D. Multiply the previous term by 5 and divide by 8.
9. Explain how you know that the rules you did not choose in Item 8 are incorrect.
10. Use this picture pattern.



   a. Draw the next shape in the pattern.
   b. Write a sequence of numbers that could be used to express the pattern.
   c. Verbally describe the pattern of the sequence.

Lesson 2-2

11. Use expressions for odd integers to confirm the conjecture that the product of two odd integers is an odd integer.

12. For the first four weeks of school, the cafeteria served either spaghetti or lasagna on Thursday. Based on this evidence, Liam states that the cafeteria will serve Italian food next Thursday. Is Liam's statement a conjecture or a theorem? Explain.

13. Use deductive reasoning to prove that $x = -3$ is not in the solution set of the inequality $-4x < 8$. Be sure to justify each step in your proof.

14. David notices this pattern.

$19 = 1 \times 9 + 1 + 9$

$29 = 2 \times 9 + 2 + 9$

$39 = 3 \times 9 + 3 + 9$

Based on this pattern, David concludes that any two-digit number ending in 9 is equal to $n \times 9 + n + 9$, where n is the tens digit of the number.

a. Is this an example of inductive or deductive reasoning? Explain.
b. Is David's conclusion correct? Support your answer.

15. The density of gold is 19.3 grams per cubic centimeter. Rachel determines that the density of a coin is 18.7 grams per cubic centimeter. Based on this evidence, she concludes that the coin is not gold or at least not entirely gold. Is this an example of inductive or deductive reasoning? Explain.

16. Consider the expression $3p + 5$. Bethany states that this expression is even for any integer p because $3p + 5 = 8$ when $p = 1$ and $3p + 5 = 20$ when $p = 5$. Is Bethany's conclusion correct? Support your answer.

17. Consider these true statements.

All amphibians are cold blooded.

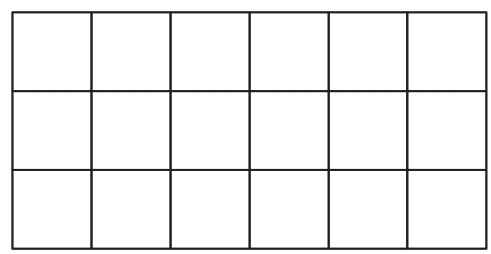
All frogs are amphibians.

Chris has a pet frog.

Based on deductive reasoning, which of the following statements is *not* necessarily true?
A. Chris has a cold-blooded pet.
B. Chris has a pet amphibian.
C. All frogs are cold blooded.
D. All amphibians are frogs.

18. Consider this conjecture: An integer that is divisible by 9 is also divisible by 3.

a. Prove the conjecture geometrically. (Hint: You can represent an integer divisible by 9 by using a rectangle composed of groups of 9 squares, as shown below.)



b. Prove the conjecture algebraically.
c. Explain the link between the geometric and the algebraic methods of the proof.

MATHEMATICAL PRACTICES
Construct Viable Arguments and Critique the Reasoning of Others

19. a. Construct a viable argument for this conjecture: If two angles are complementary, both angles must be acute.
b. What type of reasoning—inductive or deductive—did you use in your argument? Explain.
c. Compare your argument with the argument another student in your group or class has written. Do you agree with this student's argument? Why or why not?

The Axiomatic System of Geometry

ACTIVITY 3

Back to the Beginning

Lesson 3-1 Geometric Definitions and Two-Column Proofs

Learning Targets:

- Distinguish between undefined and defined terms.
- Use properties to complete algebraic two-column proofs.

SUGGESTED LEARNING STRATEGIES: Close Reading, Quickwrite, Think-Pair-Share, Vocabulary Organizer, Interactive Word Wall, Group Presentation, Discussion Groups, Self Revision/Peer Revision

Geometry is an ***axiomatic system***. That means that from a small, basic set of agreed-upon assumptions and premises, an entire structure of logic is devised. Many interactive computer games are designed with this kind of structure. A game may begin with a basic set of scenarios. From these scenarios, a gamer can devise tools and strategies to win the game.

In geometry, it is necessary to agree on clear-cut meanings, or definitions, for words used in a technical manner. For a definition to be helpful, it must be expressed in words whose meanings are already known and understood.

Compare the following definitions.

Fountain: a roundel that is barry wavy of six argent and azure

Guige: a belt that is worn over the right shoulder and used to support a shield

1. Which of the two definitions above is easier to understand? Why?

For a new vocabulary term to be helpful, it should be defined using words that have already been defined. The *first* definitions used in building a system, however, cannot be defined in terms of other vocabulary words, because no other vocabulary words have been defined yet. In geometry, it is traditional to start with the simplest and most fundamental terms—without trying to define them—and use these terms to define other terms and develop the system of geometry. These fundamental ***undefined terms*** are **point**, **line**, and **plane**.

2. Define each term using the undefined terms.

a. **Ray**

b. **Collinear points**

c. **Coplanar points**

My Notes

MATH TERMS

The term *line segment* can be defined in terms of **undefined terms:** A *line segment* is part of a line bounded by two points on the line called *endpoints*.

My Notes

MATH TIP

The common endpoint of the rays that form an angle is the *vertex* of the angle.

After a term has been defined, it can be used to define other terms. For example, an **angle** is defined as a figure formed by two rays with a common endpoint.

3. Define each term using the already defined terms.

a. Complementary angles

b. Supplementary angles

CONNECT TO ALGEBRA

Addition Property of Equality
If $a = b$ and $c = d$, then $a + c = b + d$.

Subtraction Property of Equality
If $a = b$ and $c = d$, then $a - c = b - d$.

Multiplication Property of Equality
If $a = b$, then $ca = cb$.

Division Property of Equality
If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.

Distributive Property
$a(b + c) = ab + ac$

Reflexive Property
$a = a$

Symmetric Property
If $a = b$, then $b = a$.

Transitive Property
If $a = b$ and $b = c$, then $a = c$.

Substitution Property
If $a = b$, then a can be substituted for b in any equation or inequality.

The process of deductive reasoning, or deduction, must have a starting point. A conclusion based on deduction cannot be made unless there is an established assertion to work from. To provide a starting point for the process of deduction, a number of assertions are accepted as true without proof.

When you solve algebraic equations, you are using deduction. You can use properties to support your reasoning without having to prove that the properties are true.

4. Using one operation or property per step, show how to solve the equation $4x + 9 = 18 - \frac{1}{2}x$. Name each operation or property used to justify each step.

My Notes

You can organize the steps and the reasons used to justify the steps in two columns with statements (steps) on the left and reasons (properties) on the right. This format is called a ***two-column proof***.

Example A

Given: $3(x + 2) - 1 = 5x + 11$ **Prove:** $x = -3$

Statements	Reasons
1. $3(x + 2) - 1 = 5x + 11$	**1.** Given equation
2. $3(x + 2) = 5x + 12$	**2.** Addition Property of Equality
3. $3x + 6 = 5x + 12$	**3.** Distributive Property
4. $6 = 2x + 12$	**4.** Subtraction Property of Equality
5. $-6 = 2x$	**5.** Subtraction Property of Equality
6. $-3 = x$	**6.** Division Property of Equality
7. $x = -3$	**7.** Symmetric Property of Equality

Try These A

a. Supply the reasons to justify each statement in the proof below.

Given: $\frac{x-3}{2} = \frac{6+x}{5}$ **Prove:** $x = 9$

Statements	Reasons
1. $\frac{x-3}{2} = \frac{6+x}{5}$	**1.** ____________
2. $10\left(\frac{x-3}{2}\right) = 10\left(\frac{6+x}{5}\right)$	**2.** ____________
3. $5(x - 3) = 2(6 + x)$	**3.** ____________
4. $5x - 15 = 12 + 2x$	**4.** ____________
5. $3x - 15 = 12$	**5.** ____________
6. $3x = 27$	**6.** ____________
7. $x = 9$	**7.** ____________

b. Complete the *Prove* statement and write a two-column proof for the equation given in Item 4. Number each statement and corresponding reason.

Given: $4x + 9 = 18 - \frac{1}{2}x$

Prove:

DISCUSSION GROUP TIP

As you read and discuss the two-column proof in Example A, ask and answer questions to be sure you have a clear understanding of not only all the terminology used, but also how the two-column proof is formed.

MATH TIP

In Item b, the *Prove* statement should be the solution of the given equation. In this case, it may be easier to write the statements and reasons of the proof before writing the *Prove* statement.

My Notes

Check Your Understanding

5. Explain why undefined terms are necessary in geometry.
6. **Express regularity in repeated reasoning.** What is the relationship between a conjecture, a theorem, and a two-column proof?
7. Jeffrey wrote a two-column proof to solve the equation $3(x + 5) = -6x + 6$. In addition to an incorrectly written *Prove* statement, what error did Jeffrey make in his proof? Rewrite the proof so that it correctly shows how to solve the given equation.

Given: $3(x + 5) = -6x + 6$ **Prove:** $x = -81$

Statements	Reasons
1. $3(x + 5) = -6x + 6$	**1.** Given equation
2. $3x + 15 = -6x + 6$	**2.** Distributive Property
3. $9x + 15 = 6$	**3.** Addition Property of Equality
4. $9x = -9$	**4.** Subtraction Property of Equality
5. $x = -81$	**5.** Multiplication Property of Equality

LESSON 3-1 PRACTICE

8. Identify the property that justifies the statement: If $4x - 3 = 7$, then $4x = 10$.
9. Complete the prove statement and write a two-column proof for the equation:

   Given: $x - 2 = 3(x - 4)$ Prove:
10. **Construct viable arguments.** Explain why *line segment* is considered a defined term in geometry.
11. Complete the prove statement and write a two-column proof for the equation:

    Given: $2n - 21 = \frac{n}{4}$ Prove:
12. **Look for and make use of structure.** Suppose you are given that $a = b + 2$ and $b + 2 = 5$. What can you prove by using these statements and the Transitive Property?

My Notes

Learning Targets:

- Identify the hypothesis and conclusion of a conditional statement.
- Give counterexamples for false conditional statements.

SUGGESTED LEARNING STRATEGIES: Vocabulary Organizer, Interactive Word Wall, Marking the Text, Self Revision/Peer Revision, Discussion Groups

Rules of logical reasoning involve using a set of given statements along with a valid argument to reach a conclusion. Statements to be proved are often written in if-then form. An if-then statement is called a ***conditional statement***. In such statements, the *if* clause is the ***hypothesis***, and the *then* clause is the ***conclusion***.

WRITING MATH

The letters p and q are often used to represent the hypothesis and conclusion, respectively, in a conditional statement. The basic form of an if-then statement would then be, "If p, then q."

Example A

Conditional statement: If $3(x + 2) - 1 = 5x + 11$, then $x = -3$.	
Hypothesis	**Conclusion**
$3(x + 2) - 1 = 5x + 11$	$x = -3$

Try These A

Use the conditional statement: If $x + 7 = 10$, then $x = 3$.

a. What is the hypothesis?

b. What is the conclusion?

c. State the property of equality that justifies the conclusion of the statement.

Conditional statements may not always be written in if-then form. You can restate such conditional statements in if-then form.

1. Make use of structure. Restate each conditional statement in if-then form.

a. I'll go if you go.

b. There is smoke only if there is fire.

c. $x = 4$ implies $x^2 = 16$.

READING MATH

Forms of conditional statements include:

- If p, then q.
- p only if q
- p implies q.
- q if p

My Notes

An if-then statement is false if an example can be found for which the hypothesis is true and the conclusion is false. This type of example is a ***counterexample***.

2. This is a false conditional statement.

If two numbers are odd, then their sum is odd.

a. Identify the hypothesis of the statement.

b. Identify the conclusion of the statement.

c. Give a counterexample for the conditional statement and justify your choice for this example.

Check Your Understanding

3. **Reason abstractly.** If you can find an example for which both the hypothesis and the conclusion of a conditional statement are true, is the conditional statement itself necessarily true? Explain.

4. Give an example of a true conditional statement that includes this hypothesis: An angle is named $\angle ABC$.

5. Give an example of a true conditional statement that includes this conclusion: The angles share a vertex.

6. Cesar conjectures that the quotient of any two even numbers greater than 0 is odd.

a. Write Cesar's conjecture as a conditional statement.

b. Give a counterexample to show that Cesar's conjecture is false.

7. Write the definition of *collinear points* as a conditional statement.

My Notes

LESSON 3-2 PRACTICE

8. Write the statement in if-then form:

Two angles have measures that add up to 90° only if they are complements of each other.

9. Which of the following is a counterexample of this statement?

If an angle is acute, then it measures 80°.

A. a 100° angle **B.** a 90° angle
C. an 80° angle **D.** a 70° angle

10. Identify the hypothesis and the conclusion of the statement:

If it is not raining, then I will go to the park.

11. Critique the reasoning of others. Joanna says that $4 + 7 = 11$ is a counterexample that shows that the following conditional statement is false. Is Joanna correct? Explain.

If two integers are even, then their sum is even.

12. Construct viable arguments. Why do you only need a single counterexample to show that a conditional statement is false?

My Notes

ACADEMIC VOCABULARY

When you ***interchange*** a hypothesis and a conclusion, you switch them. When you ***negate*** a hypothesis or a conclusion, you rewrite it by adding the word *not*. Note that if a hypothesis or a conclusion already includes the word *not*, you can negate it by removing *not*.

Learning Targets:

- Write and determine the truth value of the converse, inverse, and contrapositive of a conditional statement.
- Write and interpret biconditional statements.

SUGGESTED LEARNING STRATEGIES: Vocabulary Organizer, Interactive Word Wall, Think-Pair-Share, Group Presentation, Discussion Groups

Every conditional statement has three related conditionals. These are the ***converse***, the ***inverse***, and the ***contrapositive*** of the conditional statement. The converse of a conditional is formed by ***interchanging*** the hypothesis and conclusion of the statement. The inverse is formed by ***negating*** both the hypothesis and the conclusion. Finally, the contrapositive is formed by interchanging *and* negating both the hypothesis and the conclusion.

Conditional: If p, then q.

Converse: If q, then p.

Inverse: If not p, then not q.

Contrapositive: If not q, then not p.

1. Given the conditional statement:

If a figure is a triangle, then it is a polygon.

Complete the table.

Form of the statement	Write the statement	True or False?	If the statement is false, give a counterexample.
Conditional statement	If a figure is a triangle, then it is a polygon.		
Converse of the conditional statement			
Inverse of the conditional statement			
Contrapositive of the conditional statement			

Lesson 3-3
Converse, Inverse, and Contrapositive

If a given conditional statement is true, the converse and inverse are not necessarily true. However, the contrapositive of a true conditional is always true, and the contrapositive of a false conditional is always false. Likewise, the converse and inverse of a conditional are either both true or both false. Statements with the same ***truth values*** are *logically equivalent*.

2. Write a true conditional statement whose inverse is false.

3. Write a true conditional statement that is logically equivalent to its converse.

When a statement and its converse are both true, they can be combined into one statement using the words "if and only if." An "if and only if" statement is a ***biconditional statement***. All definitions you have learned can be written as biconditional statements.

4. Write the definition of perpendicular lines in biconditional form.

5. Consider the statement: *Numbers that do not end in 2 are not even.*

a. Rewrite the statement in if-then form and state whether it is true or false.

b. Write the converse and state whether it is true or false. If false, give a counterexample.

c. Write the inverse and state whether it is true or false.

d. Write the contrapositive and state whether it is true or false. If false, give a counterexample.

e. Can you write a biconditional statement for the original statement? Why or why not?

My Notes

MATH TERMS

The **truth value** of a statement is the truth or falsity of that statement.

MATH TIP

Given the biconditional statement "*p* if and only if *q*," then the following conditional statements are true.

- If *p*, then *q*.
- If *q*, then *p*.
- If not *p*, then not *q*.
- If not *q*, then not *p*.

My Notes

Check Your Understanding

6. Write four true conditional statements based on this biconditional statement.

An angle is a right angle if and only if it measures 90°.

7. A conditional statement is true, and its inverse is false. What can you conclude about the converse and the contrapositive of the conditional statement?

8. **Make use of structure.** Are these two statements logically equivalent? Explain.

If a polygon is a square, then it is a quadrilateral.

If a polygon is a quadrilateral, then it is a square.

9. **Critique the reasoning of others.** Toby says that the converse of the following statement is true. Is Toby's reasoning correct? Explain.

If a number is divisible by 6, then it is divisible by 2.

10. Consider this statement.

All birds have wings.

a. Write the statement as a conditional statement.

b. Can you write the statement as a biconditional statement? Explain.

LESSON 3-3 PRACTICE

Use the following statement for Items 11–13.

If a vehicle has four wheels, then it is a car.

11. Write the converse.

12. Write the inverse.

13. Write the contrapositive.

14. Write the definition of the vertex of an angle as a biconditional statement.

15. Give an example of a true statement that has a false converse.

16. **Reason abstractly.**

Given: (1) If X is blue, then Y is gold.
(2) Y is not gold.

Which of the following *must* be true?

A. Y is blue. **B.** Y is not blue.
C. X is not blue. **D.** X is gold.

ACTIVITY 3 PRACTICE

Write your answers on notebook paper. Show your work.

Lesson 3-1

1. Identify the property that justifies the statement: $5(x - 3) = 5x - 15$
   A. multiplication **B.** transitive
   C. subtraction **D.** distributive
2. **a.** Give an example of an undefined term and a defined term in geometry.
   b. Explain the difference between an undefined term and a defined term.
3. How is a definition different from a theorem?
4. What are the missing reasons and statements in the two-column proof?

Given: $\frac{2x}{4} = \frac{5x + 8}{2}$ **Prove:** $x = -2$

Statements	Reasons
1. ____a.____	1. Given equation
2. $2x = 2(5x + 8)$	2. ____b.____
3. ____c.____	3. Distributive Property
4. $0 = 8x + 16$	4. ____d.____
5. ____e.____	5. Subtraction Property of Equality
6. $-2 = x$	6. ____f.____
7. ____g.____	7. Symmetric Property

For Items 5 and 6, complete the prove statement and write a two-column proof for the equation.

5. Given: $5(x - 2) = 2x - 4$ Prove:
6. Given: $\frac{4c - 6}{3} = 8$ Prove:
7. Suppose you are given that $a = 5$ and $4a + b = 6$. What can you prove by using these statements and the Substitution Property?

Lesson 3-2

For Items 8–10, write each statement in if-then form.

8. Dianna will go to the movie if she finishes her homework.
9. $m\angle G = 40°$ implies $\angle G$ is acute.
10. A figure is a triangle only if it is a polygon with three sides.
11. State the hypothesis and the conclusion of this conditional statement.

    If the temperature drops below 65°F, then the swimming pool closes.
12. Given the false conditional statement, "If a vehicle is built to fly, then it is an airplane," write a counterexample.
13. Dustin says that $8 \cdot -1 = 8$ is a counterexample that shows that the following conditional statement is false. Is Dustin correct? Explain.

    If two integers are multiplied, then the product is greater than both integers.
14. Given that the hypothesis of the following conditional statement is true, which statement must also be true?

    If Margo wears gloves or a scarf, then she wears a coat.

    A. Margo is wearing a coat.
    B. Margo is wearing gloves.
    C. Margo is not wearing a coat.
    D. Margo is not wearing gloves.
15. **a.** Write a true conditional statement that includes this hypothesis: $-3x + 10 = -5$.
    b. Write a two-column proof to prove that your conditional statement is true.

Lesson 3-3

Use this statement for Items 16–19.

If today is Thursday, then tomorrow is Friday.

16. Write the converse of the statement.

17. Write the inverse of the statement.

18. Write the contrapositive of the statement.

19. Can the conditional statement be written as a biconditional statement? If so, write the biconditional statement. If not, explain why not.

20. A certain conditional statement is true. Which of the following must also be true?

A. converse
B. inverse
C. contrapositive
D. all of the above

21. Give an example of a statement that is false and logically equivalent to its inverse.

22. Compare and contrast a true conditional statement and a biconditional statement.

For Items 23–26, tell whether each statement is true or false. If it is false, give a counterexample.

23. If a number is a multiple of 8, then it is a multiple of 4.

24. the converse of the statement in Item 23

25. the inverse of the statement in Item 23

26. the contrapositive of the statement in Item 23

27. The following statement is the contrapositive of a conditional statement. What is the original conditional statement?

If a parallelogram does not have four right angles, then it is not a rectangle.

28. Consider this conditional statement.

If two numbers are negative, then their sum is less than both numbers.

Lorenzo says that the inverse of the statement can be written as follows.

If two numbers are positive, then their sum is greater than both numbers.

Is Lorenzo's reasoning correct? Explain.

Use this statement for Items 29–32.

If the sum of the measures of two angles is 90°, then the angles are complementary.

29. Write the inverse of the converse of the statement.

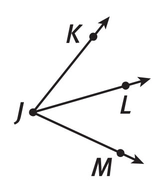
30. What is another name for the inverse of the converse?

31. Write the contrapositive of the inverse of the statement.

32. What is another name for the contrapositive of the inverse?

MATHEMATICAL PRACTICES
Attend to Precision

33. Write a clear definition of the term *adjacent angles*. Then use your definition to explain why $\angle KJL$ and $\angle LJM$ are adjacent angles but $\angle KJL$ and $\angle KJM$ are not adjacent angles.



Segment and Angle Measurement

ACTIVITY 4

It All Adds Up

Lesson 4-1 Segments and Midpoints

My Notes

Learning Targets:

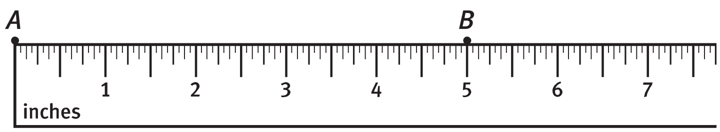
- Apply the Segment Addition Postulate to find lengths of segments.
- Use the definition of midpoint to find lengths of segments.

SUGGESTED LEARNING STRATEGIES: Close Reading, Look for a Pattern, Think-Pair-Share, Vocabulary Organizer, Interactive Word Wall, Create Representations, Marking the Text, Visualization, Identify a Subtask, Discussion Groups

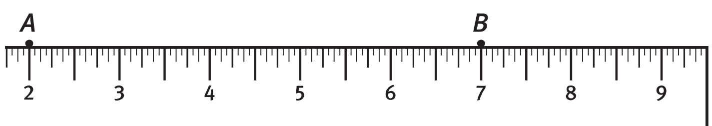
In geometry, axioms, or ***postulates***, are statements that are accepted as true without proof in order to provide a starting point for deductive reasoning.

Like *point, line,* and *plane*, *distance along a line* is an undefined term in geometry used to define other geometric terms. For example, the length of a line segment is the distance between its endpoints.

If two points are no more than 1 foot apart, you can find the distance between them by using an ordinary ruler. (The inch rulers below have been reduced to fit on the page.)



In the figure, the distance between point A and point B is 5 inches. Of course, there is no need to place the zero of the ruler on point A. In the figure below, the 2-inch mark is on point A. In this case, AB, measured in inches, is $|7 - 2| = |2 - 7| = 5$, as before.



The number obtained as a measure of distance depends on the unit of length. For example, the distance between two points in inches will be a different number than the distance between the two points in centimeters.

1. Determine the length of each segment in centimeters.

a. $DE =$ **b.** $EF =$ **c.** $DF =$

MATH TERMS

To prove a rule, at least one other rule must be used. So in order to develop geometry, some rules, called **postulates,** are accepted without proof.

READING MATH

AB denotes the distance between points A and B. If A and B are the endpoints of a segment ($\overline{AB}$), then AB denotes the length of $\overline{AB}$.

MATH TERMS

The **Ruler Postulate**

a. To every pair of points there corresponds a unique positive number called the distance between the points.

b. The points on a line can be matched with the real numbers so that the distance between any two points is the absolute value of the difference of their associated numbers.

My Notes

2. **Attend to precision.** Determine the length of each segment in centimeters (to the nearest tenth).

a. $KH =$ b. $HG =$ c. $GK =$

3. Using your results from Items 1 and 2, describe any patterns that you notice.

4. Given that N is a point between endpoints M and P of line segment MP, describe how to determine the length of $\overline{MP}$, without measuring, if you are given the lengths of $\overline{MN}$ and $\overline{NP}$.

MATH TERMS

Item 4 and your answer together form a statement of the **Segment Addition Postulate**.

5. Use the Segment Addition Postulate and the given information to complete each statement.

a. If B is between C and D, $BC = 10$ in., and $BD = 3$ in., then $CD =$ ________.

b. If Q is between R and T, $RT = 24$ cm, and $QR = 6$ cm, then $QT =$ ________.

c. If P is between L and A, $PL = x + 4$, $PA = 2x - 1$, and $LA = 5x - 3$, then $x =$ ________ and $LA =$ ________.

MATH TIP

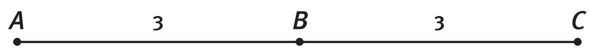
For each part of Item 5, make a sketch so that you can identify the parts of the segment.

CONNECT TO AP

You will frequently be asked to find the lengths of horizontal, vertical, and diagonal segments in the coordinate plane in AP Calculus.

My Notes

The ***midpoint*** of a segment is the point on the segment that divides it into two ***congruent*** segments. For example, if B is the midpoint of $\overline{AC}$, then $\overline{AB} \cong \overline{BC}$.



6. Given: M is the midpoint of $\overline{RS}$. Complete each statement.

a. If $RS = 10$, then $SM =$ _______.

b. If $RM = 12$, then $MS =$ _______, and $RS =$ _______.

Check Your Understanding

7. Points D and E are aligned with a ruler. Point D is at the mark for 4.5 cm, and the distance between points D and E is 3.4 cm. At which two marks on the ruler could point E be located?

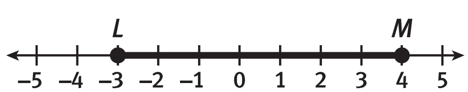
8. Point N is the midpoint of $\overline{FG}$. If $FN = 2x$, what expression represents FG?

9. **Reason abstractly.** Does a ray have a midpoint? Explain.

10. Give an example that illustrates the Segment Addition Postulate. Include a sketch with your example.

You can also use a number line to find the distance between two points.

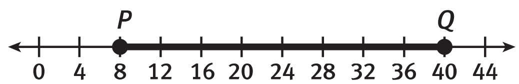
11. What is LM?

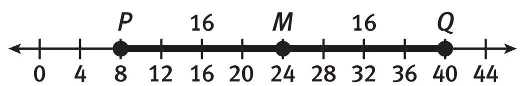


My Notes

The midpoint of a segment is halfway between its endpoints. So, if you know the coordinates of the endpoints, you can average them to find the coordinate of the midpoint.

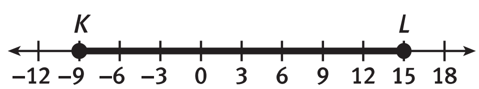
12. What is the coordinate of the midpoint M of $\overline{PQ}$?





Check Your Understanding

13. Use the number line to solve each problem.



a. What is KL?

b. What is the coordinate of the midpoint of $\overline{KL}$?

c. Point C lies between points K and L. The distance between points K and C is $\frac{1}{3}$ of KL. What is the coordinate of point C?

d. Point N lies between points C and L. The distance between points C and N is $\frac{3}{4}$ of CL. What is the coordinate of point N?

MATH TIP

A number line represents a one-dimensional coordinate system. You will explore the concepts of distance and midpoint using a two-dimensional coordinate system when you work with the coordinate plane in the next activity.

Lesson 4-1
Segments and Midpoints

My Notes

You can use the definition of midpoint and properties of algebra to determine the length of a segment.

Example A

If Q is the midpoint of $\overline{PR}$, $PQ = 4x - 5$, and $QR = 11 + 2x$, determine the length of $\overline{PQ}$.

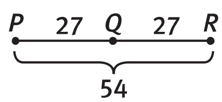
Because Q is the midpoint of $\overline{PR}$, you know that $\overline{PQ} \cong \overline{QR}$ and $PQ = QR$.

$PQ = QR$	
$4x - 5 = 11 + 2x$	Substitution Property
$2x - 5 = 11$	Subtraction Property of Equality
$2x = 16$	Addition Property of Equality
$x = 8$	Division Property of Equality

Now substitute 8 for x in the expression for PQ.

$PQ = 4x - 5 = 4(8) - 5 = 27$

Make a sketch of $\overline{PR}$ and its midpoint Q. Label the lengths of $\overline{PR}$, $\overline{PQ}$, and $\overline{QR}$.



WRITING MATH

Use $\overline{AB}$ when you talk about segment AB.

Use AB when you talk about the measure, or length, of $\overline{AB}$.

Try These A

a. If Y is the midpoint of $\overline{WZ}$, $YZ = x + 3$, and $WZ = 3x - 4$, determine the length of $\overline{WZ}$.

Given: M is the midpoint of $\overline{RS}$. Use the given information to find the missing values.

b. $RM = x + 3$ and $MS = 2x - 1$

$x =$ ______ and $RM =$ ______

c. $RM = x + 6$ and $RS = 5x + 3$

$x =$ ______ and $SM =$ ______

When you ***bisect*** a geometric figure, you divide it into two equal or congruent parts.

14. Reason abstractly. Line segment WZ bisects $\overline{XY}$ at point Z. What are two conclusions you can draw from this information?

MATH TERMS

A geometric figure that **bisects** another figure divides it into two equal or congruent parts.

My Notes

Check Your Understanding

15. Explain how to find the distance between two points on a number line.

16. Mekhi knows that $GH = 7$ and $HJ = 7$. Based on this information, he claims that point H is the midpoint of $\overline{GJ}$. Is Mekhi's claim necessarily true? Make a sketch that supports your answer.

17. Given: T is the midpoint of $\overline{JK}$, $JK = 5x - 3$, and $JT = 2x + 1$. Determine the length of $\overline{JK}$.

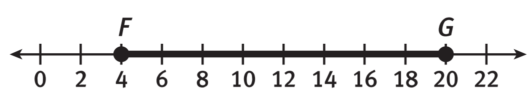
18. **Reason quantitatively.** $\overline{AB}$ lies on a number line. The coordinate of point A is -6. Given that $AB = 20$, what are the two possible coordinates for point B?

LESSON 4-1 PRACTICE

19. Given: Point K is between points H and J, $HK = x - 5$, $KJ = 5x - 12$, and $HJ = 25$. Find the value of x.

20. If B is the midpoint of $\overline{AC}$, $AB = x + 6$, and $AC = 5x - 6$, then what is BC?

21. Point P is between points F and G. The distance between points F and P is $\frac{1}{4}$ of FG. What is the coordinate of point P?



22. **Use appropriate tools strategically.** Anne has a broken ruler. It starts at the 3-inch mark and ends at the 12-inch mark. Explain how Anne could use the ruler to measure the length of a line segment in inches.

23. If P is the midpoint of $\overline{ST}$, $SP = x + 4$, and $ST = 4x$, determine the length of $\overline{ST}$.

24. Compare and contrast a postulate and a conjecture.