# More About Quadrilaterals

## A 4-gon Conclusion

## Lesson 16-1 Proving a Quadrilateral Is a Parallelogram

### Learning Targets:

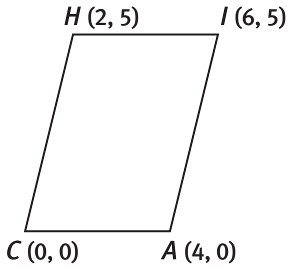
- Develop criteria for showing that a quadrilateral is a parallelogram.
- Prove that a quadrilateral is a parallelogram.

**SUGGESTED LEARNING STRATEGIES:** Think-Pair-Share, Group Presentation, Discussion Groups, Visualization

In a previous activity, the definition of a parallelogram was used to verify that a quadrilateral is a parallelogram by showing that both pairs of opposite sides are parallel.

**1.** Given quadrilateral *CHIA*:

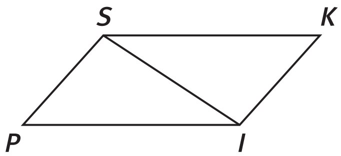
**a.** Find the slope of each side.




**b.** Use the slopes to explain how you know quadrilateral *CHIA* is a parallelogram.

**2.** Given quadrilateral *SKIP* with $SK = IP$ and $KI = SP$.

**a.** $\triangle PSI \cong$ ______________. Explain.




**b.** $\angle SIP \cong$ ____________ and $\angle PSI \cong$ ____________. Explain.

**c.** $\overline{SK} \parallel \overline{IP}$ and $\overline{KI} \parallel \overline{SP}$ because ________________.

**d.** Complete the theorem.

**Theorem** If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a ____________.

**3.** Given quadrilateral *WALK* with coordinates $W(8, 7)$, $A(11, 3)$, $L(4, 1)$, and $K(1, 5)$. Use the theorem in Item 2 to show that $\square WALK$ is a parallelogram.

My Notes

**MATH TIP**

**Slope Formula**

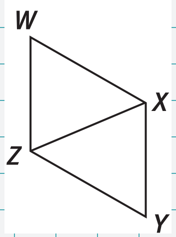
Given $A(x_1, y_1)$ and $B(x_2, y_2)$

Slope of $\overline{AB}$: $m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$

**MATH TIP**

Once a theorem has been proven, it can be used to justify other steps or statements in proofs.

My Notes




**4.** Given $\square WXYZ$ with $\overline{WX} \parallel \overline{ZY}$ and $\overline{WX} \cong \overline{ZY}$.

**a.** $\triangle WZX \cong$ _____. Explain.

**b. Construct viable arguments.** Explain why $\overline{WZ} \parallel \overline{XY}$.

**c.** Complete the theorem.

**Theorem** If one pair of opposite sides of a quadrilateral are congruent and parallel, then the quadrilateral is a _________.

**5.** Given $\square GOLD$ with coordinates $G(-1, 0)$, $O(5, 4)$, $L(9, 2)$, and $D(3, -2)$. Use the theorem in Item 4 to show that $\square GOLD$ is a parallelogram.

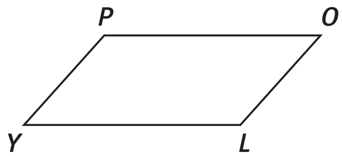
Now you can prove a theorem that can be used to show that a given quadrilateral is a parallelogram.

## Example A

**Theorem** *If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.*

**Given:** $\square POLY$ with $\angle P \cong \angle L$ and $\angle O \cong \angle Y$

**Prove:** $\square POLY$ is a parallelogram.




| Statements | Reasons |
|---|---|
| **1.** $\square POLY$ with $\angle P \cong \angle L$ and $\angle O \cong \angle Y$ | **1.** Given |
| **2.** $m\angle P = m\angle L$ and $m\angle O = m\angle Y$ | **2.** Def. of congruent angles |
| **3.** $m\angle P + m\angle O + m\angle L + m\angle Y = 360°$ | **3.** The sum of the measures of the interior angles of a quadrilateral is 360°. |
| **4.** $m\angle P + m\angle O + m\angle P + m\angle O = 360°$ | **4.** Substitution Property |
| **5.** $2m\angle P + 2m\angle O = 360°$ | **5.** Simplify. |
| **6.** $m\angle P + m\angle O = 180°$ | **6.** Division Property of Equality |
| **7.** $m\angle P + m\angle Y + m\angle P + m\angle Y = 360°$ | **7.** Substitution Property |
| **8.** $2m\angle P + 2m\angle Y = 360°$ | **8.** Simplify. |

My Notes

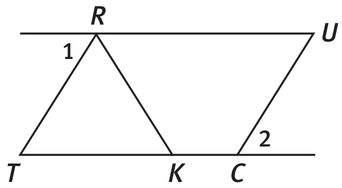
| **Statements** | **Reasons** |
|---|---|
| **9.** $m\angle P + m\angle Y = 180°$ | **9.** Division Property of Equality |
| **10.** $\overline{PY} \parallel \overline{OL}$ and $\overline{PO} \parallel \overline{YL}$ | **10.** If two lines are intersected by a transversal and a pair of consecutive interior angles are supplementary, then the lines are parallel. |
| **11.** $\square POLY$ is a parallelogram. | **11.** Def. of a parallelogram |

## Try These A

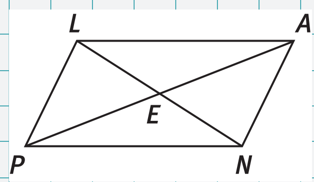
Write a proof using the theorem in Example 1 as the last reason.

**Given:** $\overline{RT} \cong \overline{RK}$
$\angle RKT \cong \angle U$
$\angle 1 \cong \angle 2$

**Prove:** $\square TRUC$




**6.** Given $\square PLAN$ whose diagonals, $\overline{PA}$ and $\overline{LN}$, bisect each other.




Complete the statements.

**a.** $\triangle LEP \cong$ _____ and $\triangle LEA \cong$ _____. Explain.

**b.** $\angle ALE \cong$ _____ and $\angle ELP \cong$ _____. Explain.

**c.** Explain how the information in part b can be used to prove that $\square PLAN$ is a parallelogram.

**d.** Complete the theorem.

**Theorem** If the diagonals of a quadrilateral _____, then the quadrilateral is a _____.

**7.** Given $\square THIN$ with coordinates $T(3, 3)$, $H(5, 9)$, $I(6, 5)$, and $N(4, -1)$.

**a.** Find the coordinates for the midpoint of each diagonal.

**b.** Do the diagonals bisect each other? Explain.

**c.** The best name for this quadrilateral is:
**A.** quadrilateral **B.** kite **C.** trapezoid **D.** parallelogram

My Notes

**8.** Summarize this part of the activity by making a list of the five ways to prove that a quadrilateral is a parallelogram.

## Check Your Understanding

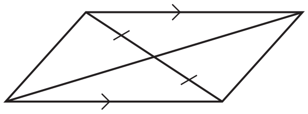
**9.** Explain why showing that only one pair of opposite sides of a quadrilateral are parallel is not sufficient for proving it is a parallelogram.

**10.** Three of the interior angle measures of a quadrilateral are 48°, 130°, and 48°. Is the quadrilateral a parallelogram? Explain.

## LESSON 16-1 PRACTICE

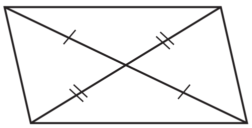
**Make use of structure.** Tell what theorem can be used to prove the quadrilateral is a parallelogram. If there is not enough information to prove it is a parallelogram, write "not enough information."

**11.**

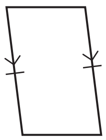


**12.**

**13.**



**14.**



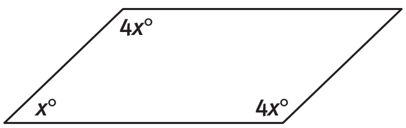
Three vertices of a parallelogram are given. Find the coordinates of the fourth vertex.

**15.** (1, 5), (3, 3), (8, 3)

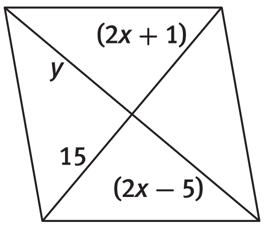
**16.** (−5, 0), (−2, −4), (3, 0)

Find the values of $x$ and $y$ that make the quadrilateral a parallelogram.

**17.**




**18.**



My Notes

**Learning Targets:**

- Develop criteria for showing that a quadrilateral is a rectangle.
- Prove that a quadrilateral is a rectangle.

**SUGGESTED LEARNING STRATEGIES:** Think-Pair-Share, Create Representations, Group Presentation, Discussion Groups

**1.** Complete the following definition.

A rectangle is a parallelogram with ________.

**2. a.** Complete the theorem.

**Theorem** If a parallelogram has one right angle, then it has ________ right angles, and it is a ________.

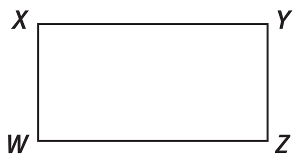
**b.** Use one or more properties of a parallelogram and the definition of a rectangle to explain why the theorem in Item 1 is true.

**3.** Given $\square WXYZ$.

**a.** If $\square WXYZ$ is equiangular, then find the measure of each angle.

**b.** Complete the theorem.

**Theorem** If a quadrilateral is equiangular, then it is a _________.




**4. Make sense of problems.** Identify the hypothesis and the conclusion of the theorem in Item 3. Use the figure in Item 3.

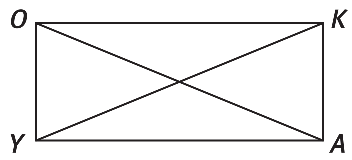
**Hypothesis:**

**Conclusion:**

**5.** Write a proof for the theorem in Item 3.

**My Notes**

**6.** Given ▱$OKAY$ with congruent diagonals, $\overline{OA}$ and $\overline{KY}$.




**a.** List the three triangles that are congruent to $\triangle OYA$, and the reason for the congruence.

**b.** List the three angles that are corresponding parts of congruent triangles and congruent to $\angle OYA$.

**c.** Find the measure of each of the angles in part b.

**d.** Complete the theorem.

**Theorem** If the diagonals of a parallelogram are _______, then the parallelogram is a _______.

**7.** Given ▱$ABCD$ with coordinates $A(1, 0)$, $B(0, 3)$, $C(6, 5)$, and $D(7, 2)$.

**a.** Show that ▱$ABCD$ is a parallelogram.

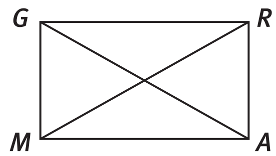
**b.** Use the theorem in Item 6 to show that ▱$ABCD$ is a rectangle.

**8.** Write a two-column proof using the theorem in Item 6 as the last reason.

**Given:** ▱$GRAM$

$\triangle GRM \cong \triangle RGA$

**Prove:** ▱$GRAM$ is a rectangle.




**9.** Summarize this part of the activity by making a list of the ways to prove that a quadrilateral (or parallelogram) is a rectangle.

My Notes

## Check Your Understanding

**10.** Jamie says a quadrilateral with one right angle is a rectangle. Find a counterexample to show that Jamie is incorrect.

**11.** Do the diagonals of a rectangle bisect each other? Justify your answer.

## LESSON 16-2 PRACTICE

Three vertices of a rectangle are given. Find the coordinates of the fourth vertex.

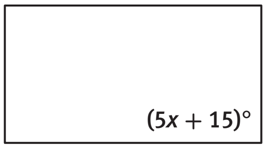
**12.** $(-3, 2), (-3, -1), (3, -1)$

**13.** $(-12, 2), (-6, -6), (4, 2)$

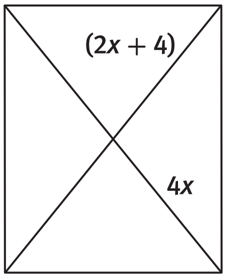
**14.** $(4, 5), (-3, -4), (6, -1)$

Find the value of $x$ that makes the parallelogram a rectangle.

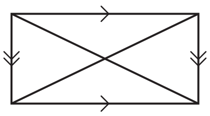
**15.**




**16.**




**17.** **Model with mathematics.** Jill is building a new gate for her yard as shown. How can she use the diagonals of the gate to determine if the gate is a rectangle?



My Notes

## Learning Targets:

- Develop criteria for showing that a quadrilateral is a rhombus.
- Prove that a quadrilateral is a rhombus.

**SUGGESTED LEARNING STRATEGIES:** Think-Pair-Share, Create Representations, Group Presentation, Discussion Groups

**1.** Complete the following definition.

A rhombus is a parallelogram with ____________.

**2. a.** Complete the theorem.

**Theorem** If a parallelogram has two consecutive congruent sides, then it has ______ congruent sides, and it is a _________.

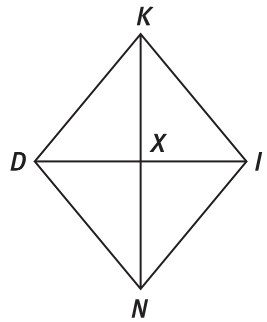
**b.** Use one or more properties of a parallelogram and the definition of a rhombus to explain why the theorem in Item 2a is true.

**3.** Complete the theorem.

**Theorem** If a quadrilateral is equilateral, then it is a _________.

**4.** Write a paragraph proof to explain why the theorem in Item 3 is true.

**5.** Given $\square KIND$ with $\overline{KN} \perp \overline{ID}$.




**a.** List the three triangles that are congruent to $\triangle KXD$, and give the reason for the congruence.

**b.** List all segments congruent to $\overline{KD}$ and explain why.

**c.** Complete the theorem.

**Theorem** If the diagonals of a parallelogram are ________, then the parallelogram is a _________.

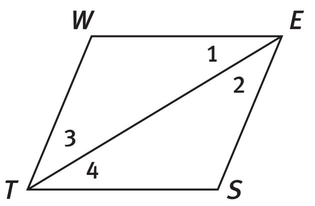
My Notes

**6.** Given ▱*BIRD* with coordinates $B(-2, -3)$, $I(1, 1)$, $R(6, 1)$, and $D(3, -3)$.

**a.** Show that ▱*BIRD* is a parallelogram.

**b.** Use the theorem in Item 5 to show ▱*BIRD* is a rhombus.

**7.** Given ▱*WEST* with $\overline{TE}$ that bisects $\angle WES$ and $\angle WTS$.




**a.** List all angles congruent to $\angle 1$ and explain why.

**b.** In $\triangle WET$, $\overline{WT} \cong$ _______. In $\triangle SET$, $\overline{ST} \cong$ _______. Explain.

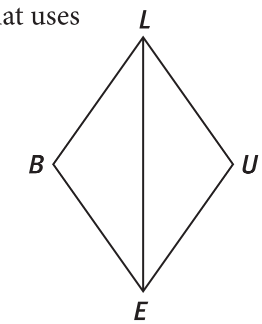
**c.** Complete the theorem.
**Theorem** If a diagonal bisects __________________ in a parallelogram, then the parallelogram is a _______________.

**8. Construct viable arguments.** Write a proof that uses the theorem in Item 7 as the last reason.

**Given:** ▱*BLUE*

$\triangle BLE \cong \triangle ULE$

**Prove:** ▱*BLUE* is a rhombus.



My Notes

**9.** Summarize this part of the activity by making a list of the ways to prove that a quadrilateral is a rhombus.

## Check Your Understanding

**10.** Can a rectangle ever be classified as a rhombus as well? Explain.

## LESSON 16-3 PRACTICE

Three vertices of a rhombus are given. Find the coordinates of the fourth vertex.

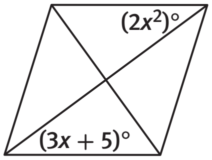
**11.** $(-2, -8)$, $(3, -3)$, $(-9, -7)$

**12.** $(-1, 2)$, $(-1, -1)$, $(2, 1)$

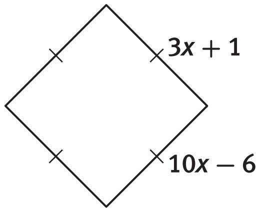
**13.** $(1, 1)$, $(-1, -2)$, $(1, -5)$

**14.** Find the value of $x$ that makes the parallelogram a rhombus.

**a.**




**b.**




**15. Reason quantitatively.** LaToya is using a coordinate plane to design a new pendant for a necklace. She wants the pendant to be a rhombus. Three of the vertices of the rhombus are $(3, 1)$, $(-1, -1)$, and $(1, -2)$. Assuming each unit of the coordinate plane represents one centimeter, what is the perimeter of the pendant? Round your answer to the nearest tenth.

My Notes

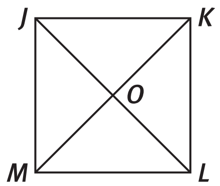
## Learning Targets:

- Develop criteria for showing that a quadrilateral is a square.
- Prove that a quadrilateral is a square.

**SUGGESTED LEARNING STRATEGIES:** Think-Pair-Share, Create Representations, Group Presentation, Discussion Groups

**1.** Given ▱*JKLM*.

**a.** What information is needed to prove that ▱*JKLM* is a square?




**b.** What additional information is needed to prove that ▱*JKLM* is a square? Explain.

**c.** What additional information is needed to prove that rectangle *JKLM* is a square? Explain.

**d.** What additional information is needed to prove that rhombus *JKLM* is a square? Explain.

**2.** Given ▱*DAVE* with coordinates $D(-1, 1)$, $A(0, 7)$, $V(6, 6)$, and $E(5, 0)$. Show that ▱*DAVE* is a square.

My Notes

**3. Critique the reasoning of others.** Several students in a class made the following statements. Decide whether you agree with each statement. If you disagree, change the statement to make it correct.

**a.** A quadrilateral with congruent diagonals must be a rectangle.

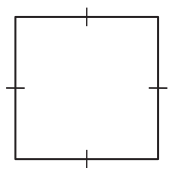
**b.** A parallelogram with two right angles must be a square.

**c.** A quadrilateral with a pair of opposite parallel sides is always a parallelogram.

**d.** A rhombus with four congruent angles is a square.

## Check Your Understanding

**4.** Elena has a garden with congruent sides, as shown below. Describe two different ways to show the garden is square.



## LESSON 16-4 PRACTICE

The coordinates of a parallelogram are given. Determine whether the figure is a square.

**5.** (−2, 3), (3, 3), (3, 0), (−2, 0)

**6.** (0, 1), (−1, 3), (1, 4), (2, 2)

**7.** (3, 6), (6, 2), (−2, 3), (−5, 7)

**8.** (3, 8), (−1, 6), (1, 2), (5, 4)

**9. Express regularity in repeated reasoning.** Find the length of the diagonal of a square with three of its vertices at (1, 0), (0, 0), and (0, 1). Then find the length of the diagonal of a square with three of its vertices at (2, 0), (0, 0), and (0, 2). Finally, find the length of the diagonal of a square with three of its vertices at (3, 0), (0, 0), and (0, 3). Use your findings to make a conjecture about the length of the diagonal of a square with three of its vertices at ($s$, 0), (0, 0), and (0, $s$).

## ACTIVITY 16 PRACTICE

**Write your answers on notebook paper.**
**Show your work.**

### Lesson 16-1

**1.** Given ▱$RSTU$ with coordinates $R(0, 0)$, $S(-2, 2)$, $T(6, 6)$, and $U(8, 4)$.
  **a.** Show that ▱$RSTU$ is a parallelogram by finding the slope of each side.
  **b.** Show that ▱$RSTU$ is a parallelogram by finding the length of each side.
  **c.** Show that ▱$RSTU$ is a parallelogram by showing that the diagonals bisect each other.

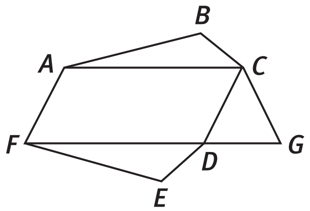
**2.** Write a proof using the theorem in Item 2 of Lesson 16-1 as the last reason.

**Given:** $\triangle ABC \cong \triangle FED$

$\overline{CD} \cong \overline{CG}$

$\overline{CG} \cong \overline{AF}$

**Prove:** ▱$ACDF$



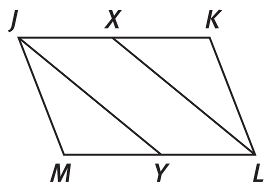

**3.** Write a proof using the theorem in Item 4 of Lesson 16-1 as the last reason.

**Given:** ▱$JKLM$

$X$ is midpt of $\overline{JK}$.

$Y$ is midpt of $\overline{ML}$.

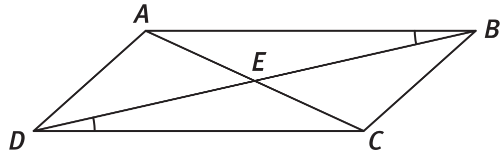
**Prove:** ▱$JXLY$




**4.** Which of the following is not a sufficient condition to prove a quadrilateral is a parallelogram?
  **A.** The diagonals bisect each other.
  **B.** One pair of opposite sides are parallel.
  **C.** Both pairs of opposite sides are congruent.
  **D.** Both pairs of opposite angles are congruent.

**5.** Show that the quadrilateral with vertices $(-2, 3)$, $(-2, -1)$, $(1, 1)$, and $(1, 5)$ is a parallelogram.

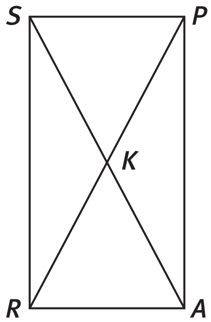
**6.** Which of the following additional pieces of information would allow you to prove that $ABCD$ is a parallelogram?




**A.** $\overline{AD} \parallel \overline{BC}$  **B.** $\overline{AD} \cong \overline{BC}$
**C.** $\overline{AB} \parallel \overline{DC}$  **D.** $\overline{AB} \cong \overline{DC}$

### Lesson 16-2

**7.** Each of the following sets of given information is sufficient to prove that ▱$SPAR$ is a rectangle *except*:



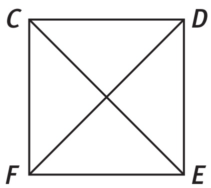

**A.** ▱$SPAR$ and $\angle SPA \cong \angle PAR$
**B.** $SK = KA = RK = KP$
**C.** ▱$SPAR$ and $\angle SKP \cong \angle PKA$
**D.** $\angle RSP \cong \angle SPA \cong \angle PAR \cong \angle ARS$

**8.** Given ▱$FOUR$ with coordinates $F(0, 6)$, $O(10, 8)$, $U(13, 3)$, and $R(3, 0)$. Show that ▱$FOUR$ is not a rectangle.

**9.** Write an indirect proof.

**Given:** $CE \neq DF$

**Prove:** ▱$CDEF$ is not a rectangle.

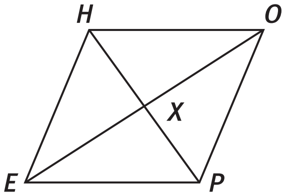



**10.** What is the best name for a quadrilateral if the diagonals are congruent and bisect each other?
  **A.** parallelogram  **B.** rectangle
  **C.** kite  **D.** trapezoid

**11.** Three vertices of a rectangle are $(-4, -3)$, $(8, 3)$, and $(5, 6)$. Show that the diagonals are congruent.

## Lesson 16-3

**12.** Each of the following sets of given information is sufficient to prove that $\square HOPE$ is a rhombus *except*:




**A.** $HX = XP = XE = XO$
**B.** $OH = OP = PE = HE$
**C.** $\square HOPE$ and $\angle HXO \cong \angle OXP$
**D.** $\square HOPE$ and $HE = PE$

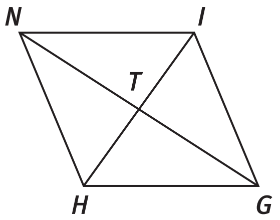
**13.** Given $\square DRUM$ with coordinates $D(-2, -2)$, $R(-3, 3)$, $U(2, 5)$, and $M(3, 0)$. Show that $\square DRUM$ is not a rhombus.

**14.** Write a proof using the theorem in Item 5 of Lesson 16-3 as the last reason.

**Given:** $\square NIGH$

$\triangle NTI \cong \triangle NTH$

**Prove:** $\square NIGH$ is a rhombus.

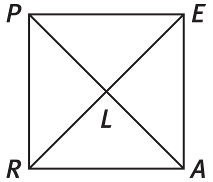



## Lesson 16-4

**15.** Write a proof.

**Given:** $\square PEAR$, $\overline{PE} \perp \overline{EA}$;
$\overline{PE} \cong \overline{AE}$

**Prove:** $\square PEAR$ is a square.




**16.** Given $\square SOPH$ with coordinates $S(-8, 0)$, $O(0, 6)$, $P(10, 6)$, and $H(2, 0)$. What is the best name for this quadrilateral?
**A.** parallelogram **B.** rectangle
**C.** rhombus **D.** square

**17.** What is the best name for an equilateral quadrilateral whose diagonals are congruent?
**A.** parallelogram **B.** rectangle
**C.** rhombus **D.** square

### MATHEMATICAL PRACTICES
#### Look For and Make Use of Structure

**18.** Why is every rhombus a parallelogram but not every parallelogram a rhombus? Why is every square a rectangle but not every rectangle a square? Why is every square a rhombus but not every rhombus a square?