Properties of Triangles

Best Two Out of Three

Lesson 13-1 Angle Relationships in Triangles

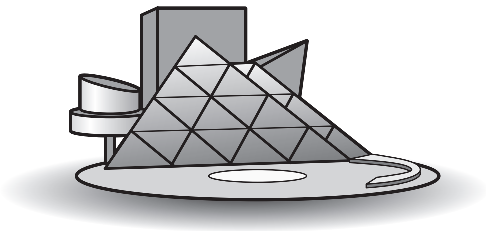
My Notes

Learning Targets:

- Prove theorems about angle measures in triangles.
- Apply theorems about angle measures in triangles.

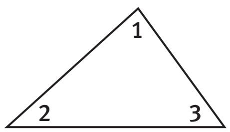
SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Use Manipulatives, Look for a Pattern, Quickwrite, Self Revision/Peer Revision

The architectural design of the Rock and Roll Hall of Fame in Cleveland, Ohio, includes triangles. Some of the glass panels on the façade need to be replaced. To cut the glass correctly, angle measures of some of the triangles will need to be determined.

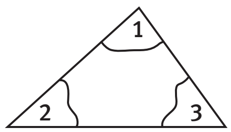


Jason owns a glass company and he will be cutting the glass for the repairs. One of the tools he will use to determine angle measures of the triangles is the Parallel Postulate. Jason cannot remember the sum of the measures of the angles of a triangle. He begins by drawing one of the triangles of glass he needs to replace.

1. Draw or trace the triangle below.



2. Tear off each corner of the triangle.



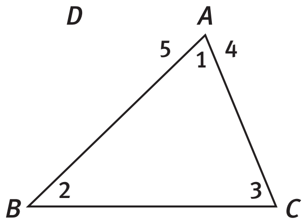
3. Arrange the corners so they are adjacent. What is formed?

4. What appears to be true about the sum of the measures of the angles of a triangle? Write a conjecture. This is known as the ***Triangle Sum Theorem***.

My Notes

You have just made a conjecture about the sum of the measures of the angles of a triangle, but the conjecture is not proven. Jason wants to prove the relationship to be absolutely certain he is correct.

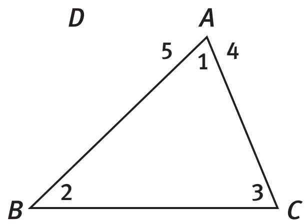
5. Make use of structure. When you are given a figure for a proof, you cannot change anything in the figure. But you can add a line to a figure to help you complete the proof. A line used in this manner is called an ***auxiliary line***. The proof of the Triangle Sum Theorem begins by drawing an auxiliary line D that intersects point A and is parallel to $\overline{BC}$. Line D forms angles 4 and 5. Explain why you can draw line D.



6. Complete the proof of the Triangle Sum Theorem.

Theorem: The sum of the angles of a triangle is 180°.

Given: $\triangle ABC$
Prove: $m\angle 2 + m\angle 1 + m\angle 3 = 180°$



Statements	Reasons
1. Through point A, draw $\overleftrightarrow{AD}$, so that $\overleftrightarrow{AD} \parallel \overleftrightarrow{BC}$.	**1.**
2. $m\angle 5 + m\angle 1 + m\angle 4 = 180°$	**2.**
3.	**3.** If parallel lines are cut by a transversal, then alternate interior angles are congruent.
4.	**4.** Definition of congruent angles
5.	**5.** Substitution Property

CONNECT TO HISTORY

The **Parallel Postulate** states that through any point *P* that is not on line *l*, there is exactly one line that can be drawn that is parallel to line *l*. The Parallel Postulate is also known as the fifth postulate in Euclid's Elements. For centuries it was believed that the Parallel Postulate was not really a postulate, but a theorem, which needed to be proved using Euclid's first four postulates. It did not seem to be obvious, as did the first four of Euclid's postulates. Many set out to prove it, but always unsuccessfully. Ultimately, non-Euclidean geometries were discovered in which the Parallel Postulate was shown to be false.

Check Your Understanding

7. In the proof of the Triangle Sum Theorem, could auxiliary line D have been drawn through vertex B or vertex C instead of vertex A? Explain.

8. How can you use the Triangle Sum Theorem to show that the measure of each angle of an equiangular triangle is 60°?

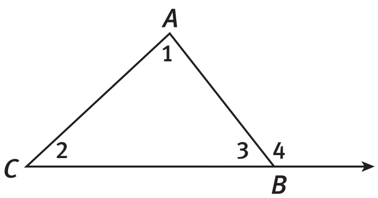
9. How can you use the Triangle Sum Theorem to show that the measures of the acute angles of a right triangle are complementary?

My Notes

An ***interior angle*** of a triangle is formed by two sides of the triangle. An ***exterior angle*** of a triangle is formed by one side of the triangle and the extension of an adjacent side. A ***remote interior angle*** of a triangle is an interior angle that is not adjacent to a given exterior angle.

Sometimes Jason will only know the measure of an exterior angle of a triangle in the façade. He needs to be able to determine the interior angles of a triangle if he knows an exterior angle measure.

10. Label each angle of the triangle using one of the following terms: interior angle, exterior angle, remote interior angle.



11. Use a protractor to measure angles 1, 2, and 4. What are their measures?

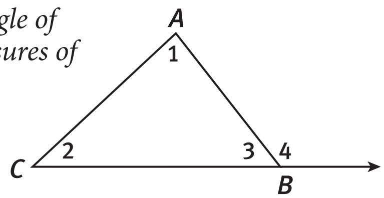
12. What appears to be true about the measure of an exterior angle of a triangle in relationship to the measures of its two remote interior angles? Write a conjecture. This is known as the ***Exterior Angle Theorem***. It is a corollary to the Triangle Sum Theorem.

13. Complete the proof of the Exterior Angle Theorem. You will use the Triangle Sum Theorem to help you prove it.

Theorem: The measure of an exterior angle of a triangle is equal to the sum of the measures of its two remote interior angles.

Given: $\triangle ABC$

Prove: $m\angle 4 = m\angle 1 + m\angle 2$

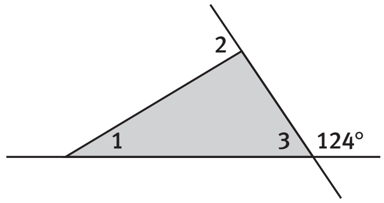


Statements	Reasons
1. $\triangle ABC$ with exterior angle 4	**1.**
2.	**2.** Triangle Sum Theorem
3. $m\angle 3 + m\angle 4 = 180°$	**3.**
4. $m\angle 4 = 180° - m\angle 3$	**4.**
5.	**5.**
6.	**6.**

My Notes

Example A

The shaded piece of glass shown below is one of the triangles Jason needs to replace.



Jason knows that in this section of the glass façade, the measure of $\angle 2$ is three times the measure of $\angle 1$. What are the measures of all the angles in the piece of glass?

By the Exterior Angle Theorem, you know that $m\angle 1 + m\angle 2 = 124°$. You also know $m\angle 2 = 3(m\angle 1)$.

Use substitution to find the measure of $\angle 1$.

$$\begin{aligned} m\angle 1 + m\angle 2 &= 124° \\ m\angle 1 + 3(m\angle 1) &= 124° \\ 4(m\angle 1) &= 124° \\ m\angle 1 &= 31° \end{aligned}$$

Now you can find the measure of $\angle 2$.

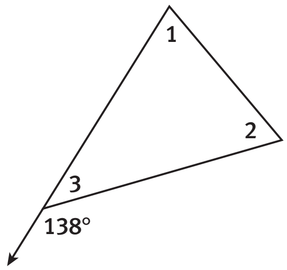
$$\begin{aligned} m\angle 1 + m\angle 2 &= 124° \\ 31° + m\angle 2 &= 124° \\ m\angle 2 &= 93° \end{aligned}$$

One way to find the measure of $\angle 3$ is to use the Triangle Sum Theorem.

$$\begin{aligned} m\angle 1 + m\angle 2 + m\angle 3 &= 180° \\ 31° + 93° + m\angle 3 &= 180° \\ m\angle 3 &= 56° \end{aligned}$$

Try These A

If $m\angle 2 = (10x + 2)°$ and $m\angle 1 = (11x - 11)°$, complete the following.



a. $m\angle 1 =$ ______

b. $m\angle 2 =$ ______

c. $m\angle 3 =$ ______

My Notes

Check Your Understanding

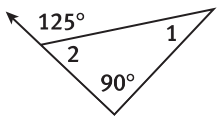
14. In an equiangular triangle, are all the measures of the exterior angles of the triangle equal? Explain.

15. Is it possible for the measure of an exterior angle of a triangle to be equal to the measure of a remote interior angle? Explain.

16. If the measures of two angles of a triangle are equal to the measures of two angles of another triangle, what can you conclude about the measures of the third angles of the triangles?

LESSON 13-1 PRACTICE

Use the figure below to find each measure.

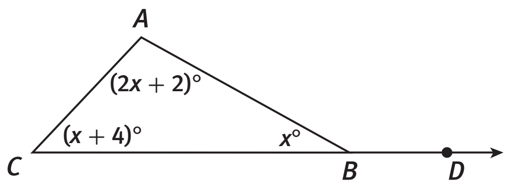


17. $m\angle 1 =$ _____

18. $m\angle 2 =$ _____

19. **Reason abstractly.** Two remote interior angles of a triangle measure 37° and 62°. What is the measure of the exterior angle associated with the remote interior angles? What is the measure of the third angle of the triangle?

Use the figure below to find each measure.



20. $m\angle A =$ _____

21. $m\angle C =$ _____

22. $m\angle ABC =$ _____

23. $m\angle ABD =$ _____

My Notes

Learning Targets:

- Develop theorems about isosceles triangles.
- Prove theorems about isosceles triangles.

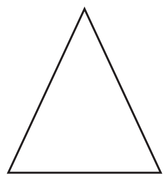
SUGGESTED LEARNING STRATEGIES: Quickwrite, Self Revision/Peer Revision, Think-Pair-Share, Group Presentation, Use Manipulatives, Look for a Pattern

Some of the triangles Jason needs to replace are isosceles triangles. Again, he thinks he knows what an isosceles triangle is, but wants to be sure he is correct.

CONNECT TO LANGUAGE

The name **isosceles** is derived from the Greek *iso* (same) and *skelos* (leg).

1. **Attend to precision.** Define isosceles triangle. On the triangle below, classify and appropriately label each side and angle using the terms vertex angle, base angle, leg, base.



2. Draw and cut out an isosceles triangle.

3. Fold your triangle so that the two legs ***coincide***.

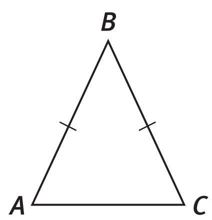
ACADEMIC VOCABULARY

Coincide means to correspond exactly.

4. List the conjectures about the relationships you observe in the angles of your isosceles triangle.

5. What appears to be true about the base angles of the isosceles triangle? Write a conjecture in if-then form. This is known as the ***Isosceles Triangle Theorem***.

6. The proof of the Isosceles Triangle Theorem begins by drawing the bisector of the vertex angle. On the diagram below, draw in the bisector of the vertex angle and write a paragraph proof of the theorem.

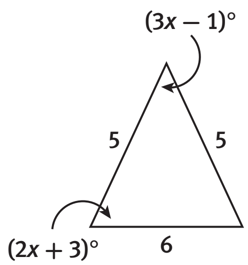


My Notes

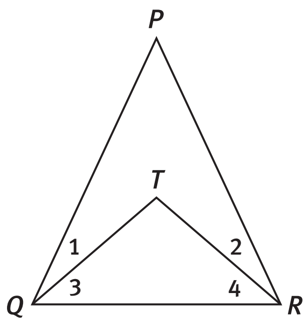
7. The measure of the vertex angle of an isosceles triangle is 80°. What is the measure of a base angle?

8. The measure of one base angle of an isosceles triangle is 25°. What is the measure of the vertex angle?

9. Solve for x.



10. Using the figure below, $\overline{PQ} \cong \overline{PR}$ and $\overline{TQ} \cong \overline{TR}$. Explain why $\angle 1 \cong \angle 2$.



11. Use the Isosceles Triangle Theorem to explain why an equilateral triangle must be equiangular.

12. Carlie said that "if an isosceles triangle is obtuse, then the obtuse angle must be the vertex." Is she correct? If so, justify her statement. If not, give a counterexample.

CONNECT TO HISTORY

Euclid's proof of the Isosceles Triangle Theorem, given as the fifth proposition in Euclid's first book of *The Elements*, is somewhat subtle and has been given the nickname *Pons asinorum*, Latin for "Bridge of Asses." The two reasons why this proposition earned its name are: (1) Euclid's diagram for this theorem looked like a bridge, and (2) this was the first proposition that tested students' ability to understand more advanced concepts in Euclidean geometry. Therefore, this proposition served as a bridge from the trivial portion of Euclidean geometry to the nontrivial portion, and the people who could not cross this bridge were considered to be unintelligent.

My Notes

13. Write the converse of the Isosceles Triangle Theorem and determine whether it will always be true. If yes, explain. If not, give a counterexample.

14. In $\triangle ABC$, $\angle A \cong \angle C$. If $AB = 4x + 25$, $BC = 2x + 45$, and $AC = 3x - 15$, determine the lengths of the three sides.

MATH TIP

Create an organized summary of the properties of triangles.

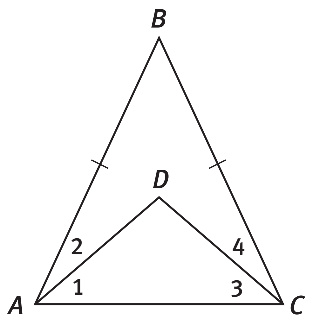
15. Look at the fold in your cutout triangle.

    a. What appears to be true about the segment determined by the fold?

    b. What geometric term(s) can be used to classify this segment?

Check Your Understanding

16. $\overline{AB} \cong \overline{BC}$ and $\angle 4 \cong \angle 2$. Explain why $\triangle ADC$ must be isosceles.



17. An isosceles triangle can be defined as a triangle with at least two congruent sides. Using this definition, what other type of triangle could be described as isosceles? Explain.

LESSON 13-2 PRACTICE

18. $\triangle ABC$ is an isosceles triangle with vertex angle B, $AB = 5x - 28$, $AC = x + 5$, and $BC = 2x + 11$. Determine the length of the base of the triangle.

19. The measure of the vertex angle of an isosceles triangle is 120°. What is the measure of a base angle?

20. Given $\triangle ABC$, $m\angle A = (x + 14)^\circ$, $m\angle B = (4x + 6)^\circ$, and $m\angle C = (15x + 40)^\circ$.
    a. Find the value of x.
    b. Determine the measure of each of the three angles.
    c. Classify $\triangle ABC$ by side length and angle measure.

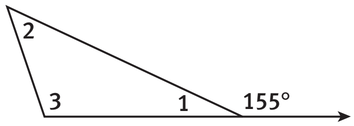
21. **Critique the reasoning of others.** Juan said that he drew a triangle with angles of measure 118°, 30°, and y°. He said that by choosing the correct value of y he could make the triangle an isosceles triangle. Do you agree or disagree? Why?

ACTIVITY 13 PRACTICE

Write your answers on notebook paper. Show your work.

Lesson 13-1

Use the figure for Items 1–3. The measure of $\angle 1$ is equal to the measure of $\angle 2$. Find the measure of the interior angle of the triangle.



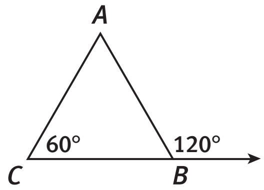
1. $m\angle 1 =$ ______

2. $m\angle 2 =$ ______

3. $m\angle 3 =$ ______

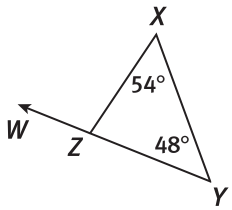
For each triangle, find the indicated angle measure.

4.



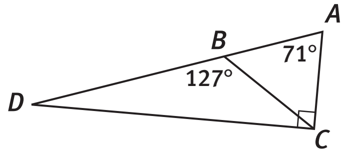
$m\angle A =$ ______

5.



$m\angle WZX =$ ______

6.



$m\angle BCD =$ ______

7. The measure of an exterior angle of a triangle is 125°. The measure of one of its remote interior angles is 65°. What is the measure of the other remote interior angle?

8. The measure of an exterior angle of a triangle is 84°. The measure of one of its remote interior angles is 22°. What are the measures of the other two interior angles of the triangle?

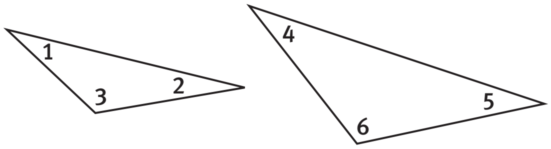
9. Prove the Third Angles Theorem by completing the two-column proof.

Third Angles Theorem: If the measures of two angles of one triangle are equal to the measures of two angles of another triangle, then the measures of the third angles are equal.

Given: $\triangle ABC$ and $\triangle DEF$

$m\angle 1 = m\angle 4$ and $m\angle 2 = m\angle 5$

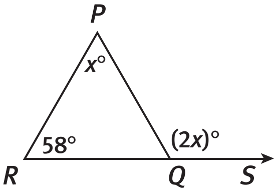
Prove: $m\angle 3 = m\angle 6$



Statements	Reasons
1.	**1.** Given
2. $m\angle 1 + m\angle 2 + m\angle 3 =$ ______	**2.** Triangle Sum Theorem
3. $m\angle 3 =$ ______	**3.** Subtraction Property of Equality
4. ______	**4.** Triangle Sum Theorem
5. $m\angle 6 =$ ______	**5.** ______
6. $m\angle 6 =$ ______	**6.** ______
7. ______	**7.** ______

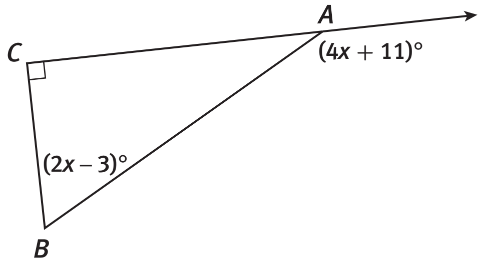
For each triangle, find the value of x. Then find the measure of each interior angle.

10.



$x =$ ______
$m\angle P =$ ______
$m\angle PQR =$ ______
$m\angle B =$ ______

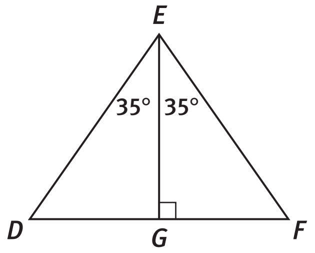
11.



$x =$ ______
$m\angle C =$ ______
$m\angle CAB =$ ______

Lesson 13-2

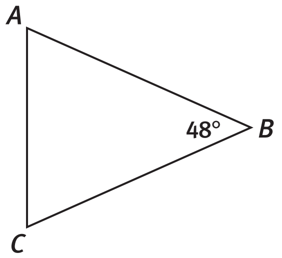
12.



$m\angle EDG =$ ______

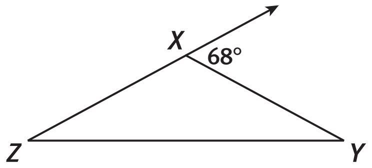
Find the measure of the base angles for each isosceles triangle.

13.



$m\angle A = m\angle C =$ ______

14.



$m\angle Y = m\angle Z =$ ______

15. The measure of each of the base angles of an isosceles triangle is 47°. What is the measure of the third angle?

16. The measure of the vertex angle of an isosceles triangle is 64°. What is the measure of each base angle?

17. What are the interior angle measures of a right isosceles triangle?

18. $\triangle ABC$ is an isosceles triangle with vertex angle B, $AB = 2x + 10$, $AC = 2x$, and $BC = 4x - 16$. Determine the length of the base of the triangle.

19. The measure of the vertex angle of an isosceles triangle is 98°. What is the measure of a base angle?

MATHEMATICAL PRACTICES
Look For and Make Use of Structure

20. You know the measure of the exterior angle of an isosceles triangle which forms a linear pair with the vertex angle. Describe two ways you can find measures of the interior angles of the triangle.