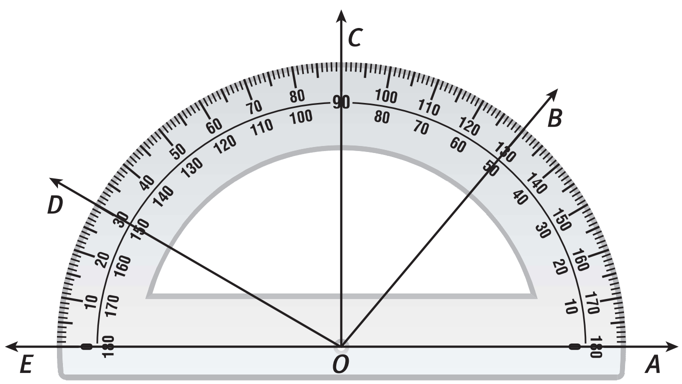
Learning Targets:

- Apply the Angle Addition Postulate to find angle measures.
- Use the definition of angle bisector to find angle measures.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Look for a Pattern, Quickwrite, Vocabulary Organizer, Interactive Word Wall, Create Representations, Debriefing, Identify a Subtask, Group Presentation

You measure angles with a protractor. The number of degrees in an angle is called its measure.

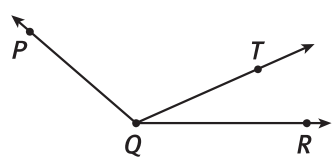


1. Use appropriate tools strategically. Determine the measure of each angle.

a. $m\angle AOB =$ 50° b. $m\angle BOC =$ ____ c. $m\angle AOC =$ ____

d. $m\angle EOD =$ ____ e. $m\angle BOD =$ ____ f. $m\angle BOE =$ ____

2. Use a protractor to determine the measure of each angle.



a. $m\angle TQP =$ ____ b. $m\angle TQR =$ ____ c. $m\angle RQP =$ ____

3. Express regularity in repeated reasoning. Using your results from Items 1 and 2, describe any patterns that you notice.

4. Given that point D is in the interior of $\angle ABC$, describe how to determine the measure of $\angle ABC$, without measuring, if you are given the measures of $\angle ABD$ and $\angle DBC$.

My Notes

CONNECT TO ASTRONOMY

The astronomer Claudius Ptolemy (about 85–165 CE) based his observations of the solar system on a unit that resulted from dividing the distance around a circle into 360 parts. This later became known as a degree.

MATH TERMS

The **Protractor Postulate**

a. To each angle there corresponds a unique real number between 0 and 180 called the measure of the angle.

b. The measure of an angle formed by a pair of rays is the absolute value of the difference of their associated numbers.

MATH TERMS

Item 4 and your answer together form a statement of the **Angle Addition Postulate**.

My Notes

5. Use the Angle Addition Postulate and the given information to complete each statement.

a. If P is in the interior of $\angle XYZ$, $m\angle XYP = 25°$, and $m\angle PYZ = 50°$, then $m\angle XYZ =$ ______.

b. If M is in the interior of $\angle RTD$, $m\angle RTM = 40°$, and $m\angle RTD = 65°$, then $m\angle MTD =$ ______.

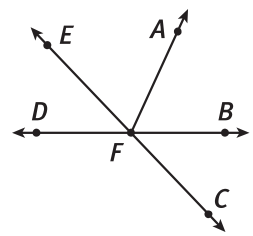
c. If H is in the interior of $\angle EFG$, $m\angle EFH = 75°$, and $m\angle HFG = (10x)°$, and $m\angle EFG = (20x - 5)°$, then $x =$ ______ and $m\angle HFG =$ ______.

d. Lines DB and EC intersect at point F. If $m\angle BFC = 44°$ and $m\angle AFB = 61°$, then

$m\angle AFC =$ ______

$m\angle AFE =$ ______

$m\angle EFD =$ ______

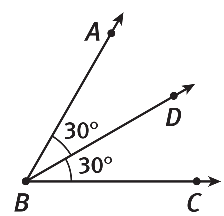


MATH TIP

Adjacent angles are coplanar angles that share a vertex and a side but no interior points. In the figure at right, $\angle ABD$ is adjacent to $\angle DBC$.

Congruent angles are angles that have the same measure. To indicate congruence, you can either write $m\angle ABD = m\angle DBC$ or $\angle ABD \cong \angle DBC$.

The ***bisector of an angle*** is a ray that divides the angle into two congruent adjacent angles. For example, if $\overrightarrow{BD}$ bisects $\angle ABC$, then $\angle ABD \cong \angle DBC$.



6. Given: $\overrightarrow{AH}$ bisects $\angle MAT$. Determine the missing measure.

a. $m\angle MAT = 70°$, $m\angle MAH =$ ______

b. $m\angle HAT = 80°$, $m\angle MAT =$ ______.

My Notes

You can use definitions, postulates, and properties of algebra to determine the measures of angles.

Example A

If $\overrightarrow{QP}$ bisects $\angle DQL$, $m\angle DQP = 5x - 7$, and $m\angle PQL = 11 + 2x$, determine the measure of $\angle DQL$.

Because $\overrightarrow{QP}$ is the angle bisector of $\angle DQL$, you know that $\angle DQP \cong \angle PQL$ and $m\angle DQP = m\angle PQL$.

$$m\angle DQP = m\angle PQL$$

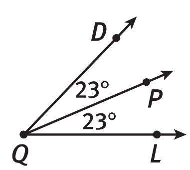
$5x - 7 = 11 + 2x$	Substitution Property
$3x - 7 = 11$	Subtraction Property of Equality
$3x = 18$	Addition Property of Equality
$x = 6$	Division Property of Equality

Substitute 6 for x in the expressions for $m\angle DQP$ and $m\angle PQL$.

$m\angle DQP = 5(6) - 7 = 23°$ $m\angle PQL = 11 + 2(6) = 23°$

By the Angle Addition Postulate, $m\angle DQL = m\angle DQP + m\angle PQL$, so $m\angle DQL = 23° + 23° = 46°$.

Make a sketch of $\angle DQL$ and its angle bisector $\overrightarrow{QP}$. Label the measures of $\angle DQP$ and $\angle PQL$.



Try These A

Given: $\overrightarrow{FL}$ bisects $\angle AFM$. Determine each missing value.

a. $m\angle LFM = 11x + 4$ and $m\angle AFL = 12x - 2$
$x =$ ________ , $m\angle LFM =$ ________, and $m\angle AFM =$ ________

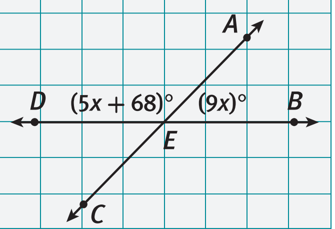
b. $m\angle AFM = 6x - 2$ and $m\angle AFL = 4x - 10$
$x =$ ________ and $m\angle LFM =$ ________

7. $\angle A$ and $\angle B$ are complementary, $m\angle A = 3x + 7$, and $m\angle B = 6x + 11$. Determine the measure of each angle.

MATH TIP

In Item 7, you know that $\angle A$ and $\angle B$ are complementary, so the sum of their measures is 90°. Use this fact to write an equation that you can solve for x. Once you know the value of x, you can find the measures of the angles.

My Notes



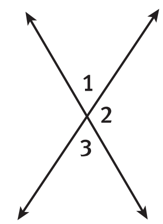
(Diagram is not drawn to scale.)

8. In the diagram at the left, $\overleftrightarrow{AC}$ and $\overleftrightarrow{DB}$ intersect as shown. Determine the measure of $\angle CEB$.

Check Your Understanding

9. $\overrightarrow{KN}$ is the angle bisector of $\angle JKL$. Explain how you could find $m\angle JKN$ if you know $m\angle JKL$.

10. In this diagram, $m\angle 1 = 4x + 30$ and $m\angle 3 = 2x + 48$. Write a step-by-step explanation for an absent classmate showing how to find $m\angle 2$.



11. **Construct viable arguments.** $\angle RST$ is adjacent to $\angle TSU$, $m\angle RST = 40°$, and $m\angle TSU = 50°$. Explain how you can use the Angle Addition Postulate to show that $\angle RSU$ is a right angle.

MATH TIP

Angles 1 and 3 are vertical angles. Vertical angles have the same measure.

LESSON 4-2 PRACTICE

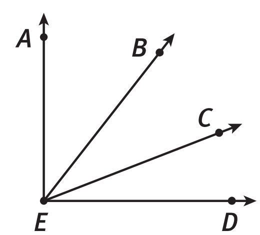
12. Point D is in the interior of $\angle ABC$, $m\angle ABC = 10x - 7$, $m\angle ABD = 6x + 5$, and $m\angle DBC = 36°$. What is $m\angle ABD$?

13. $\overrightarrow{QS}$ bisects $\angle PQR$. If $m\angle PQS = 5x$ and $m\angle RQS = 2x + 6$, then what is $m\angle PQR$?

14. $\angle L$ and $\angle M$ are complementary, $m\angle L = 2x + 25$, and $m\angle M = 4x + 11$. Determine the measure of each angle.

15. In this diagram, $\overrightarrow{EA} \perp \overrightarrow{ED}$ and $\overrightarrow{EB}$ bisects $\angle AEC$. Given that $m\angle AEB = 4x + 1$ and $m\angle CED = 3x$, determine the missing measures.

a. $x =$ ______ **b.** $m\angle BEC =$ ______



16. **Critique the reasoning of others.** Penny knows that point W is in the interior of $\angle XYZ$. Based on this information, she claims that $\angle XYW \cong \angle WYZ$. Is Penny's claim necessarily true? Explain. Make a sketch that supports your answer.

READING MATH

The symbol $\perp$ is read "is perpendicular to."

ACTIVITY 4 PRACTICE

Write your answers on notebook paper.
Show your work.

Lesson 4-1

1. If Q is between A and M and $MQ = 7.3$ and $AM = 8.5$, then $QA = \underline{\ ?\ }$.
A. 5.8 **B.** 1.2
C. 7.3 **D.** 14.6

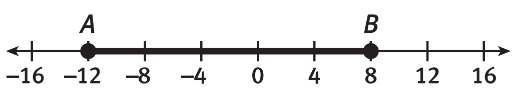
2. Given: K is between H and J, $HK = 2x - 5$, $KJ = 3x + 4$, and $HJ = 24$. What is the value of x?
A. 9 **B.** 5
C. 19 **D.** 3

3. If K is the midpoint of $\overline{HJ}$, $HK = x + 6$, and $HJ = 4x - 6$, then $KJ = \underline{\ ?\ }$.
A. 15 **B.** 9
C. 4 **D.** 10

4. State the Segment Addition Postulate in your own words.

5. Explain what *distance along a line* means as an undefined term in geometry.

Use the number line for Items 6–9.



6. What is AB?

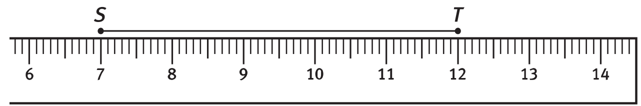
7. What is the coordinate of the midpoint of $\overline{AB}$? Explain how you found your answer.

8. Point M is the midpoint of $\overline{AB}$. What is the coordinate of the midpoint of $\overline{AM}$?

9. Point C is between points A and B. The distance between points B and C is $\frac{1}{4}$ of AB. What is the coordinate of point C?

10. $\overline{FG}$ lies on a number line. The coordinate of point F is 8. Given that $FG = 16$, what are the two possible coordinates for point G?

11. In the diagram, $\overline{ST}$ is aligned with a centimeter ruler. What is the length of $\overline{ST}$ in centimeters?



12. Compare and contrast a *postulate* and a *theorem*.

13. What are two conclusions you can draw from this statement? Support your answers.

Point P is the midpoint of $\overline{QR}$.

14. If D is the midpoint of $\overline{CE}$, $CD = x + 7$, and $CE = 5x - 1$, determine each missing value.
a. $x = \underline{\ ?\ }$
b. $CD = \underline{\ ?\ }$
c. $CE = \underline{\ ?\ }$
d. $DE = \underline{\ ?\ }$

15. Leo is running in a 5-kilometer race along a straight path. If he is at the midpoint of the path, how many kilometers does he have left to run?

16. Point S is between points R and T. Given that $\overline{RS} \cong \overline{ST}$ and $RS = 16$, what is RT?

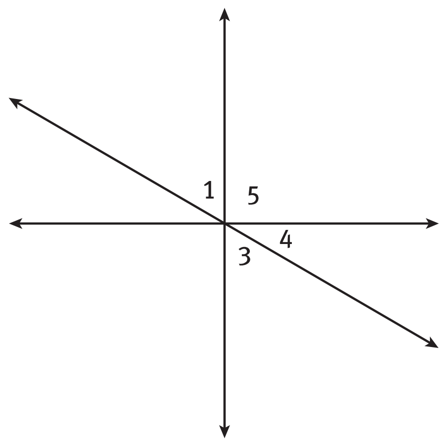
Lesson 4-2

17. P lies in the interior of $\angle RST$. $m\angle RSP = 40°$ and $m\angle TSP = 10°$. $m\angle RST = \underline{\ ?\ }$
A. 100° **B.** 50°
C. 30° **D.** 10°

18. $\overrightarrow{QS}$ bisects $\angle PQR$. If $m\angle PQS = 3x$ and $m\angle RQS = 2x + 6$, then $m\angle PQR = \underline{\ ?\ }$.
A. 18° **B.** 36°
C. 30° **D.** 6°

19. $\angle P$ and $\angle Q$ are supplementary. $m\angle P = 5x + 3$ and $m\angle Q = x + 3$. $x = \underline{\ ?\ }$
A. 14 **B.** 0
C. 29 **D.** 30

20. In this figure, $m\angle 3 = x + 18$, $m\angle 4 = x + 15$, and $m\angle 5 = 4x + 3$. Show all work for parts a, b, and c.



a. What is the value of x?
b. What is the measure of $\angle 1$?
c. Is $\angle 3$ complementary to $\angle 1$? Explain.

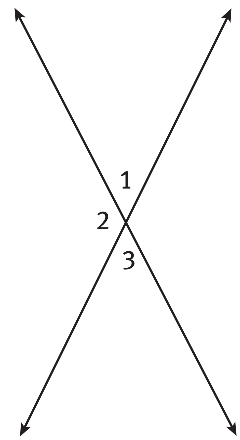
21. $\overrightarrow{MP}$ is the angle bisector of $\angle LMN$. Given that $\angle LMP$ is a right angle, what type of angle is $\angle LMN$? Explain how you know.

22. State the Angle Addition Postulate in your own words.

23. A protractor is properly aligned with the vertex of $\angle KLM$. $\overrightarrow{LK}$ passes through the mark for 38° on the protractor and $\overrightarrow{LM}$ passes through the mark for 126°. What is $m\angle KLM$?

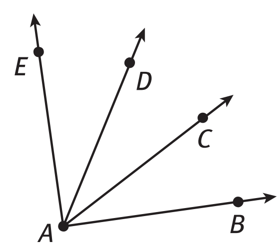
24. $\angle GFH$ and $\angle HFJ$ are adjacent and congruent. What are two conclusions you can draw from this information? Support your answers.

25. In this figure, $m\angle 1 = 4x + 50$ and $m\angle 3 = 2x + 66$. Show all work for parts a, b, and c.



a. What is the value of x?
b. What is the measure of $m\angle 3$?
c. What is the measure of $m\angle 2$?

26. Given: $\overrightarrow{AE} \perp \overrightarrow{AB}$, $\overrightarrow{AD}$ bisects $\angle EAC$, $m\angle CAB = 2x - 4$ and $m\angle CAE = 3x + 14$. Show all work for parts a and b.



a. What is the value of x?
b. What is the measure of $m\angle DAC$?

MATHEMATICAL PRACTICES
Look for and Express Regularity in Repeated Reasoning

27. $\angle CDF$ is one of the congruent angles formed by the angle bisector of $\angle CDE$. Write a formula that can be used to determine $m\angle CDF$ given $m\angle CDE$.

Distance and Midpoint Formulas

ACTIVITY 5

We ♥ Descartes

Lesson 5-1 Distance on the Coordinate Plane

My Notes

Learning Targets:

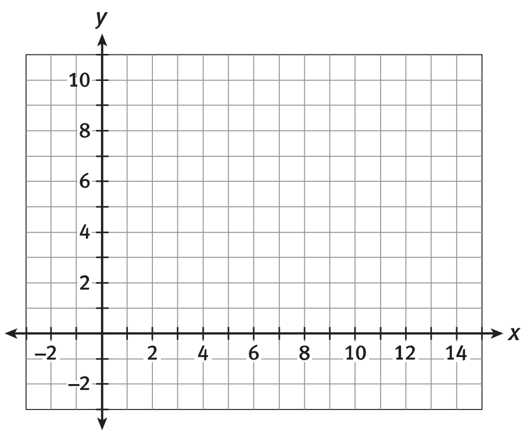
- Derive the Distance Formula.
- Use the Distance Formula to find the distance between two points on the coordinate plane.

SUGGESTED LEARNING STRATEGIES: Create Representations, Simplify the Problem, Think-Pair-Share, Think Aloud, Visualization, Look for a Pattern, Graphic Organizer, Discussion Groups, Identify a Subtask

In the previous activity, you used number lines to determine distance and to locate the midpoint of a segment. A number line is a one-dimensional coordinate system.

In this activity, you will explore the concepts of distance and midpoint on a two-dimensional coordinate system, or coordinate plane.

Use the coordinate plane and follow the steps below to determine the distance between points $P(1, 4)$ and $Q(13, 9)$.



1. **Model with mathematics.** Plot the points $P(1, 4)$ and $Q(13, 9)$ on the coordinate plane. Then draw $\overline{PQ}$.
2. Draw horizontal segment PR and vertical segment QR to create right triangle PQR, with a right angle at vertex R.
3. **Attend to precision.** What are the coordinates of point R?

CONNECT TO HISTORY

The Cartesian coordinate system was developed by the French philosopher and mathematician René Descartes in 1637 as a way to specify the position of a point or object on a plane.

READING MATH

The notation $P(1, 4)$ means point P with coordinates $(1, 4)$ on the coordinate plane.

My Notes

4. **a.** What is PR, the length of the horizontal leg of the right triangle?

   b. What is QR, the length of the vertical leg of the right triangle?

   c. Explain how you determined your answers to parts a and b.

MATH TIP

The *Pythagorean Theorem* states that the square of the length of the hypotenuse of a right triangle is equal to the sum of the squares of the lengths of the legs of the right triangle. In other words, for a right triangle with hypotenuse c and legs a and b, $c^2 = a^2 + b^2$.

5. Use the Pythagorean Theorem to find PQ. Show your work.

6. **Attend to precision.** What is the distance between points $P(1, 4)$ and $Q(13, 9)$? How do you know?

7. How do you know that the triangle you drew in Item 2 is a right triangle?

8. What relationship do you notice among the coordinates of points P, Q, and R?

Check Your Understanding

9. Explain how you would find the distance between points $J(-3, 6)$ and $K(3, 14)$.

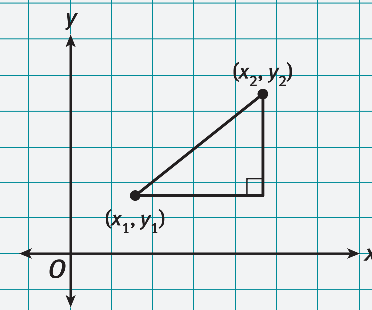
10. What is the distance between $M(9, -5)$ and $N(-11, 10)$?

Lesson 5-1
Distance on the Coordinate Plane

My Notes

Although the method you just learned for finding the distance between two points will always work, it may not be practical to plot points on a coordinate plane and draw a right triangle each time you want to find the distance between them.

Instead, you can use algebraic methods to derive a formula for finding the distance between any two points on the coordinate plane. Start with any two points (x_1, y_1) and (x_2, y_2) on the coordinate plane. Visualize using these points to draw a right triangle with a horizontal leg and a vertical leg.



11. What are the coordinates of the point at the vertex of the right angle of the triangle?

12. Write an expression for the length of the horizontal leg of the right triangle.

MATH TIP

In Item 12, you can think of the *x*-coordinates of the endpoints of the horizontal leg as values on a horizontal number line.

In Item 13, you can think of the *y*-coordinates of the endpoints of the vertical leg as values on a vertical number line.

13. Write an expression for the length of the vertical leg of the right triangle.

14. Use the Pythagorean Theorem to write an expression for the length of the hypotenuse of the right triangle.

15. **Express regularity in repeated reasoning.** Write a formula that can be used to find d, the distance between two points (x_1, y_1) and (x_2, y_2) on the coordinate plane.

MATH TIP

Squaring a difference and then taking the square root has the same effect as taking the absolute value of the difference. Both result in nonnegative quantities. That means that the absolute value symbols are not needed in your formula to find the distance between two points.

My Notes

CONNECT TO AP

The coordinate geometry formulas introduced in this activity are used frequently in AP calculus in a wide variety of applications.

MATH TIP

You can simplify $\sqrt{369}$ as follows.

$$\sqrt{369} = \sqrt{9 \cdot 41}$$
$$= \sqrt{9} \cdot \sqrt{41}$$
$$= 3\sqrt{41}$$

MATH TIP

A Venn diagram illustrates logical relationships. Think about the commonalities between the Pythagorean Theorem and the Distance Formula. These similarities are placed in the overlapping circles of the Venn diagram. The differences are shown in the nonoverlapping parts.

16. Use the formula you wrote in Item 15 to find the distance between the points with coordinates $(12, -5)$ and $(-3, 7)$.

Check Your Understanding

17. Find the distance between the points with the coordinates shown.
  a. $(-8, 5)$ and $(7, -3)$
  b. $(3, 8)$ and $(8, 3)$

18. Create a Venn diagram that compares and contrasts the Pythagorean Theorem and the Distance Formula.

19. **Reason abstractly.** Suppose two points lie on the same vertical line. Can you use the Distance Formula to find the distance between them? Explain.

20. Write and simplify a formula for the distance d between the origin and a point (x, y) on the coordinate plane.

LESSON 5-1 PRACTICE

Find the distance between the points with the given coordinates.

21. (−8, −6) and (4, 10)

22. (5, 14) and (−3, −9)

23. **Reason quantitatively.** Explain how you know that your answer to Item 22 is reasonable. Remember to use complete sentences and words such as *and, or, since, for example, therefore, because of, by the,* to make connections between your thoughts.

24. Use your Venn diagram from Item 18 to write a RAFT.

Role: Teacher

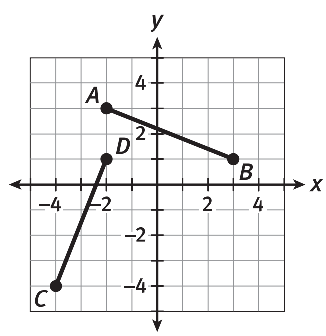
Audience: A classmate who was absent for the lesson on distance between two points

Format: Personal note

Topic: Explain the mathematical similarities and differences between the Pythagorean Theorem and the Distance Formula.

25. The vertices of $\triangle XYZ$ are $X(-3, -6)$, $Y(21, -6)$, and $Z(21, 4)$. What is the perimeter of the triangle?

26. Use the Distance Formula to show that $\overline{AB} \cong \overline{CD}$.



My Notes

ACADEMIC VOCABULARY

The ***format*** of a piece of writing can refer to its organization or style. For example, personal notes, essays, reports, and printed advertisements are all different formats for writing.

My Notes

Learning Targets:

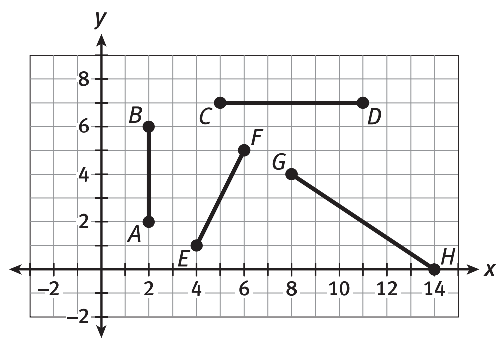
- Use inductive reasoning to determine the Midpoint Formula.
- Use the Midpoint Formula to find the coordinates of the midpoint of a segment on the coordinate plane.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Visualization, Look for a Pattern, Debriefing, Work Backward, Self Revision/Peer Revision, Discussion Groups, Identify a Subtask

In the previous activity, you defined *midpoint* as a point on a segment that divides it into two congruent segments. Follow the steps below to explore the concept of midpoint on the coordinate plane.

MATH TIP

For a segment on a number line, the coordinate of the midpoint is the average of the coordinates of the endpoints.



1. In the table below, write the coordinates of the endpoints of each segment shown on the coordinate plane.

Use appropriate tools strategically. Use a ruler to help you identify the midpoint of each segment. Then write the coordinates of the midpoint in the table.

Segment	Endpoints	Midpoint
$\overline{AB}$	A(____, ____) and B(____, ____)	(____, ____)
$\overline{CD}$	C(____, ____) and D(____, ____)	(____, ____)
$\overline{EF}$	E(____, ____) and F(____, ____)	(____, ____)
$\overline{GH}$	G(____, ____) and H(____, ____)	(____, ____)

2. Use the coordinates in the table in Item 1.

a. Compare the x-coordinates of the endpoints of each segment with the x-coordinate of the midpoint of the segment. Describe the pattern you see.

My Notes

b. Make use of structure. Compare the y-coordinates of the endpoints of each segment with the y-coordinate of the midpoint of the segment. Describe the pattern you see.

3. Use the patterns you described in Item 2 to write a formula for the coordinates of the midpoint M of a line segment with endpoints at (x_1, y_1) and (x_2, y_2).

MATH TIP

Your formula for the midpoint should have the form $M = (a, b)$, where a is an expression for the x-coordinate of the midpoint, and b is an expression for the y-coordinate of the midpoint.

Look back at the chart in Item 1 to verify the formula you wrote.

4. Use your formula to find the coordinates of the midpoint M of $\overline{AC}$ with endpoints $A(1, 4)$ and $C(11, 10)$. What can you conclude about segments AM and MC?

Check Your Understanding

5. Explain how you could check that your answer to Item 4 is reasonable.

6. Suppose a segment on the coordinate plane is vertical. Can you use the Midpoint Formula to find the coordinates of its midpoint? Explain.

7. The midpoint M of $\overline{ST}$ has coordinates (3, 6). Point S has coordinates (1, 2). What are the coordinates of point T? Explain how you determined your answer.

8. Reason quantitatively. The origin, (0, 0), is the midpoint of a segment. What conclusions can you draw about the coordinates of the endpoints of the segment?

My Notes

LESSON 5-2 PRACTICE

Find the coordinates of the midpoint of each segment with the given endpoints.

9. $Q(-3, 14)$ and $R(7, 5)$

10. $S(13, 7)$ and $T(-2, -7)$

11. $E(4, 11)$ and $F(-11, -5)$

12. $A(-5, 4)$ and $B(-5, 18)$

13. Find and explain the errors that were made in the following calculation of the coordinates of a midpoint. Then fix the errors and determine the correct answer.

Find the coordinates of the midpoint M of the segment with endpoints R(−2, 3) and S(13, −7).

$$M = \left(\frac{-2+3}{2}, \frac{13+(-7)}{2}\right)$$

$$= \left(\frac{1}{2}, \frac{6}{2}\right) = \left(\frac{1}{2}, 3\right)$$

14. Make sense of problems. $\overline{HJ}$ is graphed on a coordinate plane. Explain how you would determine the coordinates of the point on the segment that is $\frac{1}{4}$ of the distance from H to J.

ACTIVITY 5 PRACTICE

Write your answers on notebook paper.
Show your work.

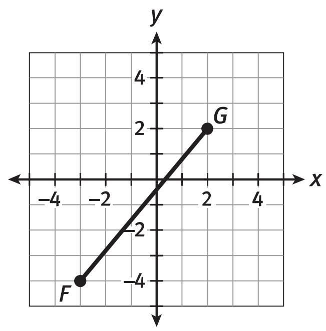
Lesson 5-1

1. Calculate the distance between the points $A(-4, 2)$ and $B(15, 6)$.

2. Calculate the distance between the points $R(1.5, 7)$ and $S(-2.3, -8)$.

3. Describe how to find the distance between two points on the coordinate plane.

4. To the nearest unit, what is FG?



A. 5 units **B.** 6 units
C. 8 units **D.** 11 units

5. $\overline{JK}$ has endpoints $J(-3, -1)$ and $K(0, 3)$. $\overline{RS}$ has endpoints $R(1, 1)$ and $S(4, 4)$. Is $\overline{JK} \cong \overline{RS}$? Explain how you know.

6. The vertices of $\triangle LMN$ are $L(7, 4)$, $M(7, 16)$, and $N(42, 4)$.
a. Find the length of each side of the triangle.
b. What is the perimeter of the triangle?
c. What is the area of the triangle? Explain how you determined your answer.

7. The distance from the origin to point P is 5 units. Give the coordinates of four possible locations for point P.

8. On a map, a trailhead is located at (5, 3) and a turnaround point is located at (15, 6). Each unit on the map represents 1 km. Ana and Larissa start at the turnaround point and walk directly toward the trailhead. If they walk at an average speed of 4.5 km/h, will they make it to the trailhead in less than 2 hours? Support your answer.

For Items 9–11, determine the length of each segment with the given endpoints.

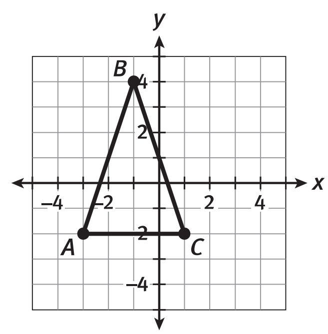
9. $C(1, 4)$ and $D(11, 28)$

10. $Y(-2, 6)$ and $Z(5, -8)$

11. $P(-7, -7)$ and $Q(9, 5)$

12. Point R has coordinates $(1, 3)$, and point S has coordinates $(6, y)$. If the distance from R to S is 13 units, what are the possible values of y?

13. Use the Distance Formula to show that $\triangle ABC$ is isosceles.



14. Draw a scalene triangle on a coordinate plane, and use the Distance Formula to demonstrate that your triangle is scalene.

Lesson 5-2

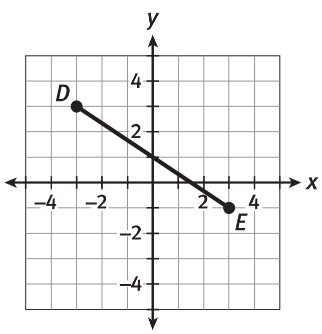
15. Determine the coordinates of the midpoint of the segment with endpoints $R(3, 16)$ and $S(7, -6)$.

16. Determine the coordinates of the midpoint of the segment with endpoints $W(-5, 10.2)$ and $X(12, 4.5)$.

17. Point C is the midpoint of $\overline{AB}$. Point A has coordinates $(2, 4)$, and point C has coordinates $(5, 0)$.
a. What are the coordinates of point B?
b. What is AB?
c. What is BC?

18. $\overline{JL}$ has endpoints $J(8, 10)$ and $L(20, 5)$. Point K has coordinates $(13, 9)$.
a. Is point K the midpoint of $\overline{JL}$? Explain how you know.
b. How could you check that your answer to part a is reasonable?

19. Find the coordinates of the midpoint of $\overline{DE}$.



20. What are the coordinates of the midpoint of the segment with endpoints at $(-3, -4)$ and $(5, 8)$?
A. $(1, 2)$ **B.** $(2, 4)$
C. $(4, 6)$ **D.** $(8, 12)$

21. A circle on the coordinate plane has a diameter with endpoints at $(6, 8)$ and $(15, 8)$.
a. What are the coordinates of the center of the circle?
b. What is the diameter of the circle?
c. What is the radius of the circle?
d. Identify the coordinates of another point on the circle. Explain how you found your answer.

22. Two explorers on an expedition to the Arctic Circle have radioed their coordinates to base camp. Explorer A is at coordinates $(-26, -15)$. Explorer B is at coordinates $(13, 21)$. The base camp is located at the origin.
a. Determine the linear distance between the two explorers.
b. Determine the midpoint between the two explorers.
c. Determine the distance between the midpoint of the explorers and the base camp.

23. A segment has endpoints with coordinates $(a, 2b)$ and $(-3a, 4b)$. Write the coordinates of the midpoint of the segment in terms of a and b.

24. Point J is the midpoint of $\overline{FG}$ with endpoints $F(1, 4)$ and $G(5, 12)$. Point K is the midpoint of $\overline{GH}$ with endpoints $G(5, 12)$ and $H(-1, 4)$. What is JK?

MATHEMATICAL PRACTICES
Look for and Express Regularity in Repeated Reasoning

25. Let (x_1, y) and (x_2, y) represent the coordinates of the endpoints of a horizontal segment.
a. Write and simplify a formula for the distance d between the endpoints of a horizontal segment.
b. Write and simplify a formula for the coordinates of the midpoint M of a horizontal segment.

Proofs about Line Segments and Angles

ACTIVITY 6

Now I'm Convinced

Lesson 6-1 Justifying Statements

My Notes

Learning Objectives

- Use definitions, properties, and theorems to justify a statement.
- Write two-column proofs to prove theorems about lines and angles.

Learning Target:

- Use properties, postulates, and definitions to justify statements.

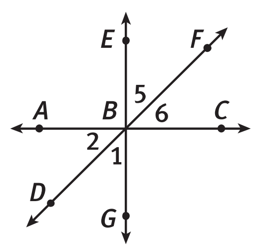
SUGGESTED LEARNING STRATEGIES: Close Reading, Activating Prior Knowledge, Think-Pair-Share, Discussion Groups

A proof is an argument, a justification, or a reason that something is true. A proof is an answer to the question "why?" when the person asking wants an argument that is indisputable.

There are three basic requirements for constructing a good proof.

- Awareness and knowledge of the definitions of the terms related to what you are trying to prove.
- Knowledge and understanding of postulates and previous proven theorems related to what you are trying to prove.
- Knowledge of the basic rules of logic.

To write a proof, you must be able to justify statements. The statements in Example A are based on the diagram to the right in which lines *AC*, *EG*, and *DF* all intersect at point *B*. Each of the statements is justified using a property, postulate, or definition.



Example A

Name the property, postulate, or definition that justifies each statement.

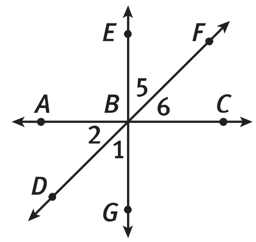
Statement	Justification
a. If $\angle ABE$ is a right angle, then $m\angle ABE = 90°$.	Definition of right angle
b. If $\angle 2 \cong \angle 1$ and $\angle 1 \cong \angle 5$, then $\angle 2 \cong \angle 5$.	Transitive Property
c. Given: B is the midpoint of $\overline{AC}$. Prove: $\overline{AB} \cong \overline{BC}$	Definition of midpoint
d. $m\angle 2 + m\angle ABE = m\angle DBE$	Angle Addition Postulate
e. If $\angle 1$ is supplementary to $\angle FBG$, then $m\angle 1 + m\angle FBG = 180°$.	Definition of supplementary angles

MATH TIP

The Reflexive, Symmetric, and Transitive Properties apply to congruence as well as to equality.

Try These A

Using the diagram from the previous page, reproduced here, name the property, postulate, or definition that justifies each statement.



Statement	Justification
a. $EB + BG = EG$	
b. If $\angle 5 \cong \angle 6$, then $\overrightarrow{BF}$ bisects $\angle EBC$.	
c. If $m\angle 1 + m\angle 6 = 90°$, then $\angle 1$ is complementary to $\angle 6$.	
d. If $m\angle 1 + m\angle 5 = m\angle 6 + m\angle 5$, then $m\angle 1 = m\angle 6$.	
e. Given: $\overleftrightarrow{AC} \perp \overleftrightarrow{EG}$ Prove: $\angle ABG$ is a right angle.	

My Notes

Check Your Understanding

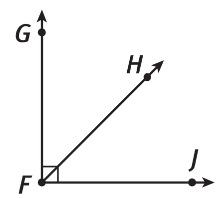
1. Explain why the following statement does not need to be justified.

The midpoint of a segment is a point on the segment that divides it into two congruent segments.

2. Given: $\overline{RS}$ and $\overline{ST}$ share endpoint S.

Critique the reasoning of others. Based on this information, Michaela says that the Segment Addition Postulate justifies the statement that $RS + ST = RT$. Is there a flaw in Michaela's reasoning, or is she correct? Explain.

3. Construct viable arguments. Write and justify two statements based on the information in the figure.

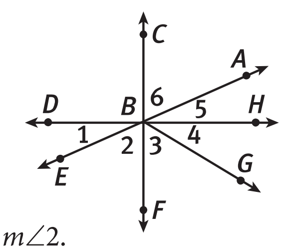


MATH TIP

Do not assume that an angle is a right angle just because it appears to measure 90°. You can only conclude that an angle is a right angle if (1) you are given this information, (2) a diagram of the angle includes a right angle symbol, or (3) you prove that the angle is a right angle.

LESSON 6-1 PRACTICE

Lines CF, DH, and EA intersect at point B. Use this figure for Items 4–8. Write the definition, postulate, or property that justifies each statement.



4. If $\angle 2$ is supplementary to $\angle CBE$, then $m\angle 2 + m\angle CBE = 180°$.

5. If $\angle 2 \cong \angle 3$, then $\overrightarrow{BF}$ bisects $\angle GBE$.

6. $CB + BF = CF$

7. If $\angle DBF$ is a right angle, then $\overleftrightarrow{HD} \perp \overleftrightarrow{CF}$.

8. If $m\angle 3 = m\angle 6$, then $m\angle 3 + m\angle 2 = m\angle 6 + m\angle 2$.

9. Reason abstractly. Write a statement related to the figure above that can be justified by the Angle Addition Postulate.

My Notes

Learning Targets:

- Complete two-column proofs to prove theorems about segments.
- Complete two-column proofs to prove theorems about angles.

SUGGESTED LEARNING STRATEGIES: Vocabulary Organizer, Think-Pair-Share, Close Reading, Discussion Groups, Self Revision/Peer Revision, Group Presentation

Earlier, you wrote two-column proofs to solve algebraic equations. You justified each statement in these proofs by using an algebraic property. Now you will use two-column proofs to prove geometric theorems. You must justify each statement by using a definition, a postulate, a property, or a previously proven theorem.

Recall that *vertical angles* are opposite angles formed by a pair of intersecting lines. In the figure below, $\angle 1$ and $\angle 2$ are vertical angles. The following example illustrates how to prove that vertical angles are congruent.

MATH TERMS

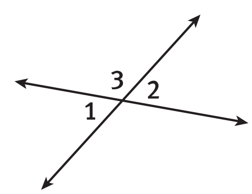
The **Vertical Angles Theorem** states that vertical angles are congruent.

Example A

Theorem: Vertical angles are congruent.

Given: $\angle 1$ and $\angle 2$ are vertical angles.

Prove: $\angle 1 \cong \angle 2$



Statements	Reasons
1. $m\angle 1 + m\angle 3 = 180°$	**1.** Definition of supplementary angles
2. $m\angle 2 + m\angle 3 = 180°$	**2.** Definition of supplementary angles
3. $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$	**3.** Substitution Property
4. $m\angle 1 = m\angle 2$	**4.** Subtraction Property of Equality
5. $\angle 1 \cong \angle 2$	**5.** Definition of congruent angles

Guided Example B

Supply the missing statements and reasons.

Theorem: All right angles are congruent.

Given: $\angle A$ and $\angle B$ are right angles.

Prove: $\angle A \cong \angle B$

Statements	Reasons
1.	**1.** Given
2. $m\angle A = 90°$; $m\angle B = 90°$	**2.** Definition of
3. $m\angle A =$	**3.** Property
4. $\cong \angle B$	**4.** Definition of

DISCUSSION GROUP TIPS

As you listen to the group discussion, take notes to aid comprehension and to help you describe your own ideas to others in your group. Ask questions to clarify ideas and to gain further understanding of key concepts.

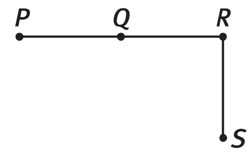
My Notes

Try These A-B

a. Complete the proof.

Given: Q is the midpoint of $\overline{PR}$.
$\overline{QR} \cong \overline{RS}$

Prove: $\overline{PQ} \cong \overline{RS}$



Statements	Reasons
1. Q is the midpoint of $\overline{PR}$.	**1.**
2. $\overline{PQ} \cong \overline{QR}$	**2.**
3. $\overline{QR} \cong \overline{RS}$	**3.**
4. $\overline{PQ} \cong \overline{RS}$	**4.**

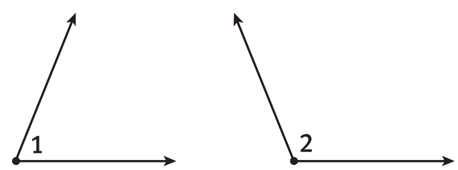
MATH TIP

More than one statement in a two-column proof can be given information.

b. Complete the proof.

Given: $\angle 1$ and $\angle 2$ are supplementary.
$m\angle 1 = 68°$

Prove: $m\angle 2 = 112°$



Statements	Reasons
1.	**1.** Given
2.	**2.** Definition of supplementary angles
3.	**3.** Given
4.	**4.** Substitution Property
5.	**5.** Subtraction Property of Equality

My Notes

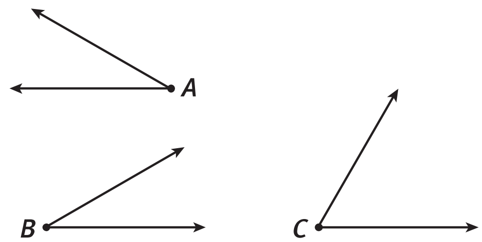
MATH TIP

The theorem stated in Example C is called the Congruent Complements Theorem.

Example C

Arrange the statements and reasons below in a logical order to complete the proof.

Theorem: If two angles are complementary to the same angle, then the two angles are congruent.



Given: $\angle A$ and $\angle B$ are each complementary to $\angle C$.

Prove: $\angle A \cong \angle B$

$m\angle A + m\angle C = m\angle B + m\angle C$	Transitive Property
$\angle A$ and $\angle B$ are each complementary to $\angle C$.	Given
$\angle A \cong \angle B$	Definition of congruent segments
$m\angle A = m\angle B$	Subtraction Property of Equality
$m\angle A + m\angle C = 90°$; $m\angle B + m\angle C = 90°$	Definition of complementary angles

Start the proof with the given information. Then decide which statement and reason follow logically from the first statement. Continue until you have proved that $\angle A \cong \angle B$.

Statements	Reasons
1. $\angle A$ and $\angle B$ are each complementary to $\angle C$.	**1.** Given
2. $m\angle A + m\angle C = 90°$ $m\angle B + m\angle C = 90°$	**2.** Definition of complementary angles
3. $m\angle A + m\angle C = m\angle B + m\angle C$	**3.** Transitive Property
4. $m\angle A = m\angle B$	**4.** Subtraction Property of Equality
5. $\angle A \cong \angle B$	**5.** Definition of congruent segments

My Notes

Try These C

a. Attend to precision. Arrange the statements and reasons below in a logical order to complete the proof.

Given: $\angle 1$ and $\angle 2$ are vertical angles; $\angle 1 \cong \angle 3$.

Prove: $\angle 2 \cong \angle 3$

$\angle 1 \cong \angle 2$	Vertical angles are congruent.
$\angle 2 \cong \angle 3$	Transitive Property
$\angle 1 \cong \angle 3$	Given
$\angle 1$ and $\angle 2$ are vertical angles.	Given

b. Write a two-column proof of the following theorem.

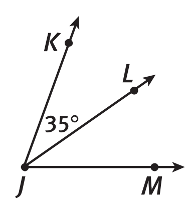
Theorem: If two angles are supplementary to the same angle, then the two angles are congruent.

Given: $\angle R$ and $\angle S$ are each supplementary to $\angle T$.

Prove: $\angle R \cong \angle S$

Check Your Understanding

1. If you know that $\angle D$ and $\angle F$ are both complementary to $\angle J$, what statement could you prove using the Congruent Complements Theorem?
2. What types of information can you list as reasons in a two-column geometric proof?
3. Kenneth completed this two-column proof. What mistake did he make? How could you correct the mistake?



Given: $\overrightarrow{JL}$ bisects $\angle KJM$; $m\angle KJL = 35°$

Prove: $m\angle LJM = 35°$

Statements	Reasons
1. $\overrightarrow{JL}$ bisects $\angle KJM$.	**1.** Given
2. $\angle KJL \cong \angle LJM$	**2.** Definition of congruent angles
3. $m\angle KJL = m\angle LJM$	**3.** Definition of angle bisector
4. $m\angle KJL = 35°$	**4.** Given
5. $m\angle LJM = 35°$	**5.** Transitive Property

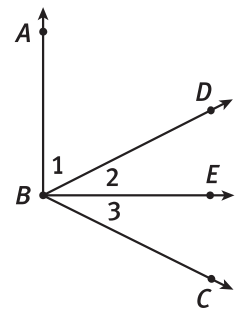
My Notes

LESSON 6-2 PRACTICE

4. Supply the missing statements and reasons.

Given: $\angle 1$ is complementary to $\angle 2$; $\overrightarrow{BE}$ bisects $\angle DBC$.

Prove: $\angle 1$ is complementary to $\angle 3$.



Statements	Reasons
1. $\overrightarrow{BE}$ bisects $\angle DBC$	1.
2. $\angle 2 \cong \angle 3$	2.
3.	3. Definition of congruent angles
4. $\angle 1$ is complementary to $\angle 2$.	4.
5. $\angle 1 + m\angle 2 =$ ____	5.
6. $m\angle 1 + m\angle 3 = 90°$	6.
7.	7. Definition of complementary angles

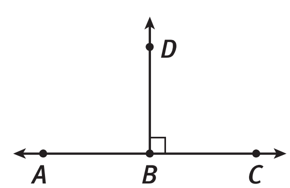
Construct viable arguments. Write a two-column proof for each of the following.

5. **Given:** M is the midpoint of $\overline{LN}$; $LM = 8$.

   Prove: $LN = 16$

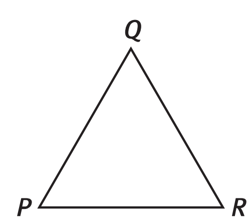
6. **Given:** $\overrightarrow{BD}$ bisects $\angle ABC$; $m\angle DBC = 90°$.

   Prove: $\angle ABC$ is a straight angle.



7. **Given:** $\overline{PQ} \cong \overline{QR}$, $QR = 14$, $PR = 14$

   Prove: $\overline{PQ} \cong \overline{PR}$



8. **Reason abstractly.** What type of triangle is shown in Item 7? Explain how you know.

MATH TIP

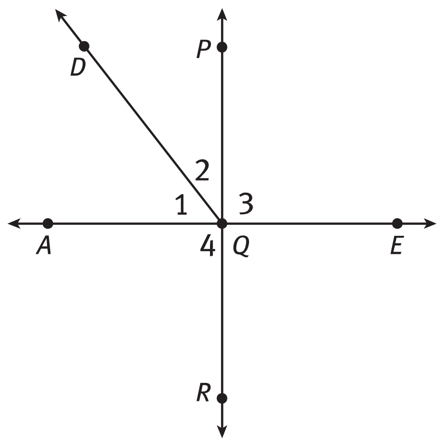
If the given information does not include a diagram, it may be helpful to make a sketch to represent the information.

ACTIVITY 6 PRACTICE

Write your answers on notebook paper.
Show your work.

Lesson 6-1

Use this diagram to identify the property, postulate, or theorem that justifies each statement in Items 1–4.



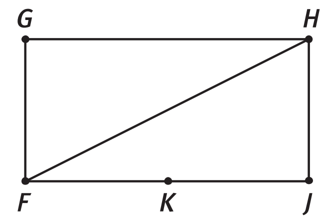
1. $PQ + QR = PR$
A. Angle Addition Postulate
B. Addition Property
C. Definition of congruent segments
D. Segment Addition Postulate

2. If Q is the midpoint of $\overline{PR}$, then $\overline{PQ} \cong \overline{QR}$.
A. Definition of midpoint
B. Definition of congruent segments
C. Definition of segment bisector
D. Segment Addition Postulate

3. $\angle 3 \cong \angle 4$
A. Definition of supplementary
B. Definition of congruent angles
C. Vertical angles are congruent
D. Definition of angle bisector

4. If $\angle 1$ is complementary to $\angle 2$, then $m\angle 1 + m\angle 2 = 90°$.
A. Angle Addition Postulate
B. Addition Property
C. Definition of perpendicular
D. Definition of complementary

Use this diagram for Items 5–8.



5. Write a statement from the diagram that can be justified by using the Angle Addition Postulate.

6. Write a statement from the diagram that can be justified by using the definition of a right angle.

7. Given that K is the midpoint of $\overline{FJ}$, write a statement that can be justified by using the definition of a midpoint.

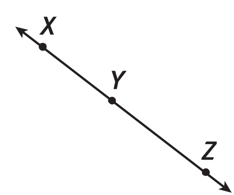
8. Given that $\angle GHF \cong \angle JFH$, write a statement that can be justified by using the definition of congruent angles.

Lesson 6-2

9. Complete the proof.

Given: $XY = 6$, $XZ = 14$

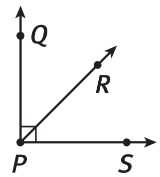
Prove: $YZ = 8$



Statements	Reasons
1. $XY = 6$, $XZ = 14$	**1.**
2. $XY + YZ = XZ$	**2.**
3. $6 + YZ = 14$	**3.**
4. $YZ = 8$	**4.**

10. Based on the given information in Item 9, can you conclude that Y is the midpoint of $\overline{XZ}$? Explain your reasoning.

11. Supply the missing statements and reasons.

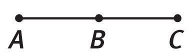


Given: $\overrightarrow{PR}$ bisects $\angle QPS$; $\angle QPS$ is a right angle.

Prove: $m\angle RPS = 45°$

Statements	Reasons
1. $\overrightarrow{PR}$ bisects $\angle QPS$.	**1.** ______
2. $m\angle QPR = m\angle RPS$	**2.** Definition of ______
3. $\angle QPS$ is a right angle.	**3.** ______
4. $m\angle QPS =$ ______	**4.** Definition of ______
5. $m\angle QPR + m\angle RPS = m\angle QPS$	**5.** ______ Postulate
6. $m\angle RPS + m\angle RPS = 90°$	**6.** ______ Property
7. $2(m\angle RPS) = 90°$	**7.** Distributive Property
8. $m\angle RPS =$ ______	**8.** ______

12. Complete the proof.



Given: $AC = 2(AB)$

Prove: B is the midpoint of $\overline{AC}$.

Statements	Reasons
1.	**1.** Given
2.	**2.** Segment Addition Postulate
3.	**3.** Transitive Property
4.	**4.** Subtraction Property of Equality
5.	**5.** Definition of congruent segments
6.	**6.** Definition of midpoint

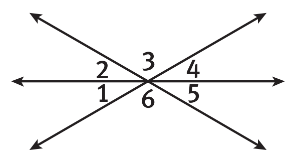
13. Arrange the statements and reasons below in a logical order to complete the proof.

Given: $\angle 1$ and $\angle 2$ are complementary; $m\angle 1 = 28°$.

Prove: $m\angle 2 = 62°$

$28° + m\angle 2 = 90°$	Substitution Property
$m\angle 1 + m\angle 2 = 90°$	Definition of complementary angles
$m\angle 1 = 28°$	Given
$m\angle 2 = 62°$	Subtraction Property of Equality
$\angle 1$ and $\angle 2$ are complementary.	Given

14. Write a two-column proof.



Given: $\angle 1 \cong \angle 2$, $m\angle 4 = 30°$

Prove: $m\angle 2 = 30°$

15. In Item 14, what are the measures of $\angle 3$, $\angle 5$, and $\angle 6$? Explain how you know.

MATHEMATICAL PRACTICES
Construct Viable Arguments and Critique the Reasoning of Others

16. Cara says that the following statement can be justified by the definition of vertical angles. Is her reasoning correct? Explain.

If $\angle G$ and $\angle H$ are vertical angles, then $\angle G \cong \angle H$.

Parallel and Perpendicular Lines

ACTIVITY 7

Patios by Madeline

Lesson 7-1 Parallel Lines and Angle Relationships

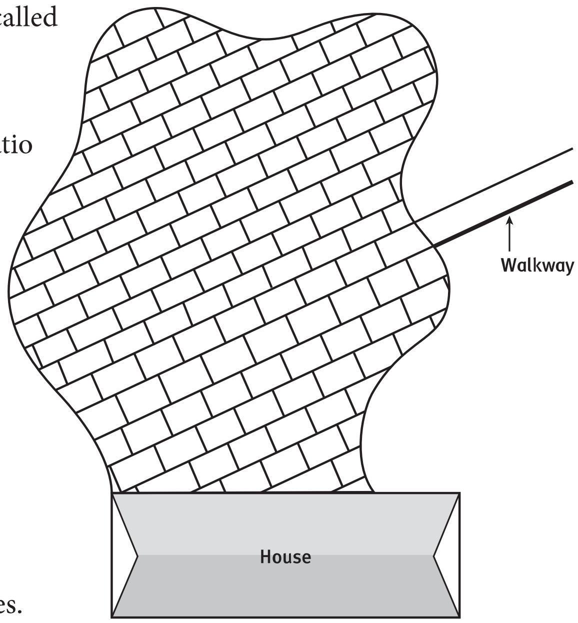
My Notes

Learning Targets:

- Make conjectures about the angles formed by a pair of parallel lines and a transversal.
- Prove theorems about these angles.

SUGGESTED LEARNING STRATEGIES: Summarizing, Paraphrasing, Vocabulary Organizer, Interactive Word Wall, Predict and Confirm, Think-Pair-Share, Discussion Groups, Group Presentation, Self Revision/Peer Revision

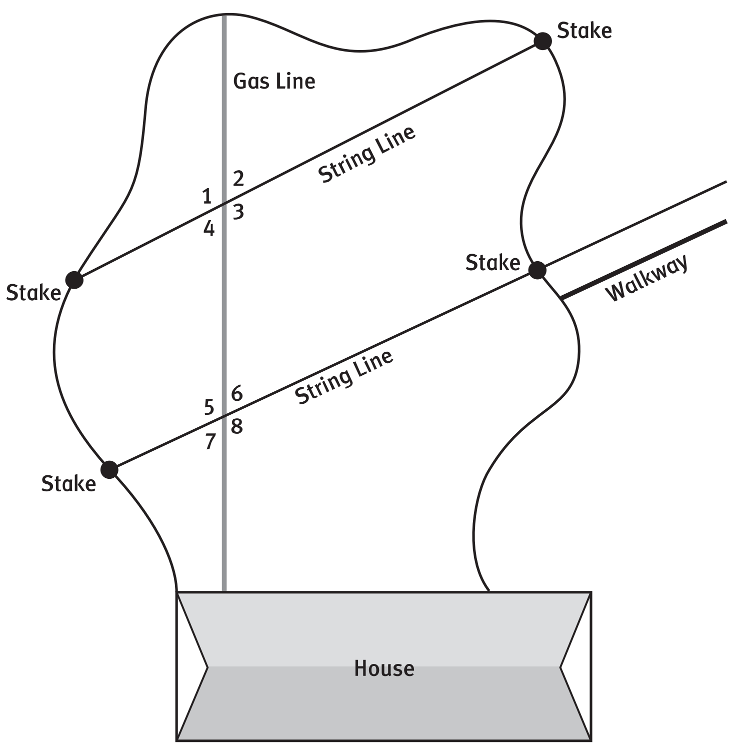
Matt works for a company called Patios by Madeline. A new customer has asked him to design a patio and walkway. The rows of bricks in the patio will be parallel to the walkway, as shown in the diagram.



So far, Matt has used stakes to tie down two ***parallel*** strings that he can use to align rows of bricks. He has also used paint to mark the underground gas line to the house so that he can avoid accidents during construction. The string lines and the gas line intersect to form eight angles.

MATH TERMS

Two lines (or parts of lines) are **parallel** if they are coplanar and do not intersect.



My Notes

MATH TERMS

A **transversal** is a line that intersects two or more coplanar lines in different points.

When two lines are intersected by a transversal,

- **Same-side interior angles** are a pair of angles that are between the two lines and on the same side of the transversal.
- **Alternate interior angles** are a pair of angles that are between the two lines and on opposite sides of the transversal.
- **Corresponding angles** are a pair of nonadjacent angles such that the angles are on the same side of the transversal and one angle is outside the two lines and one angle is between the two lines.

1. Use appropriate tools strategically. Use a protractor to measure each of the numbered angles formed by the string lines and the gas line on the previous page.

$m\angle 1 =$ $\quad$ $m\angle 2 =$

$m\angle 3 =$ $\quad$ $m\angle 4 =$

$m\angle 5 =$ $\quad$ $m\angle 6 =$

$m\angle 7 =$ $\quad$ $m\angle 8 =$

2. The gas line is a ***transversal*** of the string lines. Name the angle pairs formed by these lines that match each description.

a. Two pairs of ***same-side interior angles***

b. Two pairs of ***alternate interior angles***

c. Four pairs of ***corresponding angles***

3. Express regularity in repeated reasoning. Based on your answers to Items 1 and 2, make a conjecture about each type of angle pair formed by parallel lines and a transversal.

a. Same-side interior angles

b. Alternate interior angles

c. Corresponding angles

My Notes

4. The lines below are parallel.
   a. Draw a transversal to the parallel lines.

   b. Number the angles formed above and record the measure of each angle.

5. **Construct viable arguments.** Do your answers to Item 4 ***confirm*** the conjectures you made in Item 3? Explain. Revise your conjectures if needed.

TECHNOLOGY TIP

You can also use geometry software to draw two parallel lines and a transversal. You can then use the software to measure the angles formed by the intersecting lines.

ACADEMIC VOCABULARY

Evidence that ***confirms*** a conjecture provides support for the conjecture.

My Notes

6. The Same-Side Interior Angles Postulate involves angles formed by two parallel lines cut by a transversal.

a. **Reason abstractly.** Based on your earlier conjecture, write the Same-Side Interior Angles Postulate in if-then form.

b. Do you need to prove that this postulate is true before you can use it as a reason in a proof? Explain.

MATH TIP

You will prove these theorems later in this activity.

7. The Alternate Interior Angles Theorem and the Corresponding Angles Theorem also involve angles formed by two parallel lines cut by a transversal.

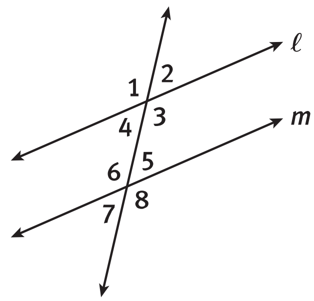
a. Based on your earlier conjecture, write the Alternate Interior Angles Theorem in if-then form.

b. Based on your earlier conjecture, write the Corresponding Angles Theorem in if-then form.

My Notes

Check Your Understanding

In the diagram, $\ell \parallel m$. Use the diagram for Items 8–10.



8. Explain how you know that $\angle 4$ and $\angle 6$ are same-side interior angles.

9. Are $\angle 2$ and $\angle 3$ corresponding angles? Explain how you know.

10. If $m\angle 5 = 65°$, then what is $m\angle 4$? Support your answer.

11. $\angle BCD$ and $\angle CGE$ are corresponding angles formed by two parallel lines cut by a transversal. Given that $m\angle BCD = 3x + 6$ and $m\angle CGE = x + 24$, complete the following.

a. What is the value of x?
b. What is $m\angle BCD$?
c. Explain how you found your answers.

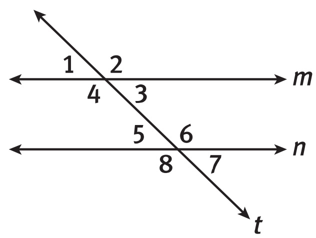
READING MATH

The symbol || means "is parallel to." So, $\ell \parallel m$ means "line ℓ is parallel to line m."

12. Reason quantitatively. Complete the following proof of the Corresponding Angles Theorem.

Given: $m \parallel n$

Prove: $\angle 1 \cong \angle 5$



Statements	Reasons
1. $m \parallel n$	**1.**
2. $m\angle 1 + m\angle 4 = 180°$	**2.** Angle Addition Postulate
3. $\angle 1$ and $\angle 4$ are supplementary.	**3.**
4. $\angle 4$ and $\angle 5$ are supplementary.	**4.**
5.	**5.** Congruent Supplements Theorem

MATH TIP

It is sufficient to prove that one pair of corresponding angles formed by a pair of parallel lines and a transversal are congruent.

MATH TIP

The Congruent Supplements Theorem states that if two angles are supplementary to the same angle, then they are congruent.

My Notes

Check Your Understanding

13. Look back at your proof in Item 12.

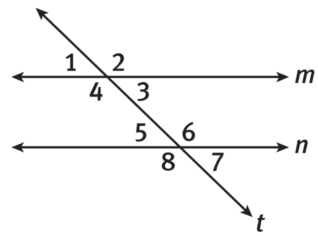
a. Explain how you know that $\angle 1$ and $\angle 5$ are corresponding angles.

b. Explain how you know that $\angle 4$ and $\angle 5$ are same-side interior angles.

14. Complete the following proof of the Alternate Interior Angles Theorem.

Given: $m \parallel n$

Prove: $\angle 4 \cong \angle 6$



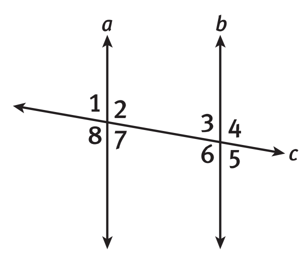
Statements	Reasons
1.	**1.**
2. $\angle 4 \cong \angle 2$	**2.**
3. $\angle 2 \cong \angle 6$	**3.**
4.	**4.**

MATH TIP

Do not assume that lines in a diagram are parallel just because they appear to be parallel. You can only conclude that lines are parallel if (1) you are given this information in the text or in a diagram, or (2) you prove that the lines are parallel.

LESSON 7-1 PRACTICE

In the diagram, $a \parallel b$. Use the diagram for Items 15–20. Determine whether each statement in Items 15–18 is true or false. Justify your response with the appropriate postulate or theorem.



15. $\angle 2$ is supplementary to $\angle 3$.

16. $\angle 8 \cong \angle 6$

17. $\angle 7$ is supplementary to $\angle 3$.

18. $\angle 6 \cong \angle 4$

19. If $m\angle 2 = 8x - 20$, and $m\angle 3 = 5x + 5$, what is $m\angle 2$? What is $m\angle 3$?

20. **Reason quantitatively.** Based on your answer to Item 19, what are the measures of the other numbered angles in the diagram? Explain your reasoning.

Lesson 7-2
Proving Lines are Parallel

My Notes

Learning Targets:

- Develop theorems to show that lines are parallel.
- Determine whether lines are parallel.

SUGGESTED LEARNING STRATEGIES: Visualization, Create Representations, Predict and Confirm, Think-Pair-Share, Debriefing, Discussion Groups, Group Presentation

MATH TIP

The converse of the statement "If p, then q" is "If q, then p." If a conditional statement is true, the converse of the statement may or may not be true.

1. **Make use of structure.** State the converse of each postulate or theorem.
   a. Same-Side Interior Angles Postulate

   b. Alternate Interior Angles Theorem

   c. Corresponding Angles Theorem

With your group, reread the problem scenario as needed. Make notes on the information provided in the problem. Respond to questions about the meaning of key information and organize the information needed to create a reasonable solution.

Matt is working on the blueprint for the new patio, as shown on the next page. Add information to the blueprint as you work through this lesson.

To ensure that the rows of bricks will be parallel to the walkway, Matt extends the string line along the edge of the walkway and labels points, X, A, and Y. He also labels points B, C, D, and E where additional string lines will cross the gas line.

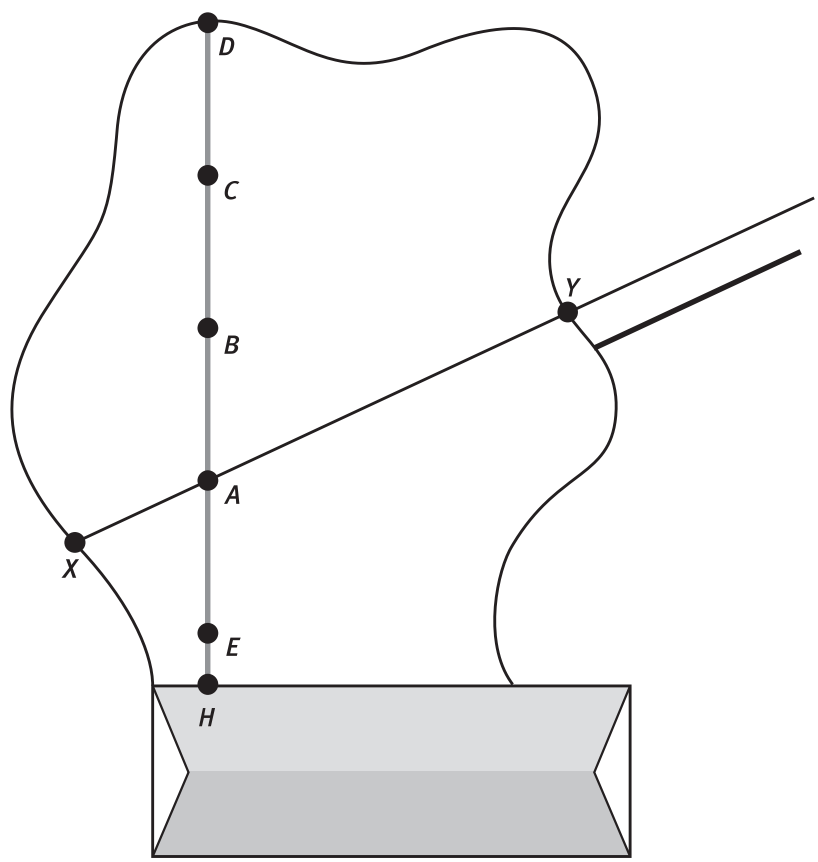
2. Follow these steps.
   a. Use a protractor to measure $\angle HAX$.

   b. Locate a stake on the left edge of the patio at point P by drawing $\overrightarrow{BP}$ so that $m\angle HBP = m\angle HAX$.

   c. Locate a stake on the right edge of the patio at point J by extending $\overrightarrow{BP}$ in the opposite direction.

My Notes

Matt's Blueprint



3. For which converse from Item 1 does your drawing from Item 2 provide support? Explain.

MATH TIP

You drew $\angle HBP$ and $\angle HAX$ so that they have the same measure. Identify the relationship between these two angles to help you answer Item 3.

My Notes

4. Matt realizes that he is not limited to using corresponding angles to draw parallel lines. Use a protractor to draw another parallel string through point *C* on the blueprint, using alternate interior angles. Mark the angles that you used to draw this new parallel line.

5. Explain how your drawing from Item 4 provides support for the converse of the Alternate Interior Angles Theorem.

6. **Use appropriate tools strategically.** Use a protractor to draw another parallel string through point *E* on the blueprint, using same-side interior angles. Mark the angles that were used to draw this new parallel line.

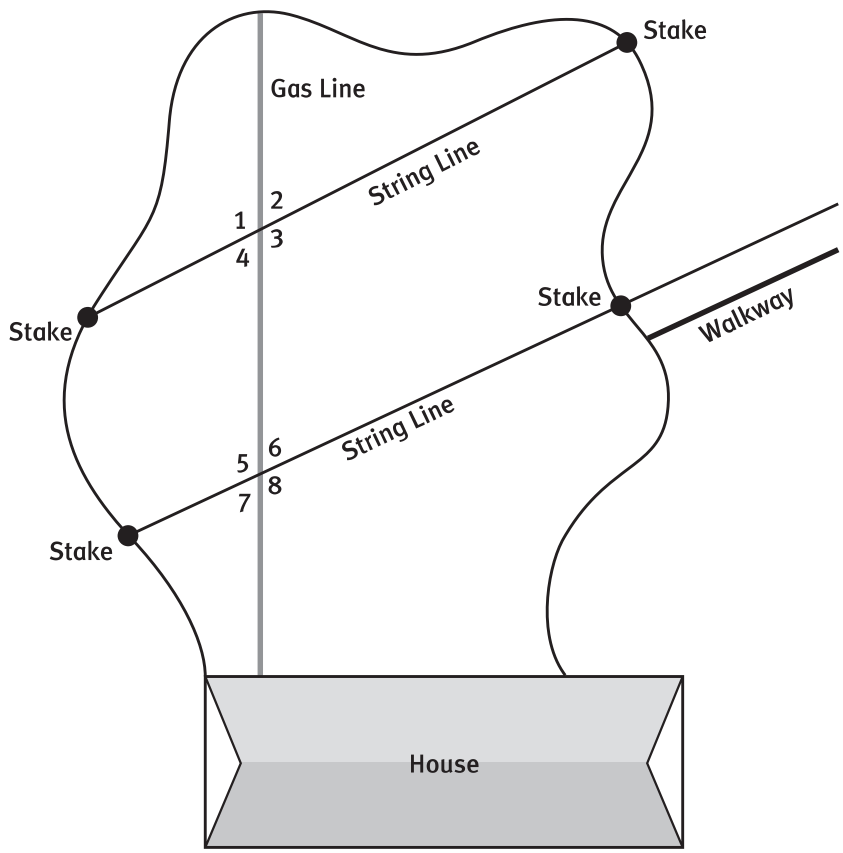
7. **Reason abstractly.** Explain how your drawing from Item 6 provides support for the converse of the Same-Side Interior Angles Theorem.

My Notes

The converses of the Same-Side Interior Angle Postulate, the Alternate Interior Angles Theorem, and the Corresponding Angles Theorem are all true, as indicated by the string lines you drew on the blueprint.

Look back at Item 1 in this lesson to review the converses of these theorems and postulate.

8. Attend to precision. Matt's assistant sets up two string lines as shown. Matt finds that $m\angle 2 = 63°$ and $m\angle 6 = 65°$. Explain how Matt can tell that the strings are not parallel.

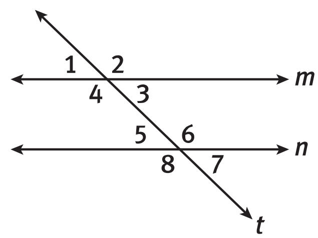


Check Your Understanding

9. Refer back to Item 8. How could Matt adjust the strings so that they are parallel?

10. What is the inverse of the Alternate Interior Angles Theorem? Is the inverse true? Explain how you know.

11. Complete the following proof of the Converse of the Corresponding Angles Theorem.



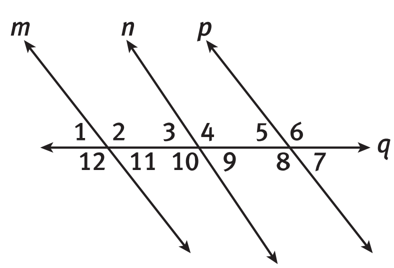
Given: $\angle 1 \cong \angle 5$

Prove: $m \parallel n$

Statements	Reasons
1.	**1.**
2. $m\angle 1 = m\angle 5$	**2.**
3. $m\angle 1 + m\angle 4 = 180°$	**3.** Linear Pair Postulate
4. $m\angle 5 + m\angle 4 = 180°$	**4.**
5. $\angle 5$ and $\angle 4$ are supplementary.	**5.**
6.	**6.** Converse of the Same-Side Interior Angles Postulate

LESSON 7-2 PRACTICE

For Items 12–14, use the diagram to answer each question. Then justify your answer.



12. Given that $m\angle 1 = 52°$ and $m\angle 7 = 52°$, is $m \parallel p$?

13. Given that $m\angle 11 = 52°$ and $m\angle 3 = 56°$, is $m \parallel n$?

14. Given that $m\angle 10 = 124°$ and $m\angle 7 = 52°$, is $n \parallel p$?

15. **Reason abstractly.** Two lines are cut by a transversal such that a pair of alternate interior angles are right angles. Are the two lines parallel? Explain.

16. **Model with mathematics.** Describe how a stadium worker can determine whether two yard lines painted on a football field are parallel. Assume that the worker has a protractor, string, and two stakes.

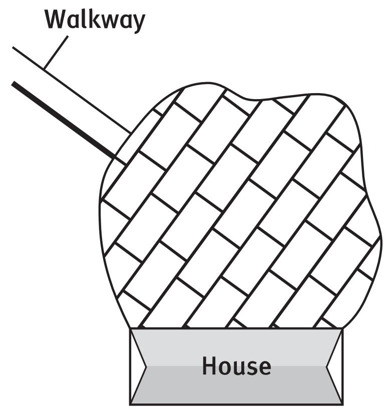
My Notes

Learning Targets:

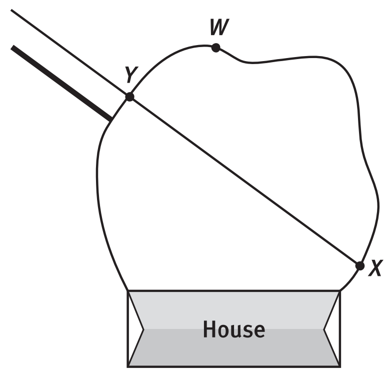
- Develop theorems to show that lines are perpendicular.
- Determine whether lines are perpendicular.

SUGGESTED LEARNING STRATEGIES: Activating Prior Knowledge, Think Aloud, Think-Pair-Share, Create Representations, RAFT

A second customer of Patios by Madeline has hired the company to build a patio with rows of bricks that are ***perpendicular*** to the walkway, as shown at right.



Matt is planning the design for this patio. So far, he has extended a line along the walkway and labeled points X and Y to mark stakes at the edge of the patio. He has also drawn point W to mark another stake at the edge of the patio, as shown below.



MATH TERMS

Two lines (or parts of lines) are **perpendicular** if they intersect to form right angles.

1. Use a protractor to draw the line perpendicular to $\overleftrightarrow{XY}$ that passes through W. Label the point at which the line intersects $\overleftrightarrow{XY}$ point Z.

2. **Use appropriate tools strategically.** Use your protractor and your knowledge of parallel lines to draw a line, across the patio, parallel to $\overleftrightarrow{WZ}$. Explain how you know the lines are parallel.

MATH TERMS

The **Perpendicular Postulate** states that if given a line and a point not on the line, then there is exactly one line through the point that is perpendicular to the given line.

Likewise, the **Parallel Postulate** states that if given a line and a point not on the line, then there is exactly one line through the point that is parallel to the given line.

3. Describe the relationship between the newly drawn line and $\overleftrightarrow{XY}$.

4. **Critique the reasoning of others.** Matt reasons that if a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other line. Show that Matt's conjecture is true. Include a diagram in your answer.

5. Matt draws $\overline{AB}$ on his design of the patio so that $\overleftrightarrow{XY}$ is the ***perpendicular bisector*** of $\overline{AB}$. $\overleftrightarrow{XY}$ intersects $\overline{AB}$ at point M. List three conclusions you can draw from this information.

My Notes

MATH TERMS

The **Perpendicular Transversal Theorem** states that if a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other line.

MATH TIP

In Item 4, start by drawing line m parallel to line n. Then draw transversal t so that it is perpendicular to m.

You want to show that $t \perp n$ given that $m \parallel n$ and $t \perp m$. Number the angles in your diagram as needed so that you can refer to them in your answer.

MATH TERMS

A **perpendicular bisector** of a segment is a line (or part of a line) that intersects the segment at its midpoint to form right angles.

My Notes

TECHNOLOGY TIP

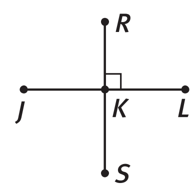
An instruction guide often includes diagrams to make the text easier to understand. You can create diagrams for your instruction guide by hand or by using geometry software.

6. Matt's boss Madeline is so impressed with Matt's work that she asks him to write an instruction guide for creating rows of bricks parallel to a patio walkway. Madeline asks that the guide provide enough information so that a bricklayer can use any pair of angles (same-side interior, alternate interior, or corresponding) to determine the location of each pair of stakes. She also requests that Matt include directions for creating rows of bricks that are perpendicular to a walkway. Write Matt's instruction guide, following Madeline's directions.

Check Your Understanding

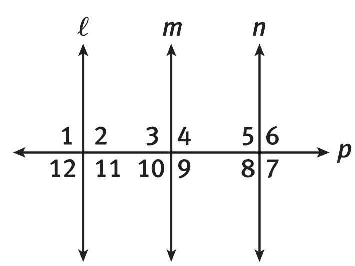
7. **Reason abstractly.** Lines a, b, and c lie in the same plane. If $a \perp b$ and $b \perp c$, can you conclude that $a \perp c$? Explain. Include a drawing to support your answer.

8. $\overline{RS}$ is the perpendicular bisector of $\overline{JL}$. Can you conclude that point K is the midpoint of $\overline{RS}$? Explain.



LESSON 7-3 PRACTICE

In the diagram, $\ell \parallel m$, $m\angle 1 = 90°$, and $\angle 5$ is a right angle. Use the diagram for Items 46–48.



9. Explain how you know that $\ell \perp p$.

10. Show that $m\angle 4 = 90°$.

11. **Make use of structure.** Show that $\ell \parallel n$.

12. The perpendicular bisector of $\overline{CD}$ intersects CD at point P. If $CP = 12$, what is CD?

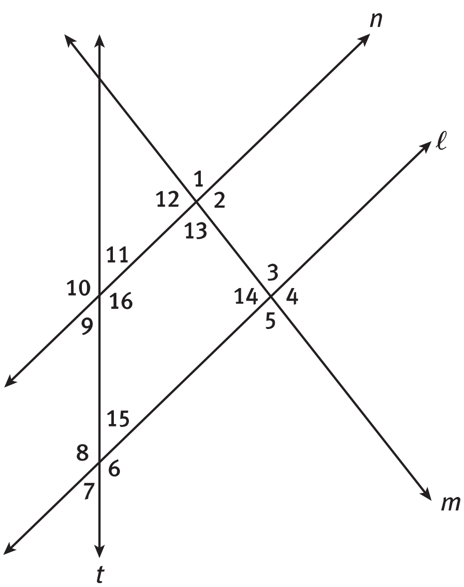
13. An angle formed by the intersection of two lines is obtuse. Could the lines be perpendicular? Explain.

ACTIVITY 7 PRACTICE

Write your answers on notebook paper.
Show your work.

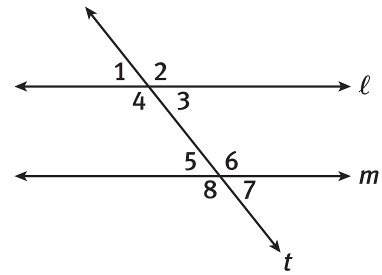
Lesson 7-1

Use this diagram for Items 1–10.



1. List all angles in the diagram that form a corresponding angle pair with $\angle 5$.
2. List all angles in the diagram that form an alternate interior angle pair with $\angle 14$.
3. List all angles in the diagram that form a same side interior angle pair with $\angle 14$.
4. Given $\ell \parallel n$. If $m\angle 6 = 120°$, then $m\angle 16 =$ __?__.
5. Given $\ell \parallel n$. If $m\angle 12 = 5x + 10$ and $m\angle 14 = 6x - 4$, then $x =$ __?__ and $m\angle 4 =$ __?__.
6. Given $\ell \parallel n$. If $m\angle 14 = 75°$, then $m\angle 2 =$ __?__.
7. Given $\ell \parallel n$. If $m\angle 9 = 3x + 12$ and $m\angle 15 = 2x + 27$, then $x =$ __?__ and $m\angle 10 =$ __?__.
8. Given $\ell \parallel n$. If $m\angle 3 = 100°$, then $m\angle 2 =$ __?__.
9. Given $\ell \parallel n$. If $m\angle 16 = 5x + 18$ and $m\angle 15 = 3x + 2$, then $x =$ __?__ and $m\angle 8 =$ __?__.
10. If $m\angle 10 = 135°$, $m\angle 12 = 75°$, and $\ell \parallel n$ determine the measure of each angle. Justify each answer.
    a. $m\angle 16$
    b. $m\angle 15$
    c. $m\angle 8$
    d. $m\angle 14$
    e. $m\angle 3$

11. Supply the missing statements and reasons.



Given: $\ell \parallel m$

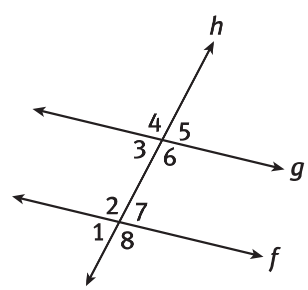
Prove: $\angle 1 \cong \angle 7$

Statements	Reasons
1.	**1.** Given
2. $\angle 1 \cong \angle 5$	**2.**
3. $\angle 5 \cong \angle 7$	**3.**
4.	**4.**

12. Two parallel lines are cut by a transversal. How are the corresponding angles and same-side interior angles alike?
    A. The angles in each pair are congruent.
    B. The angles in each pair are supplementary.
    C. The angles in each pair are between the parallel lines.
    D. The angles in each pair are on the same side of the transversal.

Lesson 7-2

Use this diagram for Items 13–15.



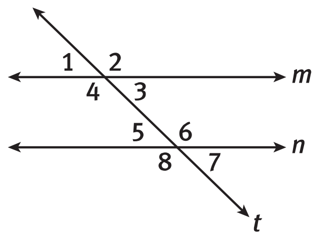
13. If $m\angle 2 = 110°$ and $m\angle 3 = 80°$, is $f \parallel g$? Justify your answer.
14. If $\angle 8 \cong \angle 4$, is $f \parallel g$? Justify your answer.
15. Given that $m\angle 5 = 12x + 4$ and $m\angle 7 = 10x + 16$, what must the value of x be in order for line f to be parallel to line g?

16. If $a \parallel b$ and $b \parallel c$, can you conclude that $a \parallel c$? Explain. Include a drawing to support your answer.

17. Arrange the statements and reasons below in a logical order to complete the proof of the Converse of the Alternate Interior Angles Theorem.

Given: $\angle 4 \cong \angle 6$

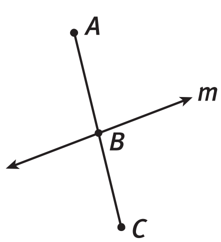
Prove: $m \parallel n$



$m\angle 5 + m\angle 6 = 180°$	Linear Pair Postulate
$\angle 4 \cong \angle 6$	Given
$m\angle 5 + m\angle 4 = 180°$	Substitution Property
$\angle 5$ and $\angle 4$ are supplementary.	Definition of supplementary angles
$m\angle 4 = m\angle 6$	Definition of congruent angles
$m \parallel n$	Converse of the Same-Side Interior Angles Postulate

Lesson 7-3

Use this diagram for Items 18–20.



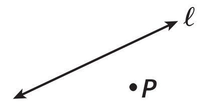
18. What information do you need to know to prove that line m is the perpendicular bisector of $\overline{AC}$?

19. Given that line m is the perpendicular bisector of $\overline{AC}$, $AB = 4x - 1$, and $BC = 2x + 7$, what is the value of x?

20. Based on your answer to Item 19, what is AC?

A. 11 **B.** 15
C. 22 **D.** 30

21. How many lines perpendicular to line l can you draw through point P? Explain how you know.



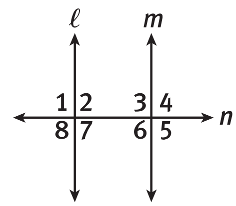
22. Complete the proof.

Theorem: If two lines in the same plane are perpendicular to the same line, then the lines are parallel to each other.

Given: $\ell \perp n$,

$m \perp n$

Prove: $\ell \parallel m$



Statements	Reasons
1.	**1.** Given
2. $\angle 2$ is a right angle; $\angle 6$ is a right angle	**2.**
3. $\angle 2 \cong \angle 6$	**3.**
4.	**4.**

MATHEMATICAL PRACTICES
Look for and Express Regularity in Repeated Reasoning

23. Suppose you are given the measure of one of the angles formed when a pair of parallel lines is cut by a transversal. Explain how to use this measure to find the measures of the other seven angles.

Equations of Parallel and Perpendicular Lines

Skateboard Geometry

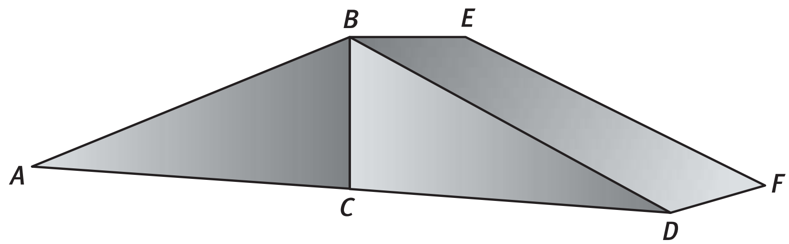
Lesson 8-1 Slopes of Parallel and Perpendicular Lines

My Notes

Learning Targets:

- Make conjectures about the slopes of parallel and perpendicular lines.
- Use slope to determine whether lines are parallel or perpendicular.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Activating Prior Knowledge, Visualization, Debriefing



The new ramp at the local skate park is shown above. In addition to the wooden ramp, an aluminum rail (not shown) is mounted to the edge of the ramp. While the image of the ramp may conjure thoughts of kickflips, nollies, and nose grinds, there are mathematical forces at work here as well.

1. Use the diagram of the ramp to complete the chart below:

Describe two parts of the ramp that appear to be parallel.	
Describe two parts of the ramp that appear to be perpendicular.	
Describe two parts of the ramp that appear to be neither parallel nor perpendicular.	

My Notes

MATH TIP

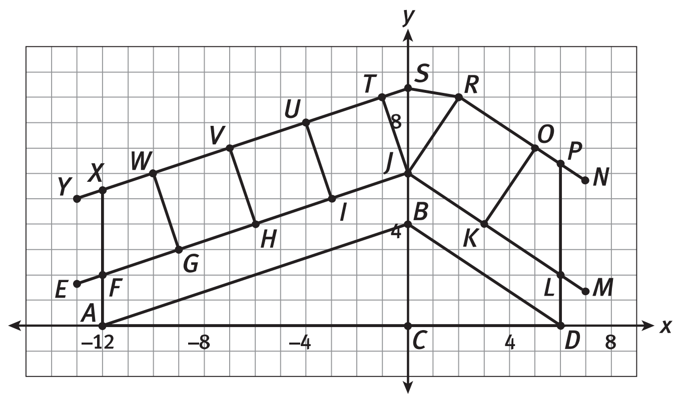
The *slope* of a line is the ratio of the rise, or vertical change, to the run, or horizontal change, between two points on the line.

CONNECT TO ALGEBRA

If (x_1, y_1) and (x_2, y_2) are two points on a line, then the slope m of the line can be calculated as follows.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Recall that the *slope* of a line is the ratio of the vertical change to the horizontal change between two points on the line.



2. The diagram above shows a cross-section of the skate ramp and its railing transposed onto a coordinate grid. Find the slopes of the following segments.

a. $\overline{AB}$

b. $\overline{XT}$

c. $\overline{WG}$

d. $\overline{TJ}$

e. $\overline{AD}$

3. **Model with mathematics.** Complete the chart by stating whether each pair of segments appear to be parallel, perpendicular, or neither. Then list the slopes of the segments.

Segments	Parallel, Perpendicular, or Neither?	Slopes
$\overline{AB}$ and $\overline{XT}$		
$\overline{WG}$ and $\overline{TJ}$		
$\overline{XT}$ and $\overline{WG}$		
$\overline{XT}$ and $\overline{TJ}$		
$\overline{AB}$ and $\overline{AD}$	neither	
$\overline{WG}$ and $\overline{AD}$	neither	

My Notes

4. **Express regularity in repeated reasoning.** Based on the chart in Item 3, make conjectures about the slopes of each type of segments.
   a. Parallel segments

   b. Perpendicular segments

   c. Segments that are neither parallel nor perpendicular

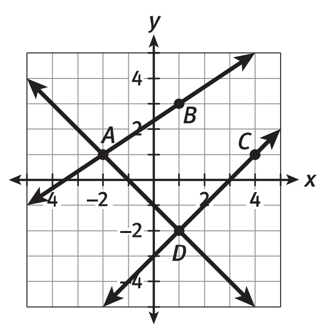
Check Your Understanding

5. Do the conjectures you wrote in Item 4 apply to lines as well as segments? Explain.
6. **Reason abstractly.** Are all horizontal lines parallel? Use slope to explain how you know.
7. $\overleftrightarrow{JK}$ has a slope of $\frac{2}{5}$, and $\overleftrightarrow{MN}$ has a slope of $\frac{5}{2}$. Are these segments parallel, perpendicular, or neither? Explain.

LESSON 8-1 PRACTICE

Use slope to support your answers to Items 8–10.

8. Is $\overleftrightarrow{AB} \parallel \overleftrightarrow{DC}$?
9. Is $\overleftrightarrow{AB} \perp \overleftrightarrow{AD}$?
10. Is $\overleftrightarrow{DC} \perp \overleftrightarrow{AD}$?
11. **Reason quantitatively.** Line ℓ passes through the origin and the point (3, 4). What is the slope of a line parallel to line ℓ?



12. $\angle RST$ is a right angle. If $\overline{RS}$ has a slope of 3, what must be the slope of $\overline{ST}$? Explain.

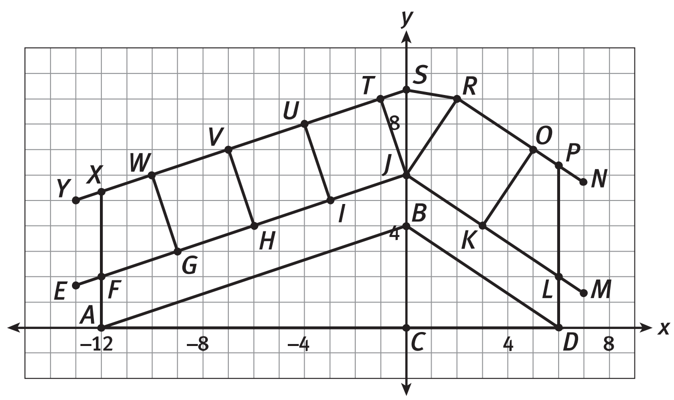
My Notes

Learning Targets:

- Write the equation of a line that is parallel to a given line.
- Write the equation of a line that is perpendicular to a given line.

SUGGESTED LEARNING STRATEGIES: Predict and Confirm, Look for a Pattern, Identify a Subtask, Discussion Groups, Create Representations, Think-Pair-Share, Self Revision/Peer Revision, Visualization, Identify a Subtask

Here again is the diagram of the skate ramp and its railing.



1. The equation of the line containing $\overline{BD}$ is $y = -\frac{2}{3}x + 4$. Identify the slope and the y-intercept of this line.

2. Based on your conjectures in Item 4 in the previous lesson, what would be the slope of the line containing $\overline{JM}$? Use the formula for slope to verify your answer or provide an explanation of the method you used to determine the slope.

3. Identify the y-intercept of the line containing $\overline{JM}$. Use the slope and the y-intercept to write the equation of this line.

4. If $\overline{JM} \perp \overline{RJ}$, identify the slope, y-intercept, and the equation for the line containing $\overline{RJ}$.

5. **Construct viable arguments.** Is $\overline{BD} \perp \overline{RJ}$? Explain your reasoning.

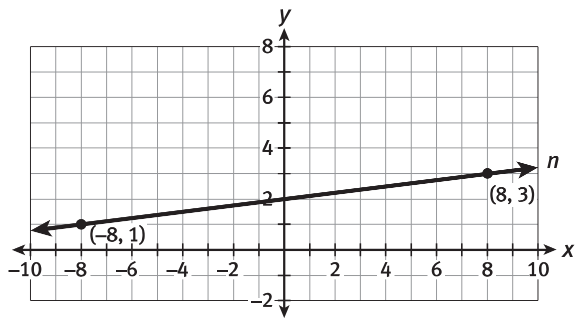
CONNECT TO ALGEBRA

The equation of a line in slope-intercept form is $y = mx + b$, where m represents the slope and b represents the y-intercept of the line.

MATH TIP

The y-intercept of a line is the y-coordinate of the point where the line crosses the y-axis.

The skate park will also have an angled grind rail. A drafter is drawing the grind rail on a coordinate plane.



6. Make sense of problems. The drafter has already drawn line n. Next, the drafter needs to draw line p so that it is parallel to line n and passes through point $(-2, 3)$.

a. What is the slope of line p? How do you know?

b. What is the equation of line p written in point-slope form?

c. What is the equation of line p written in slope-intercept form?

d. Sketch line p on the drafter's coordinate plane.

My Notes

CONNECT TO CAREERS

A drafter creates technical drawings of plans made by engineers and architects. Drafters primarily use computer software to make their drawings. The software positions items such as lines and curves by using a coordinate system.

CONNECT TO ALGEBRA

The point-slope form of a linear equation is $y - y_1 = m(x - x_1)$, where m is the slope and (x_1, y_1) is a point on the line.

My Notes

MATH TIP

If two numbers are opposite reciprocals, then their product is -1.

7. Next, the drafter needs to draw line q so that it is perpendicular to line n and passes through point $(-9, 7)$.

a. What is the slope of line q? How do you know?

b. What is the equation of line q written in point-slope form?

c. What is the equation of line q written in slope-intercept form?

d. Sketch line q on the drafter's coordinate plane.

8. Model with mathematics. Finally, the drafter needs to draw line r so that it is perpendicular to line n and passes through point $(8, 0)$.

a. What is the equation of line r written in point-slope form?

b. What is the equation of line r written in slope-intercept form?

c. Sketch line r on the drafter's coordinate plane.

My Notes

9. Reason abstractly and quantitatively. Describe the relationships among the lines n, p, q, and r.

Quadrilaterals can be classified by the presence or absence of parallel lines, and by the presence or absence of perpendicular lines.

10. Which specific quadrilateral is formed by the intersection of the four lines? Explain how you know.

Check Your Understanding

11. What is the slope of a line parallel to the line with equation $y = \frac{4}{5}x - 7$?

12. What is the slope of a line perpendicular to the line with equation $y = -2x + 1$?

13. Consider the lines described below.

line a: a line with the equation $y = \frac{3}{5}x - 2$

line b: a line containing the points $(-6, 1)$ and $(3, -14)$

line c: a line containing the points $(-1, 8)$ and $(4, 11)$

line d: a line with the equation $-5x + 3y = 6$

Use the vocabulary from the lesson to write a paragraph that describes the relationships among lines a, b, c, and d.

14. What is the slope of a line parallel to the line with equation $y = -3$? What is the slope of a line perpendicular to the line with equation $y = -3$? Explain your reasoning.

15. Construct viable arguments. Suppose you are given the equation of a line and the coordinates of a point not on the line. Explain how to write the equation of a parallel line that passes through the point.

MATH TIP

For Item 13, start by finding the slope of each line.

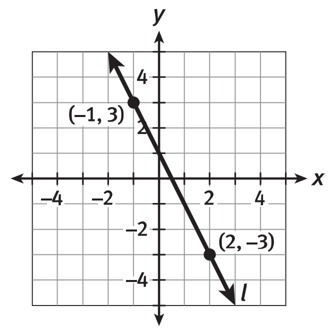
My Notes

LESSON 8-2 PRACTICE

16. What is the equation of a line parallel to $y = -3x + 5$ that passes through point (6, 8)?

17. What is the equation of a line perpendicular to $y = \frac{1}{4}x - 3$ that passes through point (−2, 4)?

Use the coordinate plane for Items 24–26.



18. What is the equation of the line parallel to line ℓ that passes through point (3, 3)?

19. What is the equation of the line perpendicular to line ℓ that passes through point (3, 3)?

20. How could you check that your answer to Item 19 is reasonable?

21. **Reason abstractly and quantitatively.** $\triangle ABC$ has vertices at the points $A(-2, 3)$, $B(7, 4)$, and $C(5, 22)$. What kind of triangle is $\triangle ABC$? Explain your answer.

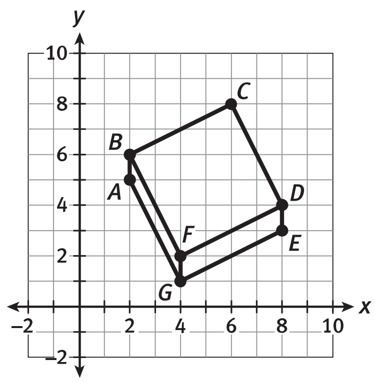
ACTIVITY 8 PRACTICE

Write your answers on notebook paper.
Show your work.

Lesson 8-1

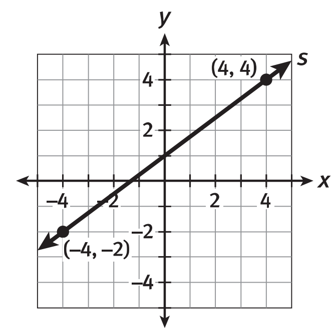
1. Determine the slope of the line that contains the points with coordinates (1, 5) and (−2, 7).
2. Line m contains the points with coordinates (−4, 1) and (5, 8), and line n contains the points with coordinates (6, −2) and (10, 7). Are the lines parallel, perpendicular, or neither? Justify your answer.

A drafter finished the first stage of a drawing of a metal part. Use the drawing for Items 3–6.



3. The drafter needs to confirm that $\overline{AG} \parallel \overline{CD}$. Are these segments parallel? Explain.
4. The drafter also needs to confirm that $\overline{BF} \perp \overline{FD}$. Are these segments perpendicular? Explain.
5. Which segments in the diagram are perpendicular to $\overline{GE}$?
6. Name a segment that is neither parallel nor perpendicular to $\overline{BC}$.
7. $\overleftrightarrow{PQ}$ has a slope of $\frac{1}{3}$. What is the slope of a line perpendicular to $\overleftrightarrow{PQ}$?
   A. -3 **B.** $-\frac{1}{3}$
   C. $\frac{1}{3}$ **D.** 3
8. $\overleftrightarrow{VW}$ has a slope of 5. What is the slope of a line parallel to $\overleftrightarrow{VW}$?
   A. -5 **B.** $-\frac{1}{5}$
   C. $\frac{1}{5}$ **D.** 5

Use the diagram for Items 9 and 10.

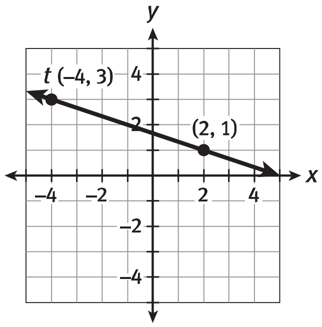


9. What is the slope of a line parallel to line s?
10. What is the slope of a line perpendicular to line s?

Lesson 8-2

11. Which of the following is NOT an equation for a line parallel to $y = \frac{1}{2}x - 6$?
    A. $y = \frac{2}{4}x + 6$ **B.** $y = 0.5x - 3$
    C. $y = \frac{1}{2}x + 1$ **D.** $y = 2x - 4$
12. Determine the slope of a line perpendicular to the line with equation $6x - 4y = 30$.

Use the diagram for Items 13 and 14.



13. Which of the following represents a line parallel to line t?

A. $y = -x + 7$ **B.** $y = -\frac{1}{3}x + 5$

C. $y = \frac{1}{3}x - 2$ **D.** $y = x - 4$

14. Which of the following represents a line perpendicular to line t?

A. $y = -3x + 4$ **B.** $y = -\frac{1}{3}x - 1$

C. $y = \frac{1}{3}x - 5$ **D.** $y = 3x + 4$

15. Consider the line with equation $y = -2x + 2$.

a. Can you write the equation of a line through point $(3, -4)$ that is parallel to the given line? Explain.

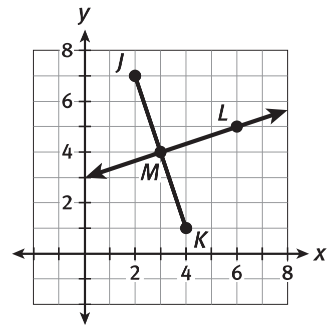
b. Can you write the equation of a line through point $(3, -4)$ that is perpendicular to the given line? Explain.

16. Suppose you are given the equation of a line and the coordinates of a point not on the line. Explain how to write the equation of a perpendicular line that passes through the point.

17. What is the equation of a line through point $(0, 5)$ that is parallel to the line with equation $y = x - 7$?

18. What is the equation of a line through point $(-4, 5)$ that is perpendicular to the line with equation $y = -6x + 4$?

Use the diagram for Items 19 and 20.



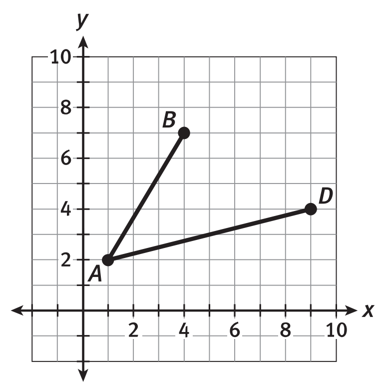
19. Use the Distance Formula to show that $\overline{JM} \cong \overline{MK}$.

20. Is $\overleftrightarrow{LM}$ the perpendicular bisector of $\overline{JK}$? Explain how you know.

MATHEMATICAL PRACTICES

Make Sense of Problems and Persevere in Solving them

21. A drafter is drawing quadrilateral $ABCD$ as part of an architectural blueprint. She needs to position point C so that $\overline{BC} \parallel \overline{AD}$ and $\overline{CD} \perp \overline{AD}$.



a. What should be the coordinates of point C? Explain how you determined your answer.

b. Describe how you can check that your answer is reasonable.