

# Basic Trigonometric Relationships

## ACTIVITY 22

### The Sine of Things to Come

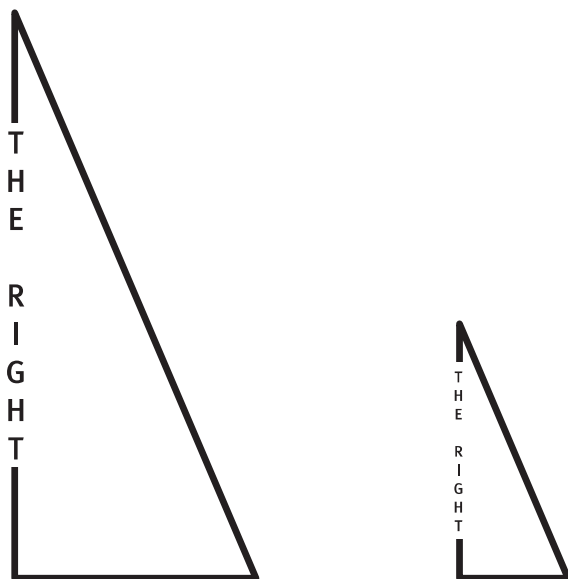
#### Lesson 22-1 Similar Right Triangles

##### Learning Targets:

- Find ratios of side lengths in similar right triangles.
- Given an acute angle of a right triangle, identify the opposite leg and adjacent leg.

**SUGGESTED LEARNING STRATEGIES:** Graphic Organizer, Marking the Text, Create Representations, Close Reading, Interactive Word Wall, Think-Pair-Share

Tricia is a commercial artist working for The Right Angle Company. The company specializes in small business public relations. Tricia creates appealing logos for client companies. In fact, she helped create the logo for her company. The Right Angle Company will use its logo in different sizes for stationery letterhead, business cards, and magazine advertisements. The advertisement and stationery letterhead-size logos are shown below.



Magazine Advertising-Size Logo

Stationery Letterhead-Size Logo

- Use appropriate tools strategically.** Measure the two acute angles and the lengths of the sides of each logo above. Measure the angles to the nearest degree and the sides to the nearest tenth of a centimeter. Be as accurate as possible. Record the results in the table below.

Logo Size	Hypotenuse	Longer Leg	Shorter Leg	Larger Acute Angle	Smaller Acute Angle
Advertisement					
Letterhead					

*continued*

## Lesson 22-1

### Similar Right Triangles

2. Although the measurements can never be exact, the lengths of the sides of any right triangle satisfy the Pythagorean Theorem. Confirm that the Pythagorean Theorem is satisfied by the measurements of the two right triangular logos on the preceding page. Show your work and results but allow for some error due to measurement limitations.
3. The logos are similar triangles. Justify this statement. Then give the scale factor of advertising logo lengths to corresponding letterhead logo lengths.
4. The triangular logo used on The Right Angle Company business cards is also similar to the logos used for advertisements and letterheads. The scale factor of letterhead logo lengths to corresponding business card logo lengths is 2.3:1.
  - a. Determine the length of each side of the business card logo.
  - b. Use a ruler to draw the business card logo to scale in the space below.

## Lesson 22-1

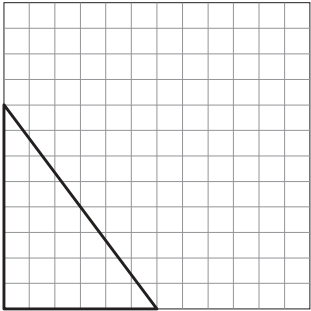
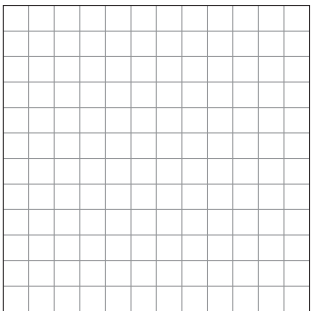
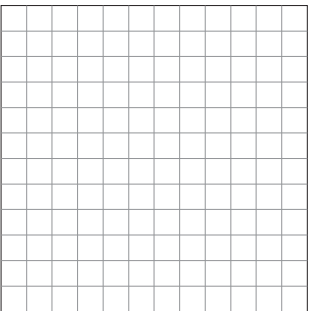
### Similar Right Triangles

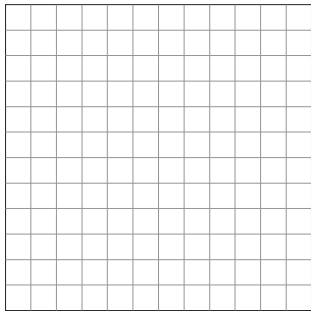
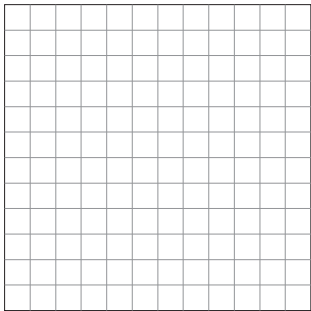
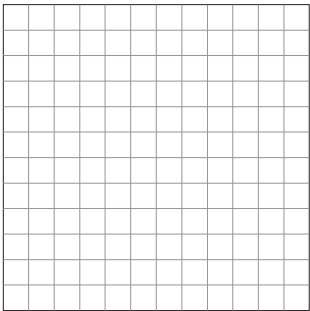
## ACTIVITY 22

continued

Tricia tries to incorporate a right triangle into many of the logos she designs for her clients. As she does, Tricia becomes aware of a relationship that exists between the measures of the acute angles and the ratios of the lengths of the sides of the right triangles.

- Use each grid below to draw a right triangle that has a longer vertical leg of  $L$  units and a shorter horizontal leg of  $S$  units. The first triangle is drawn for you. Use the Pythagorean Theorem to find the length  $H$  of the resulting hypotenuse to the nearest tenth. Record its length in the appropriate place at the bottom of each grid.

Grid A	Grid B	Grid C
$L = 8, S = 6$	$L = 5, S = 4$	$L = 4, S = 3$
		
$H =$	$H =$	$H =$

Grid D	Grid E	Grid F
$L = 10, S = 8$	$L = 12, S = 9$	$L = 6, S = 3$
		
$H =$	$H =$	$H =$

# ACTIVITY 22

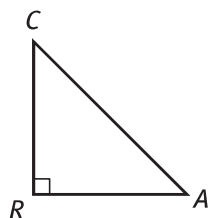
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## Lesson 22-1 Similar Right Triangles

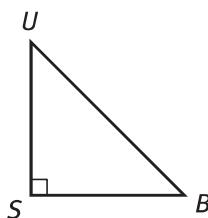
### My Notes

6. Some of the triangles in Grids A–F are similar to each other. Identify the groups of similar triangles using the grid letter and explain below how you know they are similar. You should find a group of three similar triangles and a group of two similar triangles.

In any right triangle, the hypotenuse is opposite the right angle. For each acute angle, one of the right triangle's legs is known as that angle's **opposite leg** and the remaining leg is known as that angle's **adjacent leg**. In  $\triangle CAR$  below, the hypotenuse is  $\overline{AC}$ . For acute  $\angle C$ , side  $\overline{AR}$  is its opposite leg and side  $\overline{RC}$  is its adjacent leg. For acute  $\angle A$ , side  $\overline{RC}$  is its opposite leg and side  $\overline{AR}$  is its adjacent leg.



7. In right  $\triangle BUS$ , identify the opposite leg and the adjacent leg for  $\angle U$ .



## Lesson 22-1

### Similar Right Triangles

## ACTIVITY 22

continued

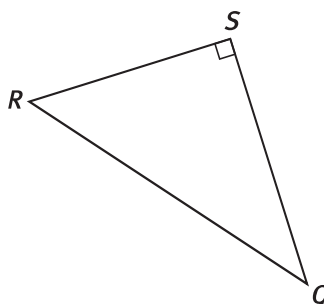
### Check Your Understanding

8. Are all isosceles right triangles similar? Explain.
9. Two right triangles are similar with a scale factor of 3:4.5. The triangle with the shorter hypotenuse has leg lengths of 6 and 8. What is the length of the longer hypotenuse?

### LESSON 22-1 PRACTICE

10. Use  $\triangle QRS$  to find the following.

- a. the leg opposite  $\angle Q$
- b. the leg adjacent to  $\angle Q$
- c. the leg opposite  $\angle R$
- d. the leg adjacent to  $\angle R$
- e. the hypotenuse



11. **Make sense of problems.** Find the scale factor and the unknown side lengths for each pair of similar triangles.

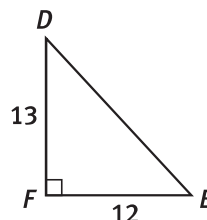
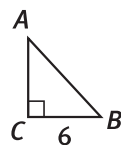
- a.  $\triangle ABC \sim \triangle DEF$

Scale factor \_\_\_\_\_

$AC =$  \_\_\_\_\_

$AB =$  \_\_\_\_\_

$DE =$  \_\_\_\_\_



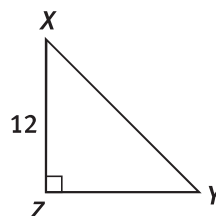
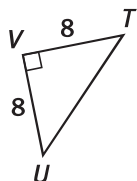
- b.  $\triangle TUV \sim \triangle XYZ$

Scale factor \_\_\_\_\_

$UT =$  \_\_\_\_\_

$YZ =$  \_\_\_\_\_

$XY =$  \_\_\_\_\_



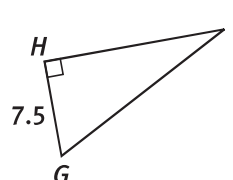
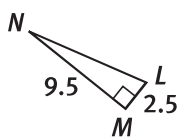
- c.  $\triangle LMN \sim \triangle GHI$

Scale factor \_\_\_\_\_

$NL =$  \_\_\_\_\_

$HI =$  \_\_\_\_\_

$GI =$  \_\_\_\_\_



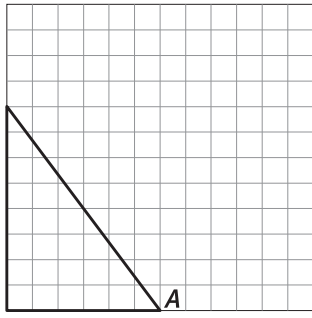
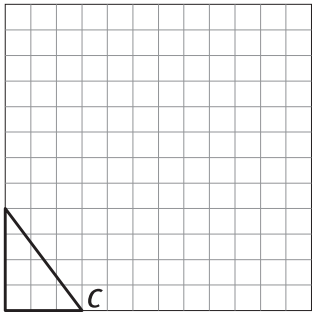
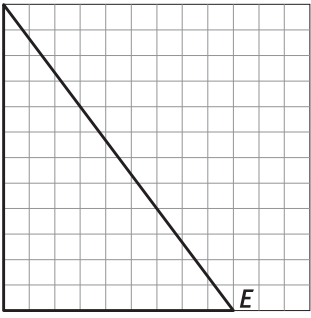
My Notes

**Learning Targets:**

- Understand the definitions of sine, cosine, and tangent ratios.
- Calculate the trigonometric ratios in a right triangle.
- Describe the relationship between the sine and cosine of complementary angles.

**SUGGESTED LEARNING STRATEGIES:** Create Representations, Graphic Organizer, Look for a Pattern, Quickwrite, Interactive Word Wall

1. One group of similar triangles, identified in Item 5 of the previous lesson, is shown on the grids below. For each right triangle, the vertex opposite the longer leg has been named with the same letter as the grid. Determine the ratios in the table and write the ratios in lowest terms.

Grid A	Grid C	Grid E
$L = 8, S = 6$	$L = 4, S = 3$	$L = 12, S = 9$
		
$H = 10$	$H = 5$	$H = 15$

	$\frac{\text{Length of Opposite Leg}}{\text{Length of Hypotenuse}}$	$\frac{\text{Length of Adjacent Leg}}{\text{Length of Hypotenuse}}$	$\frac{\text{Length of Opposite Leg}}{\text{Length of Adjacent Leg}}$
$\angle A$			
$\angle C$			
$\angle E$			

## Lesson 22-2

### Trigonometric Ratios

*continued*

2. For each of the triangles in Item 1, use your protractor to find the measure of the larger acute angle.

	$\angle A$	$\angle C$	$\angle E$
Measure of Larger Acute Angle			

3. In Item 2, you found that the measures of each of the three angles are the same. If, in another right triangle, the measure of the larger acute angle was the same as the measures of  $\angle A$ ,  $\angle C$ , and  $\angle E$ , what would you expect the following ratios to be?

$$\text{a. } \frac{\text{length of opposite leg}}{\text{length of hypotenuse}} =$$

$$\mathbf{b.} \frac{\text{length of adjacent leg}}{\text{length of hypotenuse}} =$$

c.  $\frac{\text{length of opposite leg}}{\text{length of adjacent leg}} =$

4. Explain how you reached your conclusions in Item 3.

The ratio of the lengths of two sides of a right triangle is a **trigonometric ratio**. The three basic trigonometric ratios are **sine**, **cosine**, and **tangent**, which are abbreviated **sin**, **cos**, and **tan**.

- 5. Use appropriate tools strategically.** Use a scientific or graphing calculator to evaluate each of the following to the nearest tenth. Make sure your calculator is in DEGREE mode.

**a.**  $\sin 53^\circ =$

**b.**  $\cos 53^\circ =$

**c.**  $\tan 53^\circ =$

6. In each column of the table in Item 1, the ratios that you wrote are equal. Express the ratios from the three columns as decimal numbers rounded to the nearest tenth.

$$\text{a. } \frac{\text{length of opposite leg}}{\text{length of hypotenuse}} =$$

**b.**  $\frac{\text{length of adjacent leg}}{\text{length of hypotenuse}} =$

c.  $\frac{\text{length of opposite leg}}{\text{length of adjacent leg}} =$

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## My Notes

My Notes

7. Compare your answers to Items 5 and 6. Then describe each of the ratios below in terms of sin, cos, and tan. Assume that the ratios represent sides of a right triangle in relation to acute  $\angle X$ .

a.  $\frac{\text{length of opposite leg}}{\text{length of hypotenuse}} =$

b.  $\frac{\text{length of adjacent leg}}{\text{length of hypotenuse}} =$

c.  $\frac{\text{length of opposite leg}}{\text{length of adjacent leg}} =$

8. a. For  $\triangle ABC$ , write the ratios in simplest form.

$\sin A =$

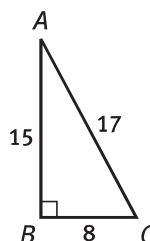
$\sin C =$

$\cos A =$

$\cos C =$

$\tan A =$

$\tan C =$



- b. How are the two acute angles of  $\triangle ABC$  related?
- c. What is the relationship between the sine and cosine of complementary angles?
- d. If you know  $\sin 68^\circ \approx 0.93$ , what other trigonometric ratio do you know?

CONNECT TO AP

Trigonometric ratios are used to define functions that model periodic phenomena like the hours of daylight.



## Lesson 22-2

### Trigonometric Ratios

## ACTIVITY 22

continued

### Check Your Understanding

- Triangle  $ABC$  is a  $30^\circ$ - $60^\circ$ - $90^\circ$  triangle. Explain how to write  $\sin 30^\circ$  as a ratio in simplest form without knowing the length of any side of the triangle.
- Suppose you know that  $\triangle RST$  is a right triangle with a right angle at  $\angle R$ . If  $\cos S = 0.67$ , what other trigonometric ratio can you write?
- Given  $\sin B = \frac{5}{13}$ , draw a right triangle  $ABC$  with right angle  $C$  and label the side lengths.
  - Determine the length of the missing side.
  - Determine  $\cos B$ .
  - What is  $\tan B$ ?

### LESSON 22-2 PRACTICE

- Use a calculator to find each of the following. Round each value to the nearest hundredth.

$$\sin 48^\circ = \underline{\hspace{2cm}}$$

$$\tan 65^\circ = \underline{\hspace{2cm}}$$

$$\cos 12^\circ = \underline{\hspace{2cm}}$$

$$\sin 90^\circ = \underline{\hspace{2cm}}$$

- Write each ratio in simplest form.

$$\sin X = \underline{\hspace{2cm}}$$

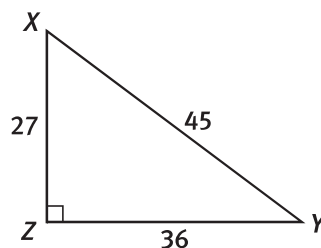
$$\cos X = \underline{\hspace{2cm}}$$

$$\sin Y = \underline{\hspace{2cm}}$$

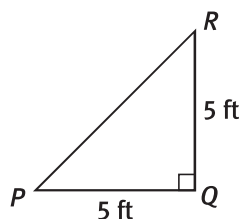
$$\cos Y = \underline{\hspace{2cm}}$$

$$\tan X = \underline{\hspace{2cm}}$$

$$\tan Y = \underline{\hspace{2cm}}$$



- Attend to precision.** Elena was asked to write an explanation of how to find  $\tan P$ . She wrote, "To find the tangent of  $\angle P$ , I found the ratio of the length of the side opposite  $\angle P$  to the length of the side adjacent to  $\angle P$ . Since these sides have the same length,  $\tan P = 1$  ft." Critique Elena's statement. Is there anything she should have written differently? If so, what?



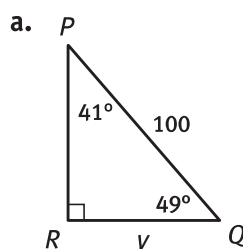
My Notes

**Learning Targets:**

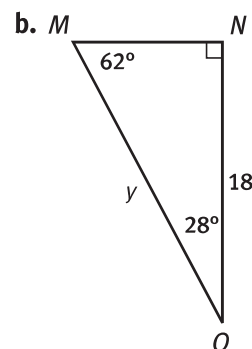
- Use trigonometric ratios to find unknown side lengths in right triangles.
- Solve real-world problems using trigonometric ratios.

**SUGGESTED LEARNING STRATEGIES:** Quickwrite, Create Representations, Identify a Subtask

1. For each of the following triangles, determine the ratios requested. Then use a scientific or graphing calculator to evaluate each trigonometric function to the nearest thousandth and solve each equation for  $y$ . Round final answers to the nearest tenth.



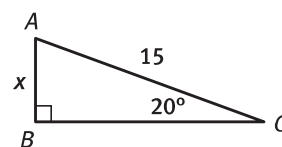
$\sin 41^\circ =$



$\cos 28^\circ =$

2. Use your knowledge of trigonometric functions to find the value of  $x$  in  $\triangle ABC$ .

- a. Choose an acute angle in  $\triangle ABC$ .



- b. Identify sides as opposite, adjacent, or hypotenuse with respect to the acute angle chosen.

- c. Use the sides to choose an appropriate trigonometric function.

- d. Write an equation using the identified sides, acute angle, and trigonometric function chosen.

- e. Solve for  $x$ .

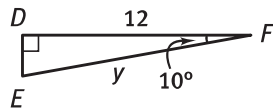
## Lesson 22-3

### Using Trigonometric Ratios

## ACTIVITY 22

continued

3. Use your knowledge of trigonometric functions to find the value of  $y$  in the triangle below.



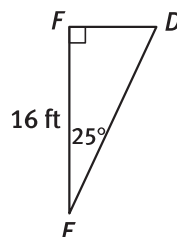
- a. Choose an acute angle in  $\triangle DEF$  and identify sides as adjacent, opposite, or hypotenuse with respect to the angle you chose.
- b. Use the sides to choose an appropriate trigonometric ratio and write an equation using the identified sides, acute angle, and trigonometric function. Then solve for  $y$ .
4. **Make sense of problems.** Tricia did such an exceptional job creating logos that she was given the task of making a banner and representing her company at a job fair. When Tricia got to the job fair, she was relieved to see there was a ladder she could use to hang the banner. While Tricia waited for someone to help her, she leaned the 12-foot ladder against the wall behind the booth. The ladder made an angle of  $75^\circ$  with the floor.
- a. Use the information above to draw and label a right triangle to illustrate the relationship between the ladder and the wall.
- b. Set up and solve an equation to find how far up the wall the top of the ladder reaches.
- c. Find the distance from the base of the wall to the base of the ladder using two different methods.

### My Notes

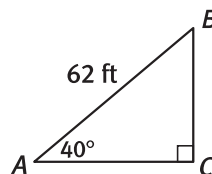
My Notes

Check Your Understanding

5. Explain how to find the length of the hypotenuse of  $\triangle DEF$  without using the Pythagorean Theorem.



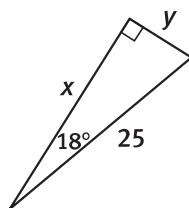
6. Jo says she can find  $BC$  using the equation  $\sin 40^\circ = \frac{BC}{62}$ . Liam says he can find  $BC$  using the equation  $\cos 50^\circ = \frac{BC}{62}$ . Who is correct? Explain.



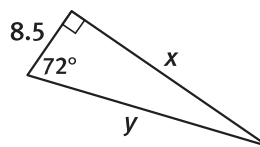
LESSON 22-3 PRACTICE

7. Find each unknown side length.

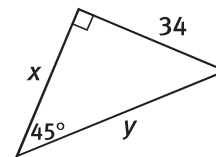
a.  $x = \underline{\hspace{1cm}}$   
 $y = \underline{\hspace{1cm}}$



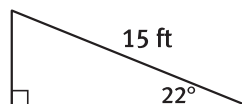
b.  $x = \underline{\hspace{1cm}}$   
 $y = \underline{\hspace{1cm}}$



c.  $x = \underline{\hspace{1cm}}$   
 $y = \underline{\hspace{1cm}}$



8. **Reason quantitatively.** Jackson has the triangular garden shown in the figure. Find the perimeter and area of the garden. Be sure to check that your answers are reasonable.



9. **Construct viable arguments.** The longer diagonal of a rhombus measures 20 cm. The rhombus has an angle that measures  $100^\circ$ . Determine the perimeter of the rhombus, to the nearest tenth. Explain how you found the answer.

**Learning Targets:**

- Calculate angle measures from trigonometric ratios.
- Solve right triangles.

**SUGGESTED LEARNING STRATEGIES:** Predict and Confirm, Summarizing, Quickwrite

- In a right triangle  $ABC$  with acute angle  $A$ , you know that  $\sin A = \frac{\sqrt{3}}{2}$ .
  - Draw a possible right triangle  $ABC$ .
  - Determine what must be true about  $\angle A$  using what you know about special right triangles.

In Item 1, you are given the sine of acute angle  $A$  and are asked to find the angle whose sine is equal to that ratio. In other words, you are finding an inverse sine function. This is written as  $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60^\circ$ . The expression  $\sin^{-1} x$  is read as “the inverse sine of  $x$ .”

The **inverse trigonometric functions** for sine, cosine, and tangent are defined as follows:

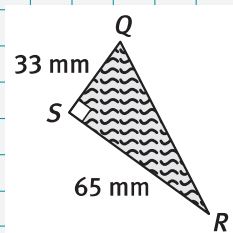
Inverse Trig Functions
If $\sin A = x$ , then $\sin^{-1} x = m\angle A$ .
If $\cos A = x$ , then $\cos^{-1} x = m\angle A$ .
If $\tan A = x$ , then $\tan^{-1} x = m\angle A$ .

- Use a scientific or graphing calculator to evaluate each of the following to the nearest tenth. Make sure your calculator is in DEGREE mode.
  - $\sin^{-1} \frac{1}{2} =$
  - $\cos^{-1} \frac{1}{2} =$
  - $\tan^{-1} \frac{1}{2} =$
  - $\sin^{-1} \frac{3}{4} =$
  - $\cos^{-1} \frac{3}{4} =$
  - $\tan^{-1} \frac{3}{4} =$

Using known measures to find all the remaining unknown measures of a right triangle is known as *solving a right triangle*.

**My Notes**

My Notes



Tricia designs the logo shown for one of her clients. She needs to know all the missing dimensions and angle measures of the logo.

3. **Model with mathematics.** Use an inverse trigonometric function to find the measure of angle  $Q$ .
4. Describe two ways to find the measure of angle  $R$ , without finding the length of the hypotenuse. Find the angle measure using both methods.
5. Describe two ways to find the length of the hypotenuse. Find the length using both methods.

Check Your Understanding

6. What is the minimal amount of information needed to solve a right triangle? Explain.
7. Explain the difference between the two expressions  $\sin 15^\circ$  and  $\sin^{-1}(0.2558)$ . How are the expressions related?

LESSON 22-4 PRACTICE

8. Angle  $X$  is an acute angle in a right triangle. What measure of angle  $X$  makes each statement true? Round angle measures to the nearest tenth.

a.  $\cos X = 0.59$

b.  $\tan X = 3.73$

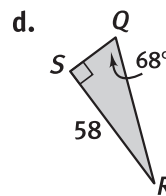
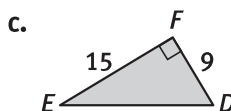
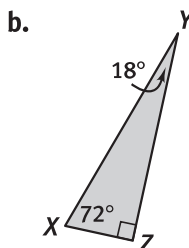
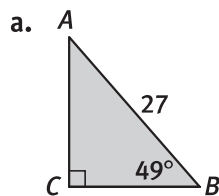
c.  $\sin X = 0.87$

d.  $\tan X = 0.18$

e.  $\cos X = 0.02$

f.  $\sin X = 0.95$

9. Solve each right triangle if possible. Round your measures to the nearest tenth.



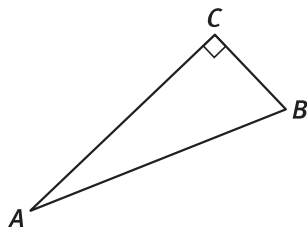
10. **Construct viable arguments.** Without using a calculator, explain how you can find the value of  $\tan^{-1}(1)$ .
11. Consider rhombus  $ABCD$ , where  $\overline{AC} = 8$  in. and  $\overline{BD} = 12$  in. What are the measures of the sides and angles of the rhombus? Round your answers to the nearest tenth.

# ACTIVITY 22 PRACTICE

Write your answers on notebook paper.  
Show your work.

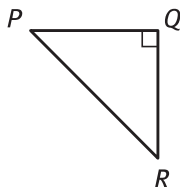
## Lesson 22-1

1. Which side of  $\triangle ABC$  is the side opposite angle  $A$ ?

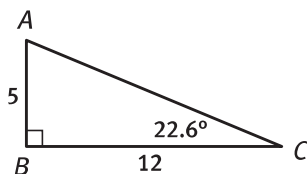


- A.  $\overline{AB}$   
B.  $\overline{AC}$   
C.  $\overline{BC}$

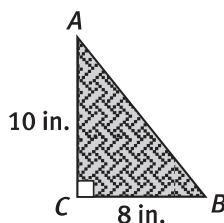
2. In  $\triangle PQR$ , identify the hypotenuse, adjacent leg, and opposite leg for  $\angle R$ .



3. a. Find the missing measures in the given triangle.

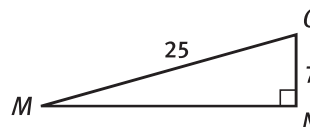


- b. Draw and label a triangle similar to the triangle given in Part a. Include each side length and angle measure.  
c. State the scale factor of the triangle given in Part a to the triangle you drew in Part b.
4. Lisa wants to make a larger bandana similar to the bandana shown. If the shorter leg of the larger bandana is 15 inches, how long is its hypotenuse?



## Lesson 22-2

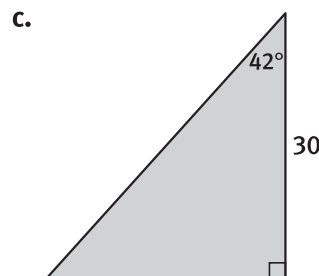
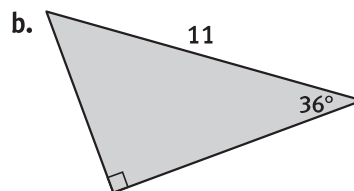
5. Use your calculator to evaluate the following. Round to 3 decimal places.  
a.  $\cos 54^\circ$   
b.  $\sin 12^\circ$   
c.  $\tan 67^\circ$
6. Find each of the following ratios. Write each ratio in simplest form.



- a.  $\sin M$   
b.  $\cos O$   
c.  $\tan M$   
d.  $\sin O$   
e.  $\cos O$   
f.  $\tan O$
7. Which expression is equivalent to  $\cos 25^\circ$ ?  
A.  $\cos 65^\circ$   
B.  $\tan 65^\circ$   
C.  $\sin 25^\circ$   
D.  $\sin 65^\circ$

## Lesson 22-3

8. Find the perimeter and area of each triangle.



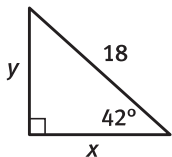
## ACTIVITY 22

continued

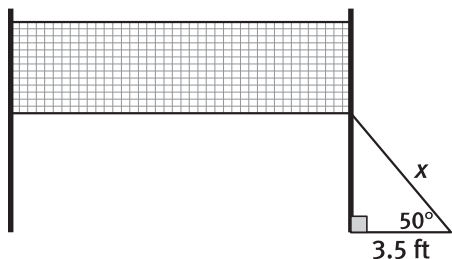
## Basic Trigonometric Relationships

### The Sine of Things to Come

9. Find  $x$  and  $y$ . Round final answers to tenths.



10. A badminton net is tethered to the ground with a strand of rope that forms a  $50^\circ$  angle with the ground. What is the length of the rope,  $x$ ?



### Lesson 22-4

11. Use a scientific or graphing calculator to evaluate each of the following to the nearest tenth.

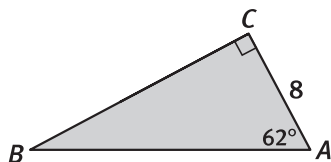
a.  $\sin^{-1} \frac{1}{2} =$

b.  $\cos^{-1} \frac{1}{4} =$

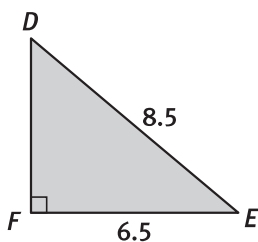
c.  $\tan^{-1} 0.6 =$

12. Solve each right triangle if possible. Round your measures to the nearest tenth.

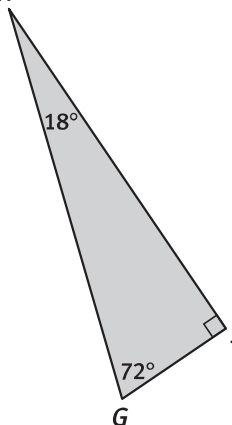
a.



b.



c. H



13. A skateboard ramp has a slope of  $\frac{4}{9}$ . What is the measure of the angle the ramp forms with the ground?

14. Which expression is equivalent to  $\sin^{-1}(0.5)$ ?

A.  $\cos^{-1}(0.5)$

B.  $\frac{\cos^{-1}(0.5)}{\tan^{-1}(0.5)}$

C.  $90^\circ - \cos^{-1}(0.5)$

D.  $90^\circ - \tan^{-1}(0.5)$

### MATHEMATICAL PRACTICES

#### Look For and Make Use of Structure

15. Compare the values of the sine and cosine ratios as the measure of an angle increases from  $0^\circ$  to  $90^\circ$ .



# The Law of Sines and Cosines

## ACTIVITY 23

### There Ought to Be a Law Lesson 23-1 The Law of Sines

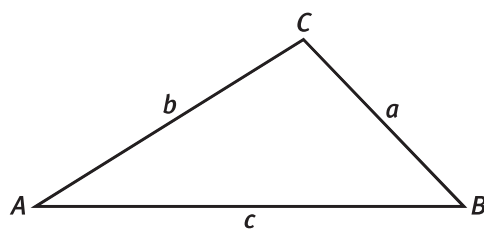
#### Learning Targets:

- Prove the Law of Sines.
- Apply the Law of Sines.

**SUGGESTED LEARNING STRATEGIES:** Create Representations, Graphic Organizer, Predict and Confirm, Visualization, Think-Pair-Share

The location of a fire spotted from two fire observation towers can be determined using the distance between the two towers and the angle measures from the towers to the fire. This process is known as **triangulation**.

You have already solved right triangles. In this lesson you will learn how to solve any triangle.



1. Begin with triangle  $ABC$ . Draw altitude  $\overline{CD}$  from vertex  $C$  to  $\overline{AB}$ . Label the altitude  $h$ .
2. What two right triangles are formed?
3. **Reason abstractly.** You can use the triangles you just formed to write some trigonometric ratios.
  - a. Write a ratio for  $\sin A$ .

b. Write a ratio for  $\sin B$ .

4. a. Solve each equation from Item 3 for  $h$ .

b. Set the values for  $h$  equal.

- c. Complete the following statement.

$$\frac{\sin A}{\square} = \frac{\sin B}{\square}$$

#### My Notes

#### ACADEMIC VOCABULARY

**Triangulation** is also used in surveying, navigation, and cellular communications.

#### MATH TIP

An altitude of a triangle is the perpendicular segment from a vertex to the opposite side.

#### MATH TIP

In a triangle, the side opposite  $\angle A$  is  $a$ , the side opposite  $\angle B$  is  $b$ , and so on.

My Notes

In a similar way, you can show  $\frac{\sin A}{a} = \frac{\sin C}{c}$  and  $\frac{\sin B}{b} = \frac{\sin C}{c}$ .

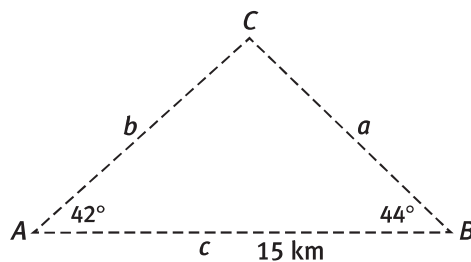
You have just derived the **Law of Sines**.

Below is a formal statement of the Law of Sines.

For any triangle  $ABC$ , with side lengths  $a$ ,  $b$ , and  $c$ ,  $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ .

**Example A**

Fire spotters at stations located at  $A$  and  $B$  notice a fire at location  $C$ . What is the distance between station  $A$  and the fire?



**Step 1:** Find  $m\angle C$ .

$$\begin{aligned} m\angle C &= 180^\circ - 42^\circ - 44^\circ \\ &= 94^\circ \end{aligned}$$

**Step 2:** Use the Law of Sines.

$$\begin{aligned} \frac{\sin B}{b} &= \frac{\sin C}{c} \\ \frac{\sin 44^\circ}{b} &= \frac{\sin 94^\circ}{15} && \text{Substitute.} \\ 15 \sin 44^\circ &= b \cdot \sin 94^\circ && \text{Cross Products Property} \\ b &= \frac{15 \sin 44^\circ}{\sin 94^\circ} && \text{Solve for } b = AC. \\ b &\approx 10.4 && \text{Simplify.} \end{aligned}$$

**Solution:** The distance between station  $A$  and the fire is about 10.4 km.

**MATH TIP**

The Cross Products Property states that, for real numbers  $a$ ,  $b$ ,  $c$ , and  $d$ ,  $b \neq 0$  and  $c \neq 0$ , if  $\frac{a}{b} = \frac{c}{d}$ , then  $ad = bc$ .

## Lesson 23-1

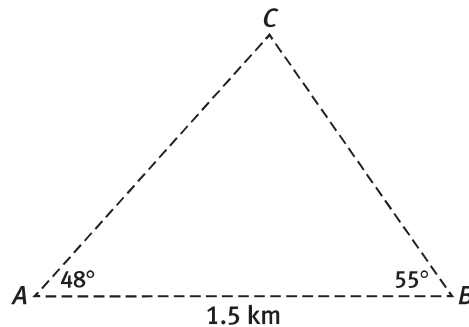
### The Law of Sines

## ACTIVITY 23

continued

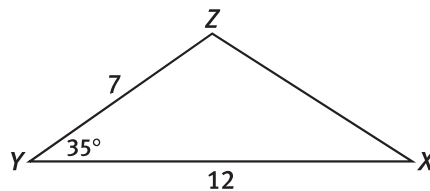
### Try These A

Two boaters located at points  $A$  and  $B$  notice a lighthouse at location  $C$ . What is the distance between the boater located at point  $B$  and the lighthouse? Round to the nearest tenth.



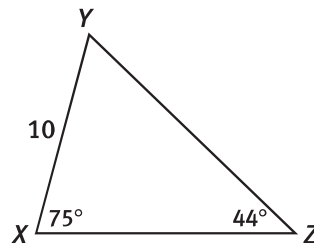
### Check Your Understanding

- Reason quantitatively.** Show that the Law of Sines is true for a  $45^\circ$ - $45^\circ$ - $90^\circ$  triangle with leg lengths of 1.
- Can you use the Law of Sines to find  $ZX$ ? Explain.

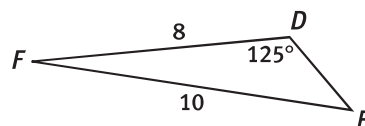


### LESSON 23-1 PRACTICE

- Find each measure. Round to the nearest tenth.
  - $YZ = \underline{\hspace{2cm}}$



- $DE = \underline{\hspace{2cm}}$



- Construct viable arguments.** When can you use the Law of Sines to find the measure of an angle of a triangle? Explain your thinking.

### My Notes

### MATH TIP

You may need to use the Law of Sines to find an angle measure *before* finding the measure of a side. This may require the use of an inverse trigonometric function.

My Notes

TECHNOLOGY TIP

You can also use geometry software to complete Item 1.

Learning Targets:

- Understand when the ambiguous case of the Law of Sines occurs.
- Solve problems using the Law of Sines.

**SUGGESTED LEARNING STRATEGIES:** Create Representations, Use Manipulatives, Predict and Confirm

Fire spotters at stations located at  $Q$  and  $R$  notice a fire at location  $S$ . The distance between stations  $Q$  and  $R$  is 15 km, and the distance between station  $R$  and the fire is 12 km. Station  $Q$  forms a  $38^\circ$  angle with station  $R$  and the fire. What is the distance between station  $Q$  and the fire?

1. **Use appropriate tools strategically.** Cut straws or coffee stirrers that are 15 cm and 12 cm long to model the distances between the locations. Use a protractor to form a  $38^\circ$  angle at vertex  $Q$ . How many different triangles can you make using a  $38^\circ$  angle and side lengths of 15 cm and 12 cm? Sketch the triangles.

2. Why could you form two different triangles in Item 1?

3. Is there an SSA criterion for proving two triangles are congruent? How does this help explain your answer to Item 2?

The problem you just explored is an example of the ambiguous case of the Law of Sines. If you know an acute angle measure of a triangle and the side that is opposite the angle is shorter than the other side length you know, then two triangles can be formed.

4. You know two side lengths and an angle measure. Use the Law of Sines to complete the equation to find the measure of angle  $S$ . Round to the nearest degree.

a.  $\frac{\sin 38^\circ}{\square} = \frac{\sin S^\circ}{\square}$

b.  $m\angle S \approx \square^\circ$

## Lesson 23-2

### The Ambiguous Case

## ACTIVITY 23

continued

5. There are two angles between  $0^\circ$  and  $180^\circ$  whose sine is 0.7660, one acute and one obtuse. Your calculator gives you the measure of the acute angle,  $50^\circ$ . The obtuse angle uses  $50^\circ$  as a reference angle.

- a. Find the measure of the obtuse angle whose sine is 0.7660.

$$m\angle S = 180^\circ - 50^\circ = \boxed{\phantom{00}}^\circ$$

- b. Now find the measure of  $\angle R$  for the two possible triangles.

$$m\angle R = 180^\circ - 38^\circ - \boxed{\phantom{00}}^\circ = \boxed{\phantom{00}}^\circ$$

$$m\angle R = 180^\circ - 38^\circ - \boxed{\phantom{00}}^\circ = \boxed{\phantom{00}}^\circ$$

- c. Use the Law of Sines to complete the equations to find the two possible values of  $QS$ . Round to the nearest tenth.

$$\frac{\sin 38^\circ}{12} = \frac{\sin \boxed{\phantom{00}}^\circ}{QS} \quad \frac{\sin 38^\circ}{12} = \frac{\sin \boxed{\phantom{00}}^\circ}{QS}$$

$$QS = \underline{\hspace{2cm}} \text{ or } QS = \underline{\hspace{2cm}}$$

### Check Your Understanding

6. Given:  $\triangle ABC$  with  $m\angle B = 40^\circ$ ,  $AB = 12$ ,  $AC = 8$ . Find two possible values for each measure.
- $m\angle C$
  - $m\angle A$
  - $BC$

### LESSON 23-2 PRACTICE

7. Given:  $\triangle ABC$  with  $m\angle A = 70^\circ$ ,  $BC = 85$ ,  $AB = 88$ . Find two possible values for each measure.
- $m\angle C$
  - $AC$
8. Given:  $\triangle ABC$  with  $m\angle A = 40^\circ$ ,  $BC = 26$ ,  $AB = 32$ . Find two possible values for each measure.
- $m\angle C$
  - $AC$
9. **Reason abstractly.** Three radio towers are positioned so that the angle formed at vertex  $A$  is  $65^\circ$ . The distance between tower  $A$  and tower  $B$  is 28 miles. The distance between tower  $B$  and tower  $C$  is 26 miles. What are the two possible distances between tower  $A$  and tower  $C$ ? Draw a diagram to support your answers. Round to the nearest whole number.

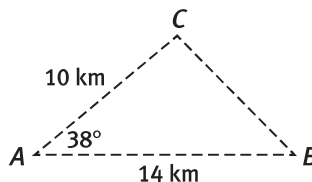
My Notes

**Learning Targets:**

- Prove the Law of Cosines.
- Solve problems using the Law of Cosines.

**SUGGESTED LEARNING STRATEGIES:** Create Representations, Predict and Confirm, Think-Pair-Share

Fire spotters at stations located at  $A$  and  $B$  notice a fire at location  $C$ . What is the distance between station  $B$  and the fire?



1. Can you use the Law of Sines to solve the problem? Explain.

You need a different relationship to solve this problem. You can solve the problem using the **Law of Cosines**.

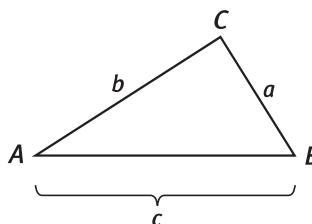
For any triangle  $ABC$ , with side lengths  $a$ ,  $b$ , and  $c$ ,

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Follow these steps to prove the Law of Cosines.



2. Begin with triangle  $ABC$ . Draw altitude  $\overline{CD}$  from vertex  $C$  to  $\overline{AB}$ .
  - a. Label the altitude  $h$ .
  - b. Label  $\overline{AB}$  as  $c$ .
  - c. Label  $\overline{AD}$  in terms of  $x$  and  $c$ .

3. Complete the following ratio.

$$\cos A = \frac{x}{\square}$$

## Lesson 23-3

### The Law of Cosines

## ACTIVITY 23

continued

### My Notes

### MATH TIP

The *Pythagorean Theorem* states that the square of the length of the hypotenuse of a right triangle is equal to the sum of the squares of the lengths of the legs of the right triangle. In other words, for a right triangle with hypotenuse  $c$  and legs  $a$  and  $b$ ,  $c^2 = a^2 + b^2$ .

4. Solve the equation from Item 3 for  $x$ .
5. Use the Pythagorean Theorem to complete the following statements.
  - a.  $x^2 + \square = b^2$
  - b.  $(\square)^2 + h^2 = a^2$
6. **Make sense of problems.** Solve for  $h^2$  in each equation. Expand the equation from Item 5 and set them equal to each other. Then use substitution to prove the Law of Cosines.
7. Suppose  $\angle B$  was a right angle. Show how the Law of Cosines would relate to the Pythagorean Theorem.

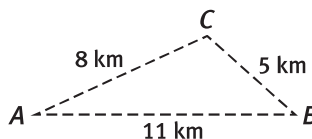
The Law of Cosines is a generalization of the Pythagorean Theorem.

Now you can solve the problem that was posed at the beginning of the lesson.

8. Fire spotters at stations located at  $A$  and  $B$  notice a fire at location  $C$ . What is the distance between station  $B$  and the fire? Round to the nearest tenth.
  - a. Use the Law of Cosines to complete the equation.
 
$$CB^2 = 10^2 + \square^2 - 2(\square)(\square)\cos(\square^\circ)$$
  - b. Solve the equation for  $CB$ . Round to the nearest tenth.  
The distance between station  $B$  and the fire is \_\_\_\_\_.

My Notes

9. Fire station towers are located at points  $A$ ,  $B$ , and  $C$ . The distances between the towers are known. Spotters need to know the measures of angles  $A$ ,  $B$ , and  $C$ . What is the measure of angle  $C$ ? Round to the nearest degree.



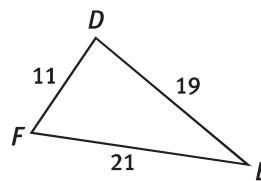
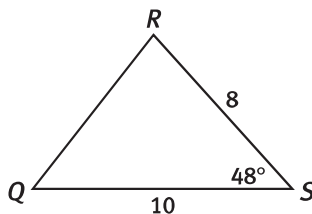
- Which equation of the Law of Cosines do you use to find the measure of angle  $C$ ?
- Write the equation.
- What is the measure of angle  $C$ ?

Check Your Understanding

- Given  $\triangle XYZ$  and the lengths of  $\overline{XY}$  and  $\overline{XZ}$ , do you have enough information to use the Law of Cosines to find  $m\angle X$ ? Explain.
- An equilateral triangle has sides of length 1. Use the Law of Cosines to show that each angle of the triangle measures  $60^\circ$ .

LESSON 23-3 PRACTICE

- Make use of structure.** Once you know the measure of angle  $C$  in Item 9, describe two ways to find the measures of the remaining angles of the triangle.
- Find each measure. Round to the nearest tenth.
  - $QR =$  \_\_\_\_\_
  - $m\angle D =$  \_\_\_\_\_





**Learning Targets:**

- Determine when to use the Law of Sines and when to use the Law of Cosines.
- Solve problems using the Law of Cosines and/or the Law of Sines.

**SUGGESTED LEARNING STRATEGIES:** Create Representations, Predict and Confirm, Think-Pair-Share, Visualization

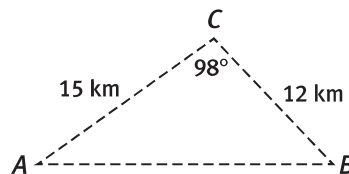
To solve a triangle, you find all its angle measures and all its side lengths. It is important that you know when you can use the Law of Sines and when you can use the Law of Cosines to solve triangles.

1. Complete the table below and use it as a reference.

Case	Description	Use Law of Sines or Law of Cosines to solve?
AAS	Two angle measures and the length of a nonincluded side are known.	
ASA	Two angle measures and the length of the included side are known.	
SSA	The lengths of two sides and the angle opposite are known.	
SAS	Two side lengths and the included angle measure are known.	
SSS	Three side lengths are known.	

2. Fire spotters at stations located at  $B$  and  $C$  notice a fire at location  $A$ . Solve the triangle.

- a. What information do you know?



- b. Describe the first step in solving the problem.

- c. **Make sense of problems.** Draw a flowchart to plan a solution pathway for the problem.

My Notes

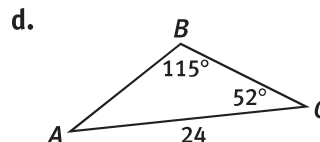
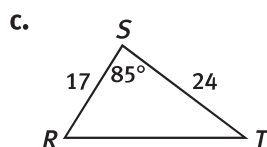
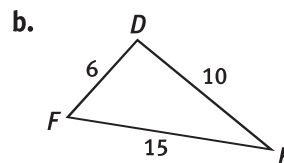
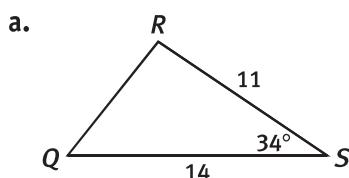
- d. Does your plan include using the Law of Sines, the Law of Cosines, or both? Explain.
- e. Solve the triangle. Round all side measures to the nearest tenth and angle measures to the nearest degree.

Check Your Understanding

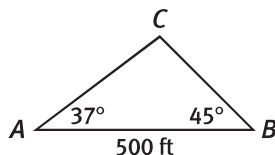
3. Suppose you know the lengths of the three sides of a triangle. Describe how you would find the measure of each angle of the triangle.
4. Suppose you know the measures of the three angles of a triangle. Can you use the Law of Sines and/or the Law of Cosines to solve the triangle? Explain.

LESSON 23-4 PRACTICE

5. Solve each triangle. Round all side measures to the nearest tenth and angle measures to the nearest degree.



6. **Reason quantitatively.** Two lifeguards on the towers at vertex A and vertex B are watching a swimmer in the ocean at vertex C. Which lifeguard is closer to the swimmer? Show your work.



### ACTIVITY 23 PRACTICE

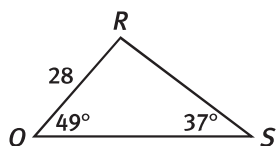
Write your answers on notebook paper.

Show your work.

Unless otherwise indicated, round all side measures to the nearest tenth and angle measures to the nearest degree.

#### Lesson 23-1

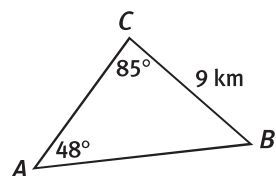
- Solve for  $x$ :  $\frac{\sin 28^\circ}{x} = \frac{\sin 52^\circ}{15}$
- Solve for  $A$ :  $\frac{\sin 82^\circ}{28} = \frac{\sin A^\circ}{8}$
- Describe how to use the Law of Sines to find the lengths of the other two sides of a triangle, if you know the measures of two angles and the included side.
- To the nearest tenth, what is  $RS$ ?



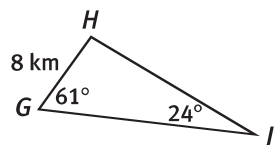
- |         |       |
|---------|-------|
| A. 21.2 | B. 29 |
| C. 35.1 | D. 37 |

- Find each measure.

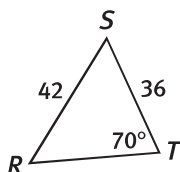
a.  $AC = \underline{\hspace{2cm}}$



b.  $HJ = \underline{\hspace{2cm}}$



c.  $m\angle R = \underline{\hspace{2cm}}$



#### Lesson 23-2

- Given:  $\triangle ABC$  with  $m\angle A = 28^\circ$ ,  $BC = 6$ ,  $AB = 11$ . Find two possible values for each measure.
  - $m\angle C$
  - $AC$
- Given:  $\triangle ABC$  with  $m\angle A = 65^\circ$ ,  $BC = 19$ ,  $AB = 21$ . Find two possible values for each measure.
  - $m\angle C$
  - $AC$
- Three fire towers are positioned so that the angle formed at vertex  $R$  is  $31^\circ$ . The distance between tower  $R$  and tower  $S$  is 8 miles. The distance between tower  $S$  and tower  $T$  is 6 miles. What are the two possible distances between tower  $R$  and tower  $T$ ?

#### Lesson 23-3

- Solve for  $b$ :  

$$b^2 = 22^2 + 28^2 - 2(22)(28)\cos 36^\circ$$
- Solve for  $C$ :  

$$5^2 = 8^2 + 11^2 - 2(8)(11)\cos C^\circ$$

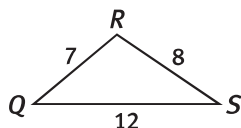
# ACTIVITY 23

continued

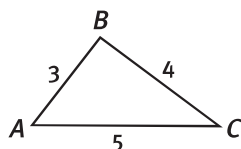
## The Law of Sines and Cosines

There Ought to Be a Law

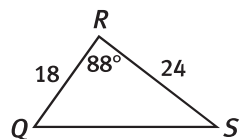
11. To the nearest degree, what is  $m\angle S$ ?



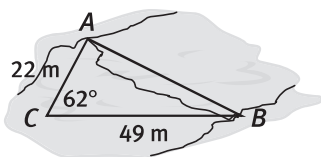
- A.  $34^\circ$   
 B.  $40^\circ$   
 C.  $50^\circ$   
 D.  $106^\circ$
12. Find each measure.  
 a.  $m\angle C = \underline{\hspace{2cm}}$



- b.  $m\angle S = \underline{\hspace{2cm}}$



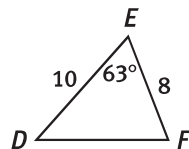
13. What is the distance across the lake,  $AB$ ?



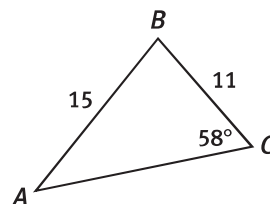
## Lesson 23-4

14. Solve each triangle.

a.



b.



## MATHEMATICAL PRACTICES

### Construct Viable Arguments and Critique the Reasoning of Others

15. Chandra wants to find the measure of angle  $B$  in triangle  $ABC$ . Is her work correct? Explain. If it is not, then fix the error and find the measure of angle  $B$ .

$$\cos B = \frac{36^2 - 20^2 + 30^2}{2(20)(30)}$$

