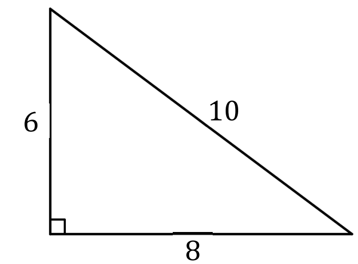
Section 8: Right Triangles

The following Mathematics Florida Standards will be covered in this section:	
MAFS.912.G-CO.2.8	Explain how the criteria for triangle congruence (ASA, SAS, SSS, and Hypotenuse-Leg) follow from the definition of congruence in terms of rigid motions.
MAFS.912.G-CO.3.10	Prove theorems about triangles; use theorems about triangles to solve problems. (Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.)
MAFS.912.G-SRT.1.2	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
MAFS.912.G-SRT.2.4	Prove theorems about triangles, *[such as]: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity*.
MAFS.912.G-SRT.2.5	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
MAFS.912.G-SRT.3.6	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
MAFS.912.G-SRT.3.7	Explain and use the relationship between the sine and cosine of complementary angles.
MAFS.912.G-SRT.3.8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Topics in this Section

Section 8 – Topic 1
The Pythagorean Theorem

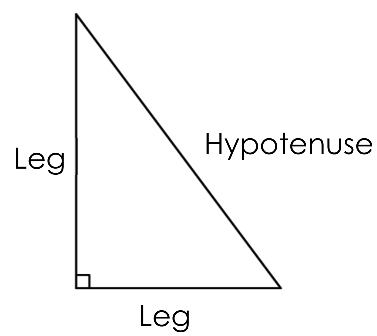
Consider the triangle below.

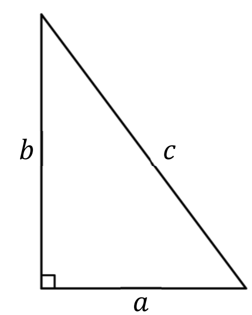


What relationship exists between the length of the hypotenuse and the length of the legs?

TAKE NOTE! *Postulates & Theorems*

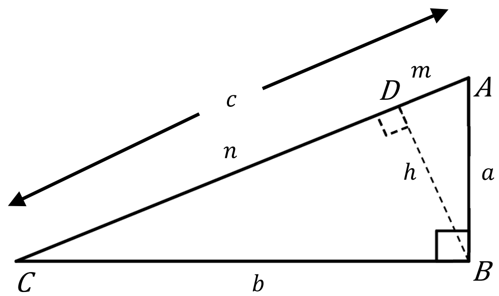
Pythagorean Theorem
In a right triangle, the square of the hypotenuse (the side opposite to the right angle) is equal to the sum of the squares of the other two sides.





$$a^2 + b^2 = c^2$$

Consider the following diagram and complete the two-column proof below.



Given: $\triangle ABC \sim \triangle ADB \sim \triangle BDC$
Prove: $a^2 + b^2 = c^2$ using triangle similarity.

Statements	Reasons
1.	**1.** Given
2.	**2.** Corresponding sides of similar triangles are proportional
3.	**3.** Multiplication Property Of Equality
4. $a^2 + b^2 = cm + cn$	**4.**
5.	**5.** Distributive Property
6. $AD + DC = AC$ or $m + n = c$	**6.**
7. $a^2 + b^2 = c^2$	**7.**

Let's Practice!

1. A business park has several office spaces for rent. Each office is in the shape of a right triangle. If one side of the office is 11 feet long and the hypotenuse is 15 feet long, what is the length of the other side?

Try It!

2. Mr. Roosevelt is leaning a ladder against the side of his son's tree house to repair the roof. The top of the ladder reaches the roof, which is 18 feet from the ground. The base of the ladder is 5 feet away from the tree. How long is the ladder?

BEAT THE TEST!

1. A baseball diamond is actually a square with sides of 90 feet.

   Part A: If a runner tries to steal second base, how far must the catcher, who is at home plate, throw the ball to get the runner out?

   Part B: Explain why runners more often try to steal second base rather than third base.

Section 8 – Topic 2
The Converse of the Pythagorean Theorem

Suppose we are given a triangle and the lengths of the sides. How can you determine if the triangle is a right triangle?

A Pythagorean triple is a set of positive integers a, b and c that satisfy the Pythagorean theorem, $a^2 + b^2 = c^2$.

The side lengths of a right triangle, 3, 4 and 5, form a Pythagorean triple. Prove that each example below is a Pythagorean triple.

- $5, 12, 13$
- $8, 15, 17$
- $7, 24, 25$

Hypothesize if multiples of Pythagorean triples are still Pythagorean triples. Justify your answer.

TAKE NOTE! *Postulates & Theorems*

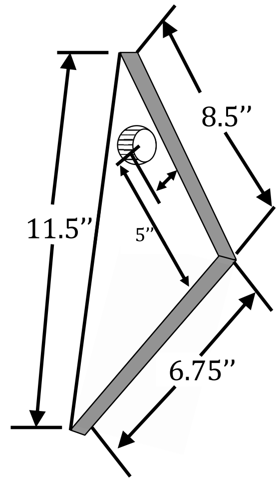
Converse Pythagorean Theorem

If the square of one side of a triangle is equal to the sum of the squares of the other two sides, then the triangle is a right triangle.

Let's Practice!

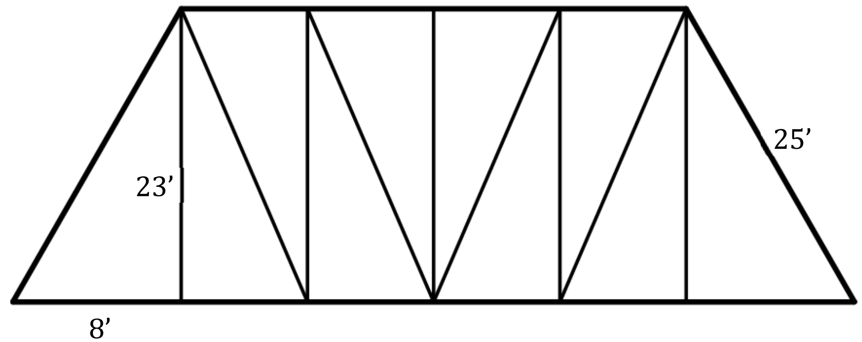
1. Zully is designing a birdcage that her husband will build for the little birds that come to eat in the mornings. The birdcage must be a right triangle. The first draft of her design is displayed to the right.

   Does this design contain a right triangle? Justify your answer.



Try It!

2. Mr. Chris designed a Pratt Truss bridge with a structure that slanted towards the center of the bridge. The Pratt Truss contains right triangles in its design. However, the construction firm he submitted the design to rejected his draft because the design failed to meet the requirements.



   a. Consider the representation of the bridge Mr. Chris designed (above). Justify why it did not meet the Pratt Truss bridge requirement.

   b. What options does Mr. Chris have to fix the design? Justify your answer.

BEAT THE TEST!

1. Clay designs roofs that form 2 congruent right triangles. His designs are flawless. He submitted his latest design to a firm along with three other contractors and the firm selected Clay's plan. Which of the following designs is Clay's design?

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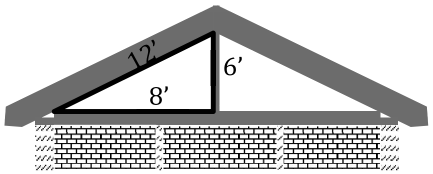

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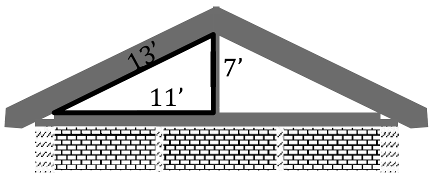

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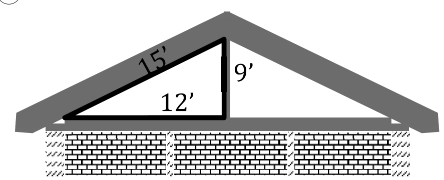

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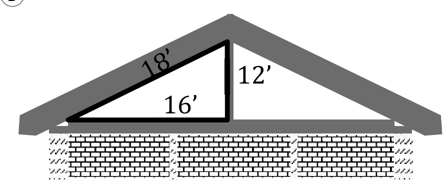


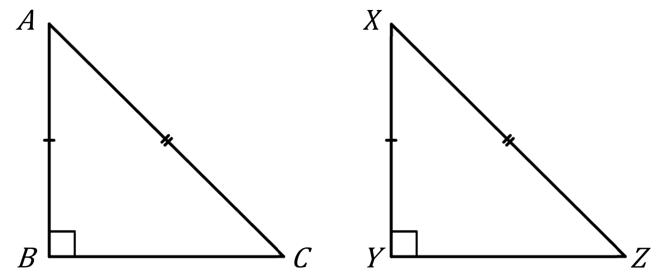
Section 8 – Topic 3
Proving Right Triangles Congruent

Let's review the four postulates that can be used to prove triangles congruent.

Hypotenuse-Leg (HL) Theorem is another way to prove triangles congruent.

TAKE NOTE! Postulates & Theorems	**The Hypotenuse-Leg (HL) Theorem** Two right triangles are said to be congruent if their corresponding hypotenuse and one of the two remaining sides are congruent.

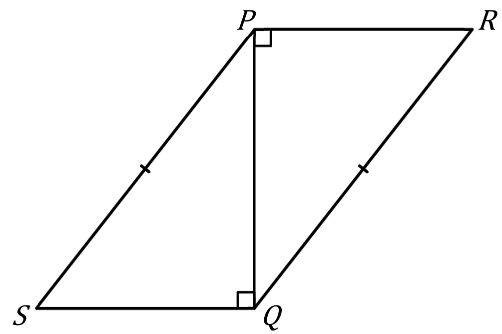
Consider the diagram below.



List three statements that prove the triangles congruent by HL Theorem.

Let's Practice

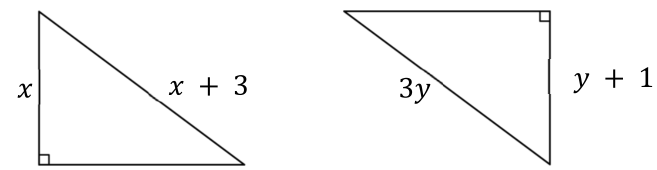
1. Consider ΔSQP and ΔRPQ in the diagram below. Complete the two column proof.



Given: ΔSQP and ΔRPQ are right triangles and $\overline{SP} \cong \overline{QR}$.
Prove: $\Delta SQP \cong \Delta RPQ$

Statements	Reasons
1. ΔSQP and ΔRPQ are right triangles.	**1.**
2.	**2.** Given
3.	**3.** Reflexive Property of congruence
4. $\Delta SQP \cong \Delta RPQ$	**4.**

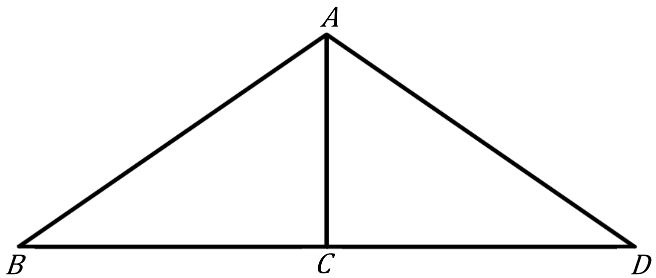
2. Consider the following diagrams.



Find the values of x and y that prove the two triangles congruent by the HL Theorem.

Try It!

3. Consider ΔABC and ΔADC in the diagram below. Complete the two column proof.

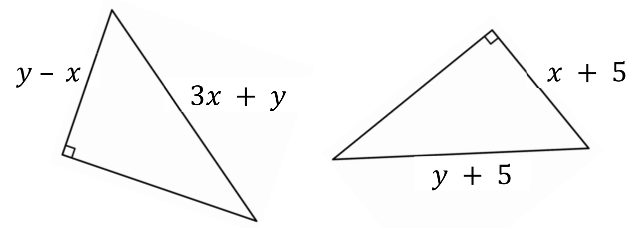


Given: $\overline{AC}$ is perpendicular to $\overline{BD}$; $\overline{AB} \cong \overline{AD}$

Prove: $\Delta ABC \cong \Delta ADC$

Statements	Reasons
1. $\overline{AC}$ is perpendicular to $\overline{BD}$.	**1.** Given
2. $\angle ACB$ and $\angle ACD$ are right angles.	**2.**
3. $\overline{AB} \cong \overline{AD}$	**3.** Given
4. $\overline{AC} \cong \overline{AC}$	**4.**
5. $\Delta ABC \cong \Delta ADC$	**5.**

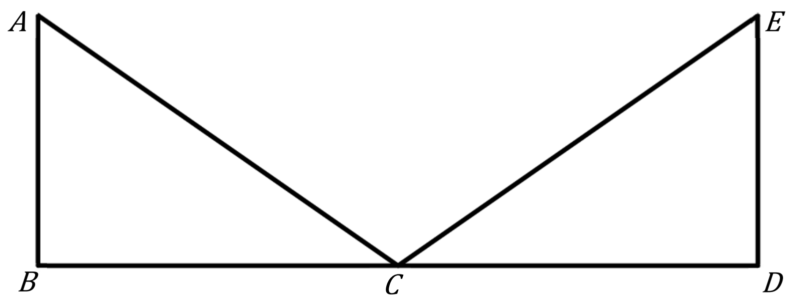
4. Consider the diagrams below.



Find the values of x and y so that the right triangles above are congruent.

BEAT THE TEST!

1. Engineers are designing a new bridge to cross the Intracoastal Waterway. Below is a diagram that represents a partial side view of the bridge. The bridge must be designed so that $\Delta ABC \cong \Delta EDC$. Engineers have measured the support beams, represented by $\overline{AC}$ and $\overline{EC}$ in the diagram, and found they are both $120\ ft$ long. The engineers also determined beams $\overline{AB}$ and $\overline{ED}$ are perpendicular to the bridge, $\overline{BD}$. Point C represents the midpoint of $\overline{BD}$.

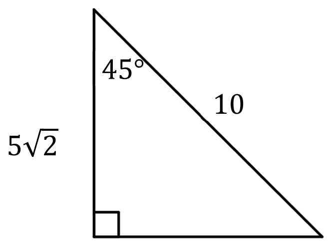


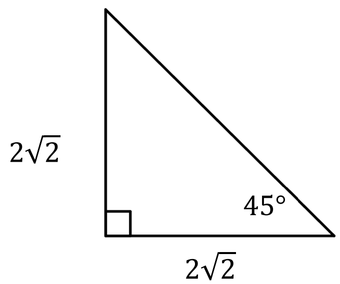
Complete the two-column proof on the next page to prove $\Delta ABC \cong \Delta EDC$.

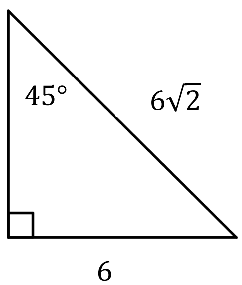
Statements	Reasons
1.	**1.** Given
2. $\overline{AB} \perp \overline{BD}$ and $\overline{ED} \perp \overline{BD}$	**2.**
3.	**3.** Definition of a midpoint
4. $\angle ABC$ and $\angle EDC$ are right angles.	**4.**
5. $\Delta ABC \cong \Delta EDC$	**5.**

Section 8 – Topic 4
Special Right Triangles: $45° - 45° - 90°$

Use the Pythagorean Theorem to find the missing lengths of the following triangles.



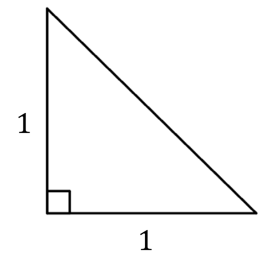




Choose three patterns that you observe in the three right triangles above and list them below.

Let's Practice!

1. Consider the following $45 - 45 - 90$ triangle. Prove that the ratio of the hypotenuse to one of the legs is $\sqrt{2}: 1$.



2. Find the hypotenuse of a $45 - 45 - 90$ triangle with legs equal to $5\ cm$.

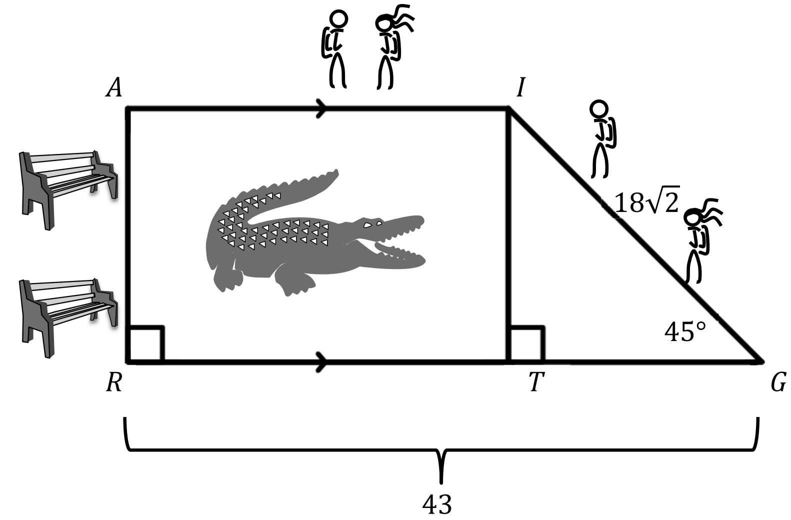
Try It!

3. Find the length of the sides of a square with a diagonal of $25\,\frac{2}{3}$ meters.

4. The Tilley household wants to build a patio deck in the shape of a $45-45-90$ triangle in a nice corner section of their backyard in front of the lake. They have enough room for a triangle with a leg of 36 feet. What will the length of the hypotenuse be?

BEAT THE TEST!

1. Consider the figure below.

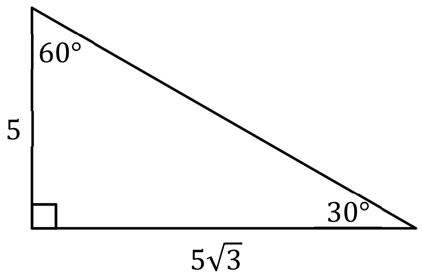


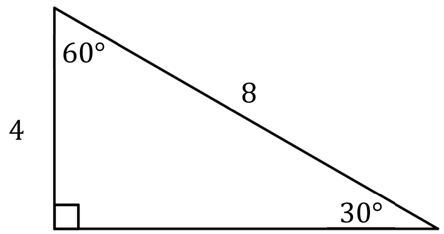
Part A: What is the perimeter of the above figure?

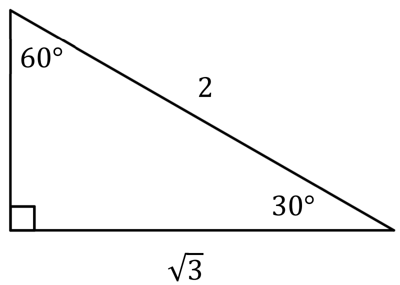
Part B: Write a 3 - sentence short story explaining the above figure and the calculations made in *Part A*.

Section 8 – Topic 5
Special Right Triangles: $30° - 60° - 90°$

Use the Pythagorean Theorem to find the missing lengths of the following triangles.







Choose three patterns that you observe in the three right triangles above and list them below.

Let's Practice!

1. The length of a hypotenuse of a $30\text{–}60 - 90$ right triangle is $17\ yds$. Find the other two lengths.

Try It!

2. A right triangle has a leg with a length of 34 and a hypotenuse with a length of 68. A student notices that the hypotenuse is twice the length of the given leg, which means it is a $30\text{–}60\text{–}90$ triangle. If the student is correct, what should the length of the remaining leg be? Explain your answer. Confirm your answer using the Pythagorean Theorem.

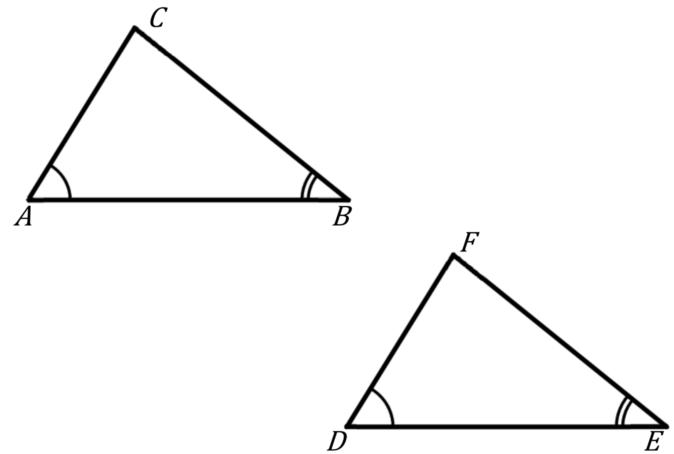
BEAT THE TEST!

1. The base of the engineering building at Lenovo Tech Industries is approximately a $30-60-90$ triangle with a hypotenuse of about 294 feet. The base of the engineering building at Asus Tech Industries is approximately an isosceles right triangle with a side about $144.5\sqrt{2}$ feet.

   What is the difference between the perimeters of both triangles? Round your answer to the nearest hundredth.

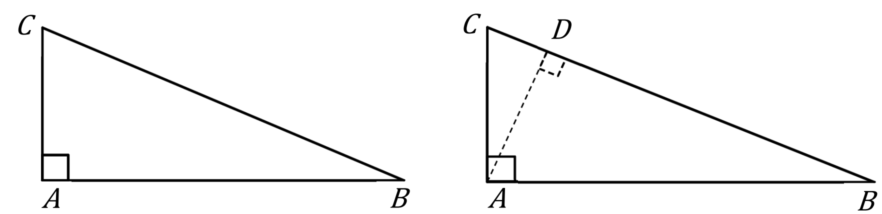
Section 8 – Topic 6
Right Triangles Similarity – Part 1

Make observations about the following triangles.



These triangles are similar by the ____________________.

Consider the diagram below.



Make observations about ΔABD and ΔACD.

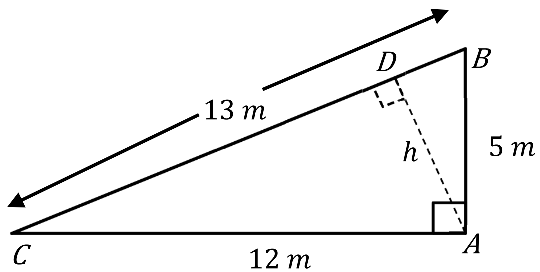
TAKE NOTE! Postulates & Theorems

Right Triangle Altitude Theorem

If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.

Let's Practice!

1. Consider the following diagram.

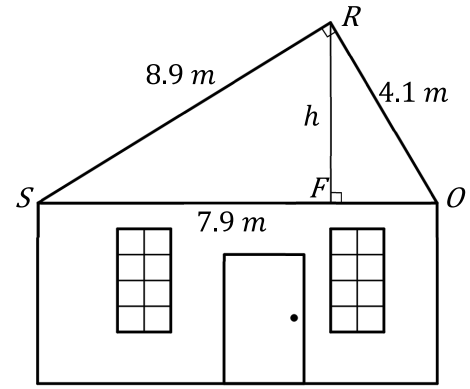


a. Identify the similar triangles in the above diagram.

b. Find h in the above diagram.

Try It!

2. A roof has a cross section that forms a right angle. Consider the diagram below that shows the approximate dimensions of this cross section.



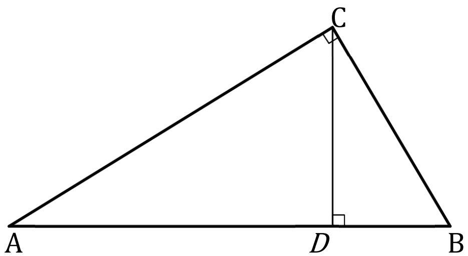
a. Identify the similar triangles represented in the above figure.

b. Find the height h of the roof represented above.

TAKE NOTE! *Postulates & Theorems*

Geometric Mean Theorem: Altitude Rule

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments. The length of the altitude is the geometric mean of the lengths of the two segments.

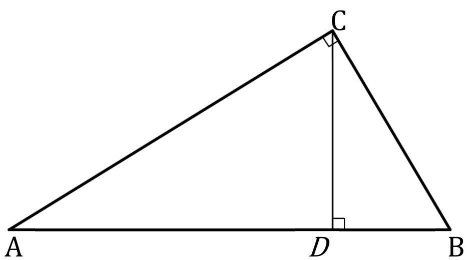


$$\frac{\overline{AD}}{\square} = \frac{\square}{\overline{DB}}$$

TAKE NOTE! *Postulates & Theorems*

Geometric Mean Theorem: Leg Rule

In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments. The length of each leg of the right triangle is the geometric mean of the lengths of the hypotenuse and the segment of the hypotenuse that is adjacent to the leg.

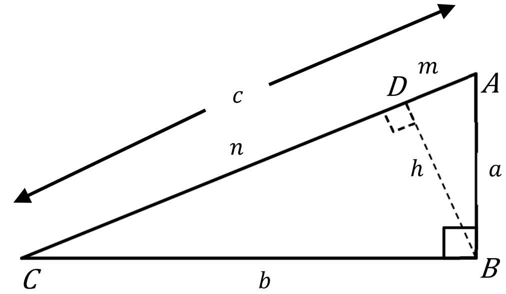


$$\frac{\overline{AB}}{\square} = \frac{\square}{\overline{DB}}$$

or

$$\frac{\overline{AB}}{\square} = \frac{\square}{\overline{AD}}$$

Consider the following diagram.



Can we accept $\triangle ADB \sim \triangle BDC$ as a given statement? Justify your answer.

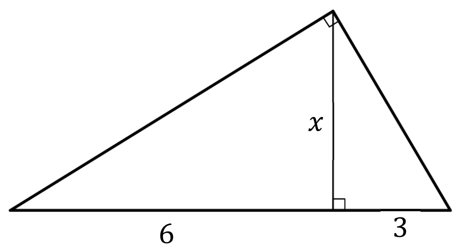
Complete the following two-column proof to prove that $h = \sqrt{mn}$.

Statements	Reasons
1.	**1.** Given
2. $\frac{m}{h} = \frac{h}{n}$	**2.**
3.	**3.** Multiplication Property Of Equality
4. $h = \sqrt{mn}$	**4.**

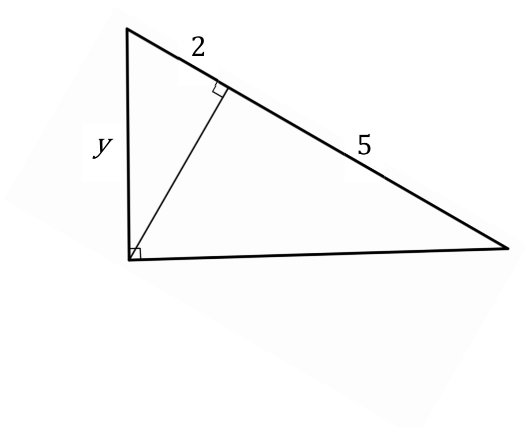
Section 8 – Topic 7
Right Triangles Similarity – Part 2

Let's Practice!

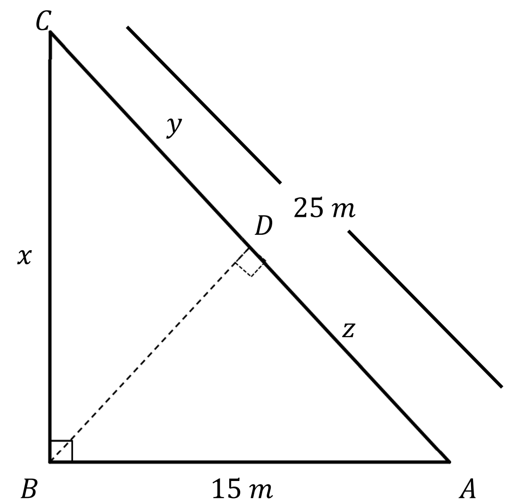
1. Consider the diagram below and find the value of x.



2. Consider the diagram below and find the value of y.



3. Consider the diagram below.

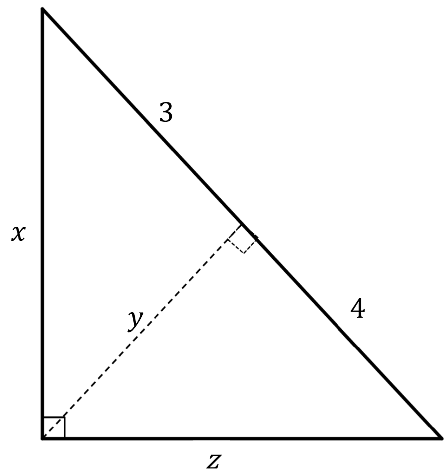


Given: $\Delta CDB \sim \Delta ADB$

Prove: $x = 20\ m$
$y = 16\ m$
$z = 9\ m$

Try It!

4. Consider the diagram below.



Find the values of x, y, and z to the nearest tenth.

5. A cruise port, a business park, and a federally protected forest are located at the vertices of a right triangle formed by three highways. The port and business park are 6.0 miles apart. The distance between the port and the forest is 3.6 miles, and the distance between the business park and the forest is 4.8 miles.

   A service road will be constructed from the main entrance of the forest to the highway that connects the port and business park. What is the shortest possible length for the service road? Round your answer to the nearest tenth.

BEAT THE TEST!

1. Consider the statement below.

   In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments. The length of the altitude is the geometric mean of the lengths of the two segments.

   Which of the following figures is a counterexample of the statement above?

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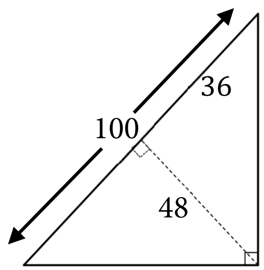

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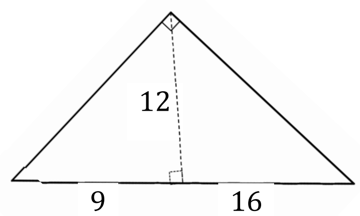

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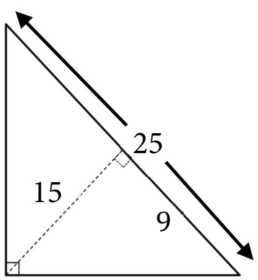

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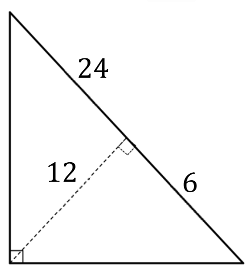


2. A shopping center has the shape of a right triangle with sides measuring $600\sqrt{3}\ meters$, $600\ meters$, and $1200\ meters$. During the holidays and busy seasons, the shopping center is so crowded that it needs another walkway. The owners will construct the walkway from the right angle to the hypotenuse. They want to use the shortest possible length for the walkway.

   a. Determine the length of the segment of the hypotenuse adjacent to the shorter leg.

      meters

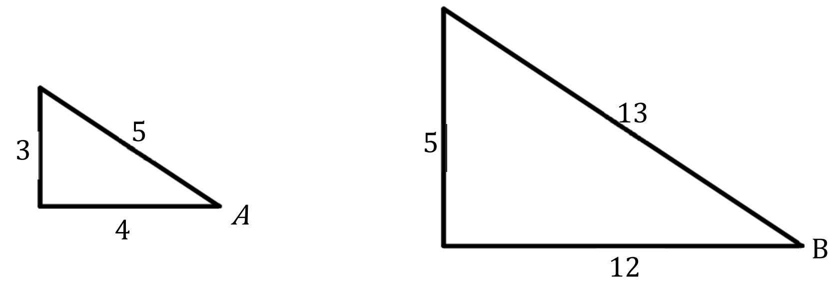
   b. Determine the length of the new walkway.

      meters

Section 8 – Topic 8
Introduction to Trigonometry – Part 1

In previous lessons, we learned that the lengths of the sides of a right triangle have a certain relationship, which allows us to use the ______________________________________.

Consider the following right triangles.



For angle A, find the ratio of the opposite leg to the hypotenuse. $\frac{Opposite}{Hypotenuse}$ = ____

Find the same ratio for angle B. _____ = _____

The ratio of the lengths of any 2 sides of a right triangle is a ______________________ _______________.

Let's examine the three main trigonometric ratios. Complete each of the statements below with the most appropriate answer.

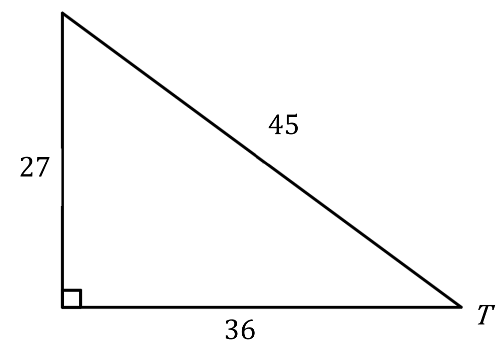
_______ $= \frac{leg\ opposite\ to\ the\ angle}{hypotenuse}$

_______ $= \frac{leg\ adjacent\ to\ the\ angle}{hypotenuse}$

_______ $= \frac{leg\ opposite\ to\ the\ angle}{leg\ adjacent\ to\ the\ angle}$

Let's Practice!

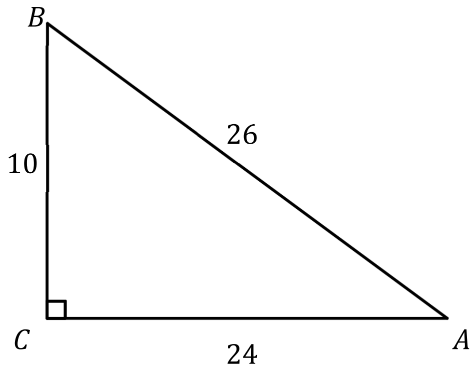
1. Consider the figure below.



Find the sine, cosine, and tangent of $\angle T$ for the above figure.

Try It!

2. Consider the figure below.

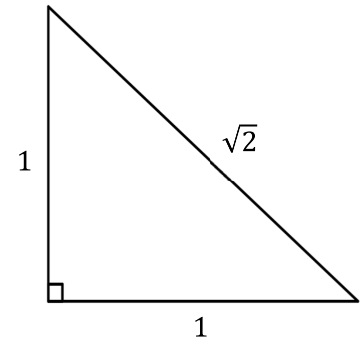


a. Find $sinA$ for the above triangle.

b. Find $cosB$ for the above triangle.

c. What do notice about the values of $sinA$ and $cosB$?

Now, let's consider the figure below.



The triangle above is a special right triangle known as the ______ ______ ______ triangle. We know that the two non-right angles measure ______.

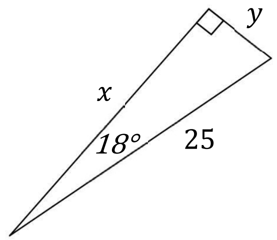
Write proportions for sin, cos, and tan of the acute angles of the triangle.

Use a calculator to verify the proportions.

If there is an unknown length, we can set up an equation to find it.

Let's Practice!

3. Consider the following figure.



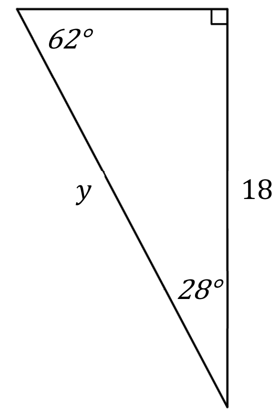
   a. Which trigonometric function should you use to find the value of x?

   b. Write an equation to find x in the above figure.

   c. Find the value of x in the above figure.

Try It!

4. Consider the figure below.



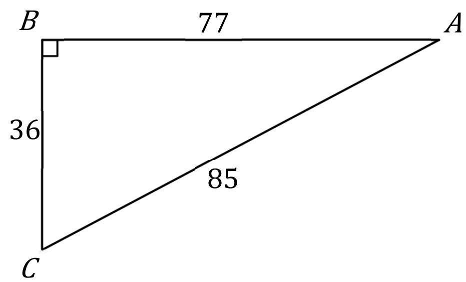
   Determine the value of y.

Section 8 – Topic 9
Introduction to Trigonometry – Part 2

Given the lengths of sides, we can use "trig" functions to find missing angles by using their inverses: sin^{-1}, cos^{-1}, and tan^{-1}.

Let's Practice!

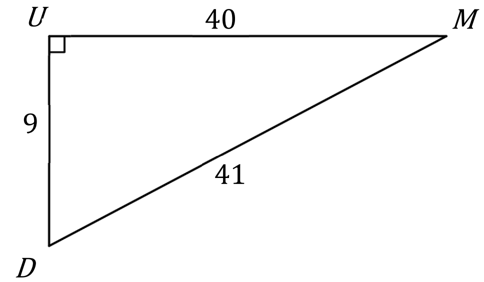
1. Consider the triangle below.



Find $cosC$, $sinA$, $m\angle A$ and $m\angle C$ for the above triangle.

Try It!

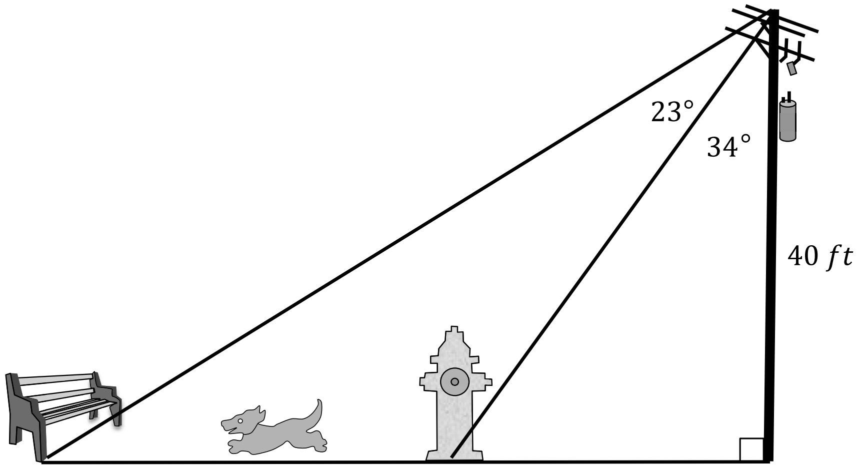
2. Consider the triangle below.



Find $tanM$, $cosD$, $m\angle D$ and $sinM$ for the above triangle.

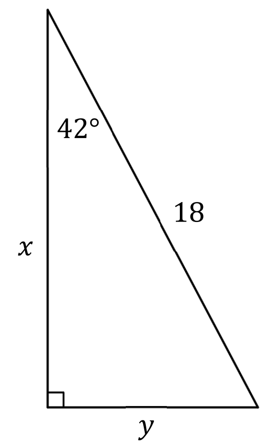
BEAT THE TEST!

1. The picture below shows the path that Puppy Liz is running. The electrical post is 40 feet tall. Puppy Liz usually starts at the bench post and runs until she gets to the fire hydrant, rests, and then she runs back to the bench. How far does Puppy Liz run to get to the fire hydrant?



Puppy Liz runs ______ feet.

2. Consider the triangle below.



Which of the following measurements represents the perimeter and area of the triangle above?

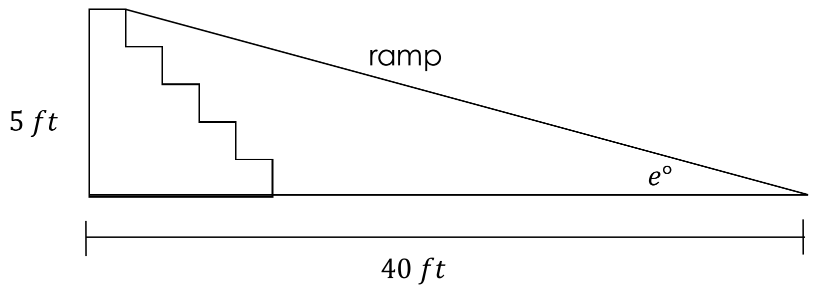
Ⓐ Perimeter: 80.55 units
Area: 43.42 square units

Ⓑ Perimeter: 43.42 units
Area: 80.55 square units

Ⓒ Perimeter: 21.71 units
Area: 161.03 square units

Ⓓ Perimeter: 161.03 units
Area: 21.71 square units

3. Yandel will place a ramp over a set of stairs at the backyard entrance so that one end is 5 feet off the ground. The other end is at a point that is a horizontal distance of 40 feet away, as shown in the diagram. The angle of elevation of the ramp is represented by $e°$. Each step of the stair is one foot long.



What is the angle of elevation to the nearest tenth of a degree?