Unit 1: Area and Surface Area

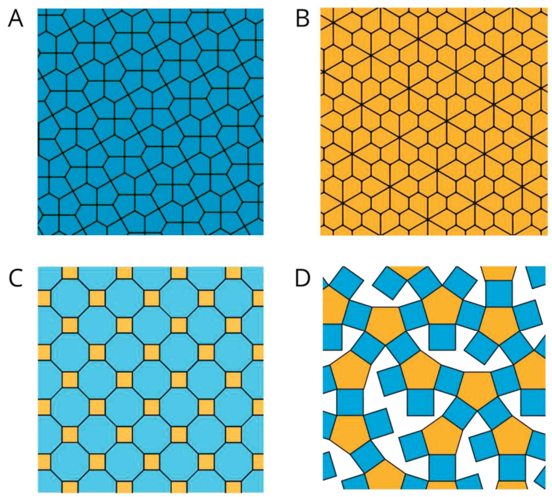
Adapted from Open Up Resources 6-8 Math (authored by Illustrative Mathematics).
See inside front cover for details.

Unit 1, Lesson 1: Tiling the Plane

Let's look at tiling patterns and think about area.

1.1.1 Warm-Up: Which One Doesn't Belong: Tilings

Which pattern doesn't belong?

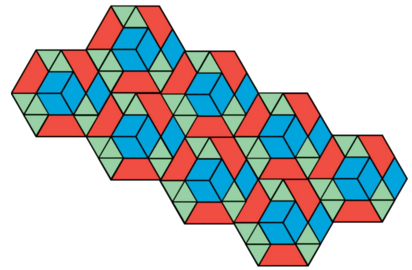


1.1.2a Exploration Activity: More Red, Green, or Blue?

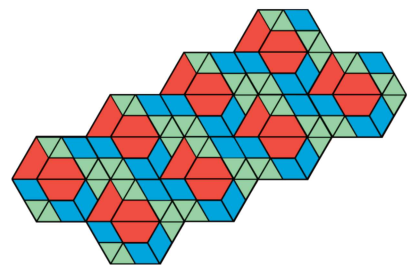
Your teacher will assign you to look at Pattern A or Pattern B.

In your pattern, which shape covers more of the plane: blue rhombuses, red trapezoids, or green triangles? Explain how you know.

Pattern A



Pattern B



1.1.2b Exploration Extension: Are you ready for more?

On graph paper, create a tiling pattern so that:

- The pattern has at least two different shapes
- The same amount of the plane is covered by each type of shape.

Lesson 1 Sum ary

In this lesson, we learned about *tiling* the plane, which means covering a two-dimensional region with copies of the same shape or shapes such that there are no gaps or overlaps.

Then, we compared tiling patterns and the shapes in them. In thinking about which patterns and shapes cover more of the plane, we have started to reason abo **area**.

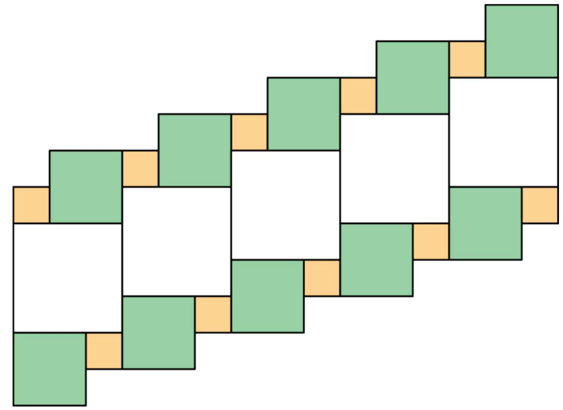
We will continue this work, and to learn how to use mathematical tools strategically to help us do mathematic

1.1.3 Cool-Down

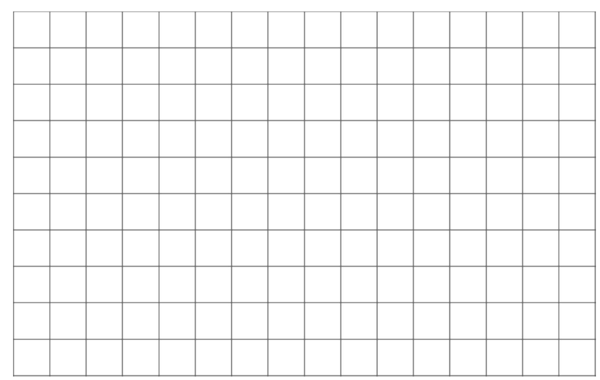
Think about your work today, and write your best definition of area.

1.1.4 Practice Problems

1. Which square—large, medium, or small—covers more of the plane? Explain your reasoning.

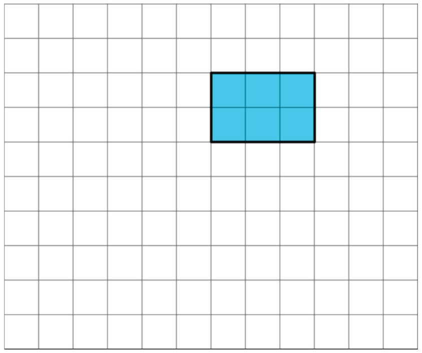


2. Draw three different quadrilaterals, each with an area of 12 square units.

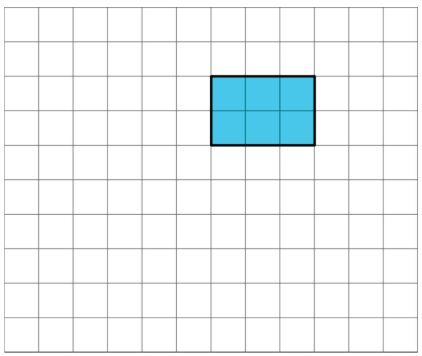


3. Use copies of the rectangle to show how a rectangle could:

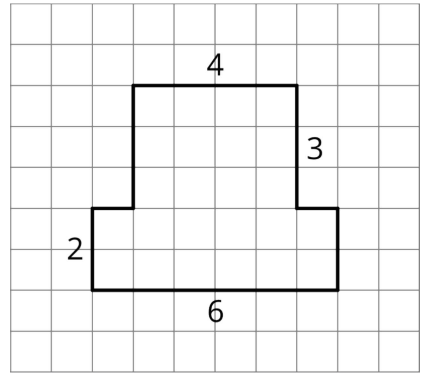
   a. tile the plane.



   b. *not* tile the plane.

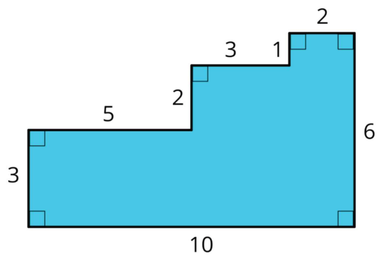


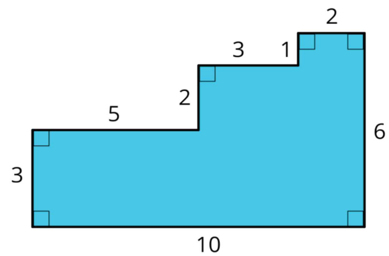
4. The area of this shape is 24 square units. Which of these statements is true about the area? Select **all** that apply.



- ☐ The area can be found by counting the number of squares that touch the edge of the shape.
- ☐ It takes 24 grid squares to cover the shape without gaps and overlaps.
- ☐ The area can be found by multiplying the sides lengths that are 6 units and 4 units.
- ☐ The area can be found by counting the grid squares inside the shape.
- ☐ The area can be found by adding 4×3 and 6×2.

5. Here are two copies of the same figure. Show two different ways for finding the area of the shaded region. All angles are right angle





6. Which shape has a larger area: a rectangle that is 7 inches by $\frac{3}{4}$ inch, or a square with side length of $2\frac{1}{2}$ inches? Show your reasoning.

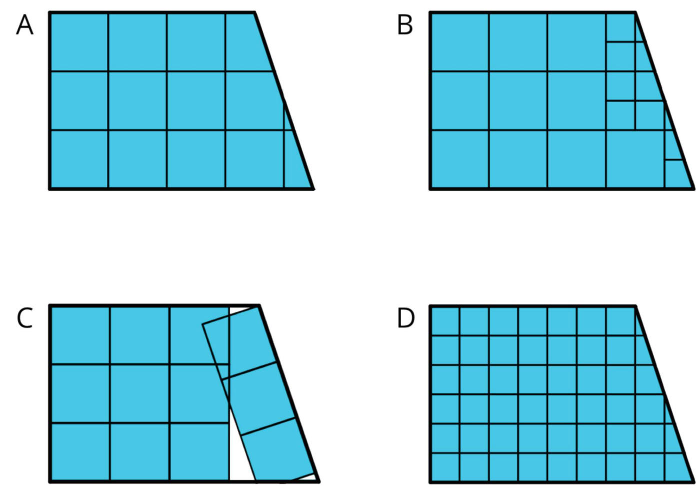
Unit 1, Lesson 2: Finding Area by Decomposing and Rearranging

Let's create shapes and find their areas.

1.2.1 Warm-Up: What is Area?

You may recall that the term **area** tells us something about the number of squares inside a two-dimensional shape.

1. Here are four drawings that each show squares inside a shape. Select **all** drawings whose squares could be used to find the area of the shape. Be prepared to explain your reasoning.



2. Write a definition of area that includes all the information that you think is important.

1.2.2a Exploration Activity: Composing Shapes

Your teacher will give you one square and some small, medium, and large right triangles. The area of the square is 1 square unit.

1. Notice that you can put together two small triangles to make a square. What is the area of the square composed of two small triangles? Be prepared to explain your reasoning.

2. Use your shapes to create a new shape with an area of 1 square unit that is not a square. Trace your shape.

3. Use your shapes to create a new shape with an area of 2 square units. Trace your shape.

4. Use your shapes to create a *different* shape with an area of 2 square units. Trace your shape.

5. Use your shapes to create a new shape with an area of 4 square units. Trace your shape.

1.2.2b Exploration Extension: Are you ready for more?

Find a way to use all of your pieces to compose a single large square. What is the area of this large squa

1.2.3 Exploration Activity: Tangram Triangles

Recall that the area of the square you saw earlier is 1 square unit. Complete each statement and explain your reasoning.

1. The area of the small triangle is _________ square units. I know this because . . .

2. The area of the medium triangle is _________ square units. I know this because . .

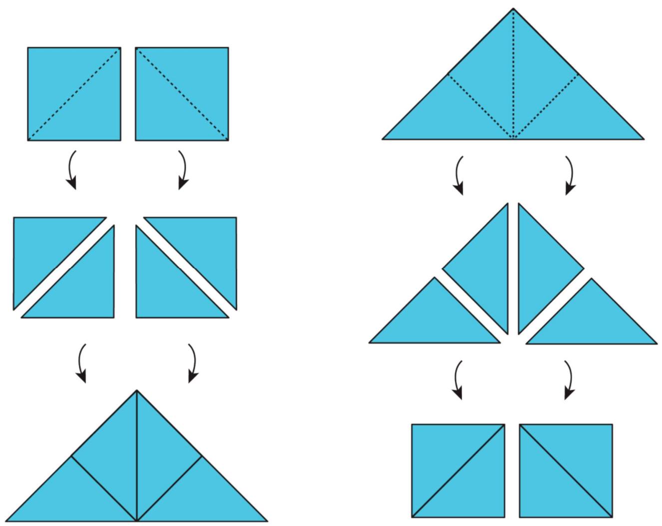
3. The area of the large triangle is _________ square units. I know this because . .

Lesson 2 Summary

Here are two important principles for finding **area**:

1. If two figures can be placed one on top of the other so that they match up exactly, then they have the *same area*.
2. We can **decompose** a figure (break a figure into pieces) and **rearrange** the ieces (move the pieces around) to find its area.

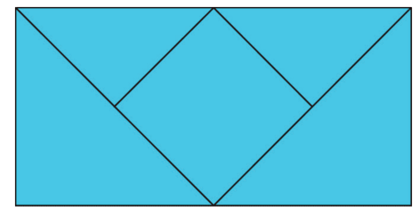
Here are illustrations of the tw inciples.



- Each square on the left can be decomposed into 2 triangles. These triangles can be rearranged into a large triangle. So the large triangle has t *same area* as the 2 squares.
- Similarly, the large triangle on the right can be decomposed into 4 equal triangles. The t iangles can be rearranged to form 2 squares. If each square has an area of 1 square unit, then the area of the large triangle is 2 square units. We also can say that each small triangle has an area of $\frac{1}{2}$ square unit.

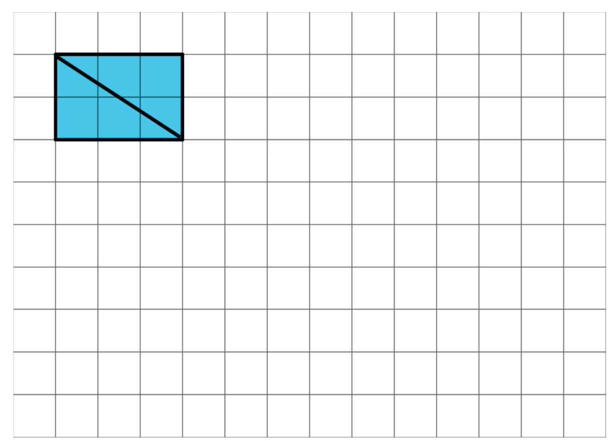
1.2.4 Cool-Down

The square in the middle has an area of 1 square unit. What is the area of the entire rectangle in square units? Explain your reasoning.



1.2.5 Practice Problems

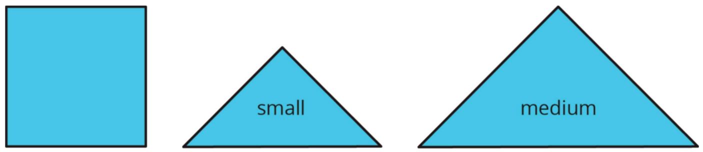
1. The diagonal of a rectangle is shown.



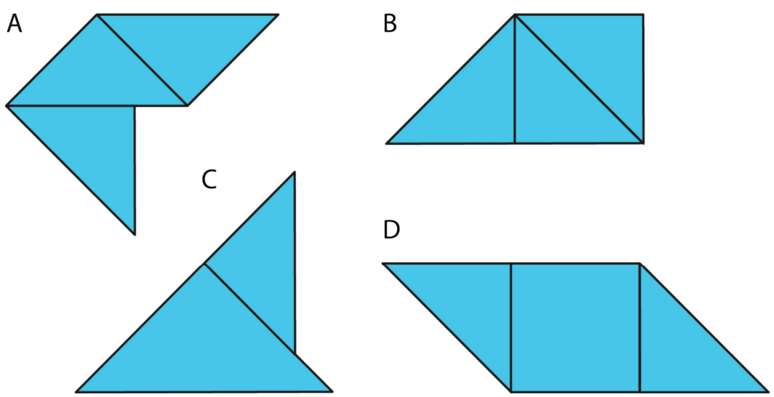
   a. Decompose the rectangle along the diagonal, and recompose the two pieces to make a *different* shape.

b. How does the area of this new shape compare to the area of the original rectangle? Explain how you know.

2. The area of the square is 1 square unit. Two small triangles can be put together to make a square or to make a medium triangle.

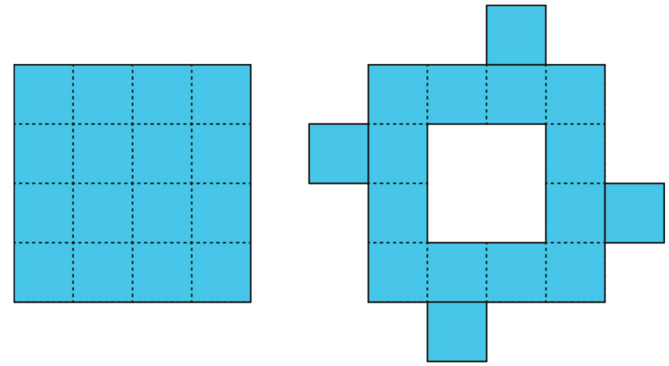


Which figure also has an area of $1\frac{1}{2}$ square units? Select **all** that apply.



3. Priya decomposed a square into 16 smaller, equal-size squares and then cut out 4 of the small squares and attached them around the outside of original square to make

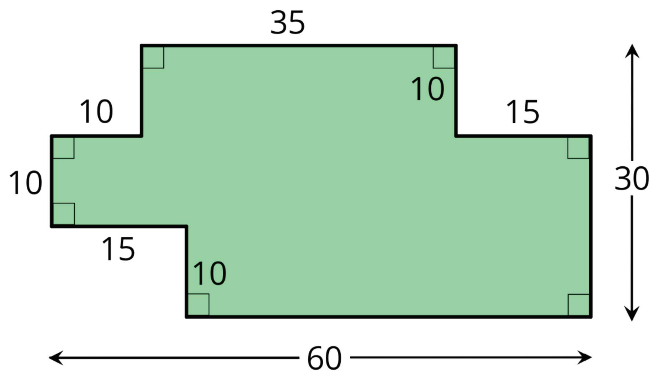
   How does the area of her new figure compare with that of the original square?



   A. The area of the new figure is greater.
   B. The two figures have the same area.
   C. The area of the original square is greater.
   D. We don't know because neither the side length nor the area of the original square is known.

4. The area of a rectangular playground is 78 square meters. If the length of the playground is 13 meters, what is its width?

5. A student said, "We can't find the area of the shaded region because the shape has many different measurements, instead of just a length and a width that we could multiply."



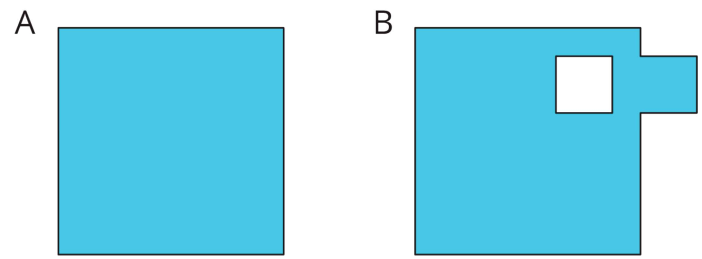
Explain why the student's statement about area is incorrect.

Unit 1, Lesson 3: Reasoning to Find Area

Let's decompose and rearrange shapes to find th ir areas.

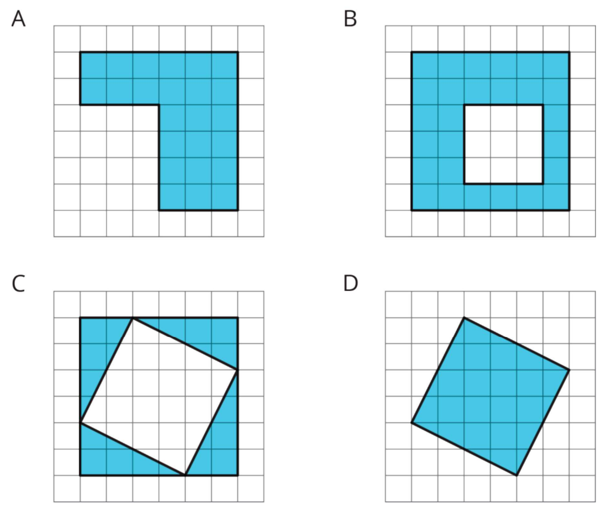
1.3.1 Warm-Up: Comparing Regions

Is the area of Figure A greater than, less than, or equal to the area of the shaded region in Figure B? Be prepared to explain your reasoning.



1.3.2a Exploration Activity: On the Grid

Each grid square is 1 square unit. Find the area, in square units, of each shaded region without counting every square. Be prepared to explain your reasoning.

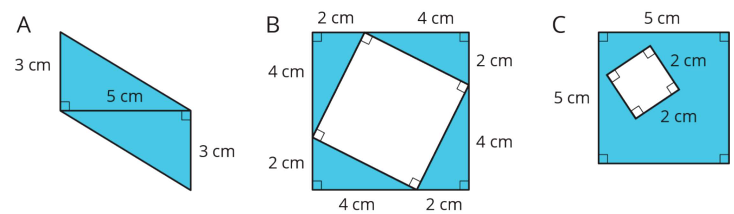


1.3.2b Exploration Extension: Are you ready for more?

Rearrange the triangles from Figure C so they fit inside Figure D. Draw and color a diagram of your work on the grid.

1.3.3 Exploration Activity: Off the Grid

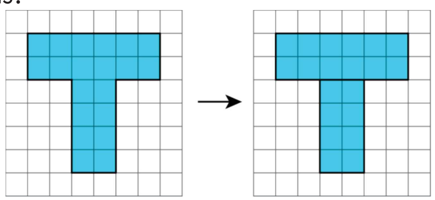
Find the area of the shaded region(s) of each figure. Explain or show your reasoning.



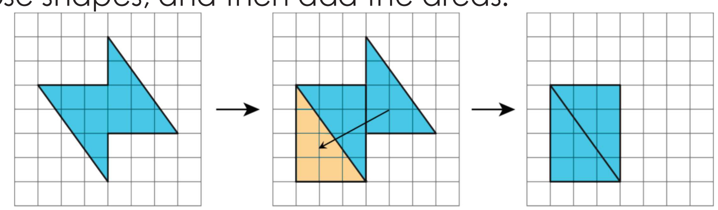
Lesson 3 Summary

There are different strategies we can use to find the area of a region. We can:

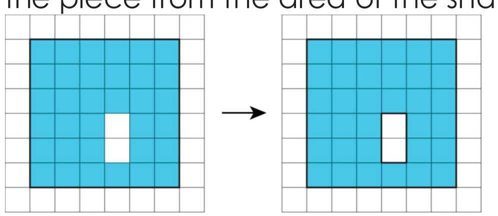
- Decompose it into shapes whose areas you know how to calcu ind the area of each of those shapes, and then add the areas.



- Decompose it and rearrange the pieces into shapes whose areas you know how to calculate; ind the area of each of those shapes, and then add the areas.

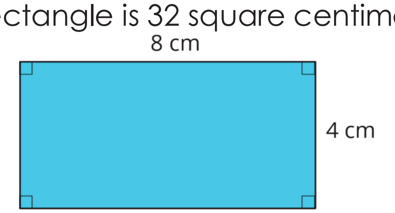


- Consider it as a shape with a missing piece; calculate the area of the shape and the missing piece, and then subtract the area of the piece from the area of the shape.



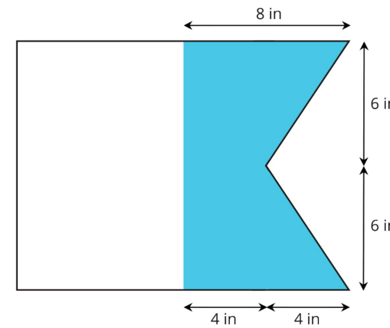
The area of a figure is always measured in square units. When both side lengths of a rectangle are given in centimeters, then the area is given in square centimeters.

The area of this rectangle is 32 square centimeters.



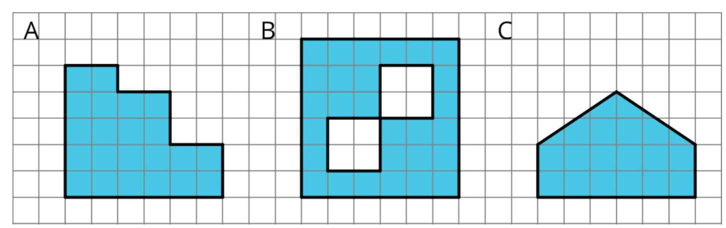
1.3.4 Cool-Down

A maritime flag is shown. What is the area of the shaded part of the flag? Explain or show your reasoning.

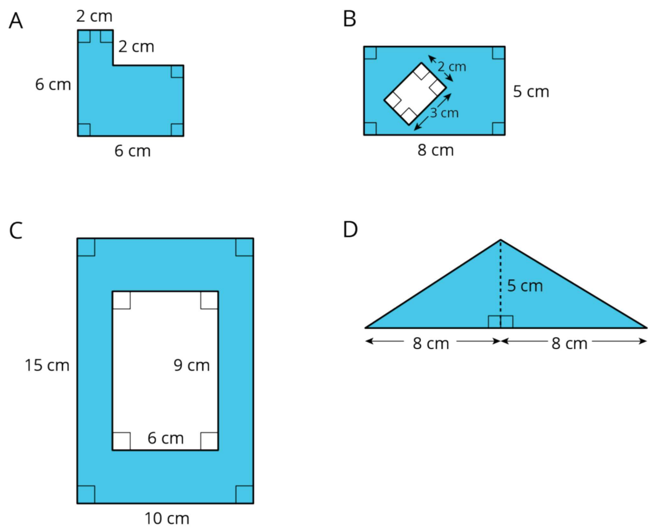


1.3.5 Practice Problems

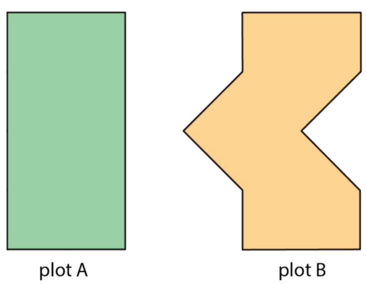
1. Find the area of each shaded region. Show your reasoning.



2. Find the area of each shaded region. Show or explain your reasoning.



3. Two plots of land have very different shapes. Noah said that both plots of land have the same area.



Do you agree with Noah? Explain your reas

4. A homeowner is deciding on the size of tiles to use to fully tile a rectangular wall in her bathroom that is 80 inches by 40 inches. The tiles are squares and come in three side lengths: 8 inches, 4 inches, and 2 inches. State if you agree with each statement about the tiles. Explain your reasoning.

   a. Regardless of the size she chooses, she will need the same number of tile

   b. Regardless of the size she chooses, the area of the wall that is being tiled is the same.

   c. She will need two 2-inch tiles to cover the same area as one 4-inch tile

   d. She will need four 4-inch tiles to cover the same area as one 8-inch t

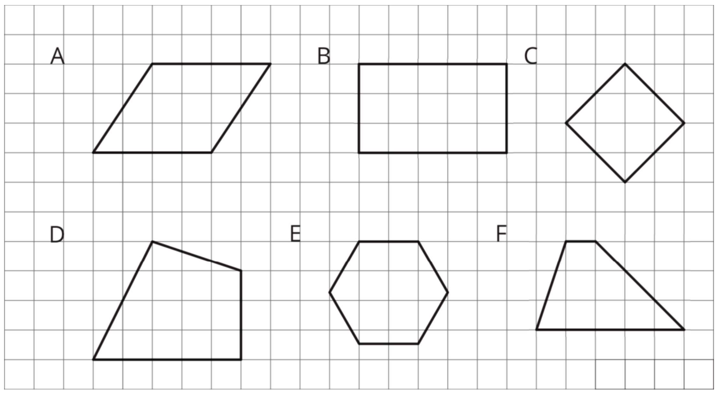
   e. If she chooses the 8-inch tiles, she will need a quarter as many tiles as she would with 2-inch tiles.

Unit 1, Lesson 4: Parallelograms

Let's investigate the features and area of parallelograms.

1.4.1 Warm-Up: Features of a Parallelogram

Figures A, B, and C are **parallelograms**. Figures D, E, and F are *not* parallelograms.



Study the examples and non-examples. What do you notice about:

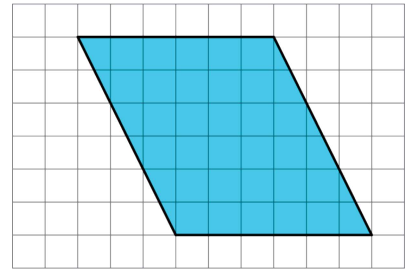
1. the number of sides that a parallelogram has?

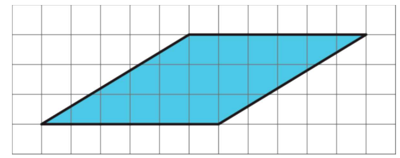
2. opposite sides of a parallelogram?

3. opposite angles of a parallelogram?

1.4.2 Exploration Activity: Area of a Parallelogram

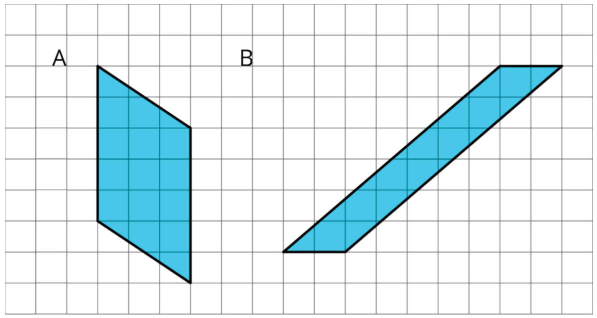
Find the area of each parallelogram. Show your reasoning.



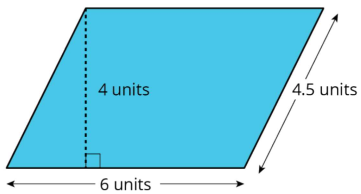


1.4.3 Exploration Activity: Lots of Parallelograms

Find the area of the following parallelograms. Show your reasoning.



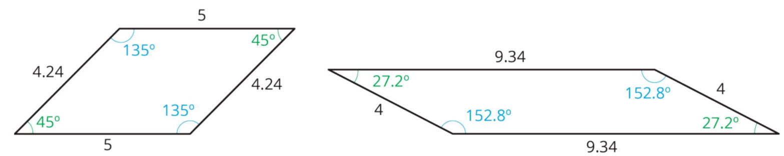
C



Lesson 4 Summary

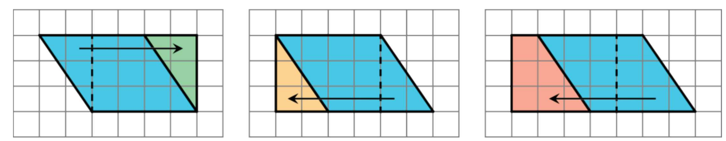
A **parallelogram** is a quadrilateral (it has four sides). The opposite sides of a parallelogram are parallel. It is also true that:

- The opposite sides of a parallelogram have equal length.
- The opposite angles of a parallelogram have equal measure.

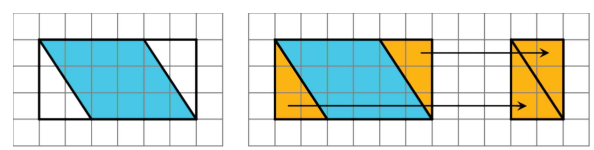


There are several strategies for finding the area of a **parallelogram**.

- We can decompose and rearrange a parallelogram to form a rectangle. Here are three ways

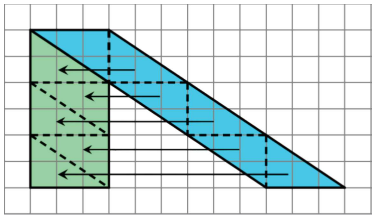


- We can enclose the parallelogram and then subtract t area of the two triangles in the corner



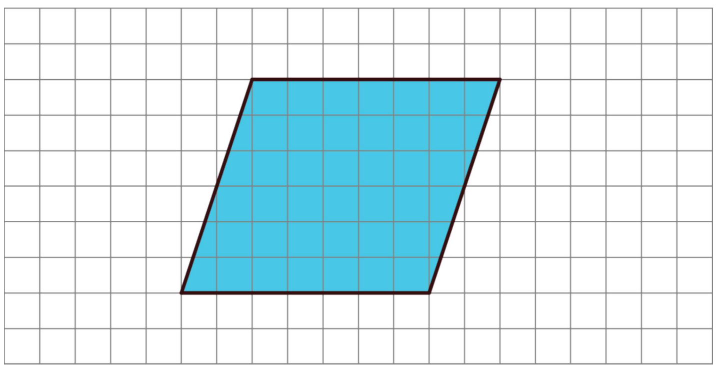
Both of these ways will work for any parallelogram

For some parallelograms, however, the process of decomposing and rearranging requires a lot more steps than if we enclose the parallelogram with a rectangle and subtract the combined area of the two triangles in the corners. Here is an example.



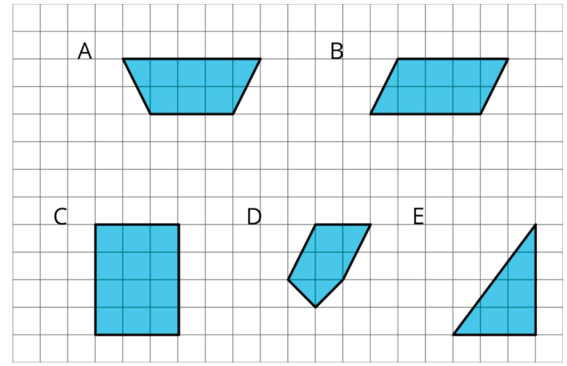
1.4.4 Cool-Down

How would you find the area of this parallelogram? Describe your strategy.

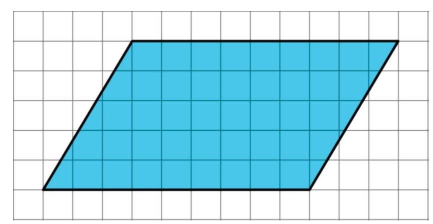


1.4.5 Practice Problems

1. Select **all** of the parallelograms. For each figure that is *not* selected, explain how you know it is not a parallelogram.

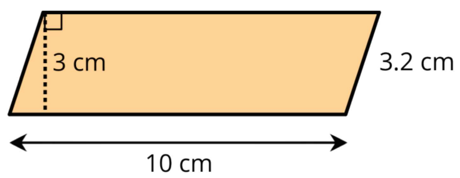


2. a. Decompose and rearrange this parallelogram to make a rectangle.

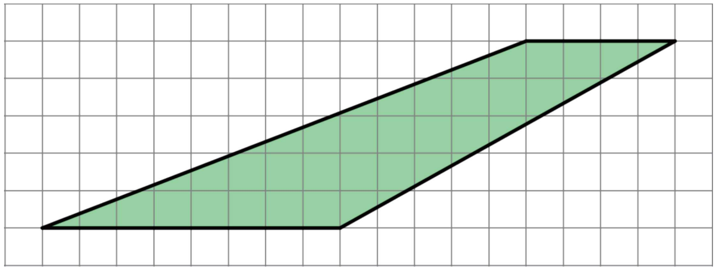


b. What is the area of the parallelogram? Explain your reasoning.

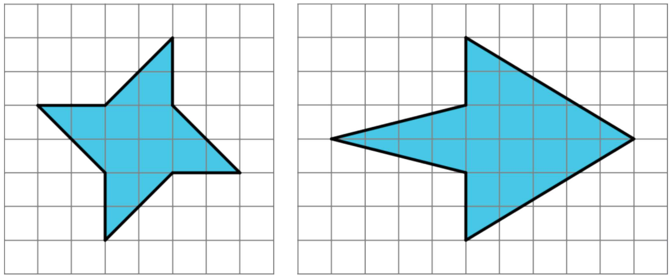
3. Find the area of the parallelogram



4. Explain why this quadrilateral is *not* a parallelogram.



5. Find the area of each shape. Show your reasoning.



6. Find the areas of the rectangles with the following side lengths.

a. 5 in and $\frac{1}{3}$ in

c. $\frac{5}{2}$ in and $\frac{4}{3}$ in

b. 5 in and $\frac{4}{3}$ in

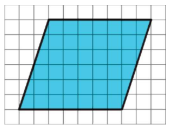
d. $\frac{7}{6}$ in and $\frac{6}{7}$ in

Unit 1, Lesson 5: Bases and Heights of Parallelograms

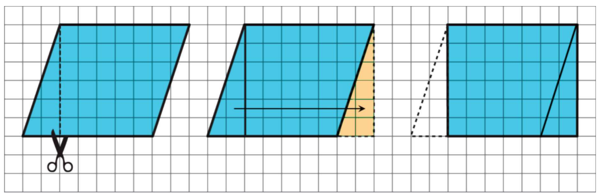
Let's investigate the area of parallelograms some more.

1.5.1 Warm-Up: A Parallelogram and Its Rectangles

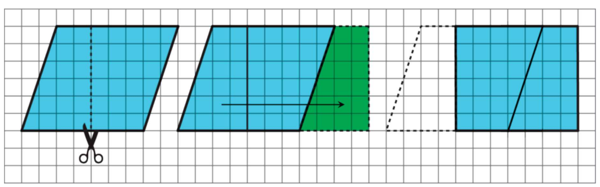
Elena and Tyler were finding the area of this parallelogram:



Here is how Elena did it:



Here is how Tyler did it:



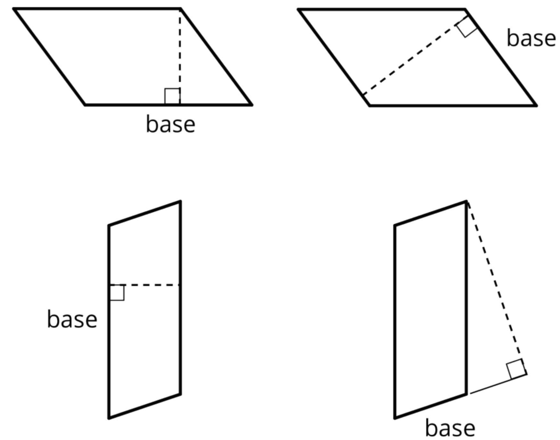
How are the two strategies for finding the area of a parallelogram the same? How they are differe

1.5.2 Exploration Activity: The Right Height?

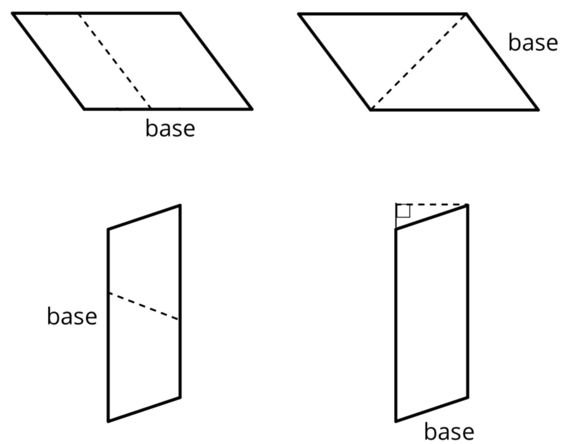
1. Each parallelogram has a side that is labeled "base."

   Study the examples and non-examples of **bases** and **heights** of parallelograms. Then, answer the questions that follow.

   Examples: The dashed segment in each drawing represents the corresponding height for the given base.



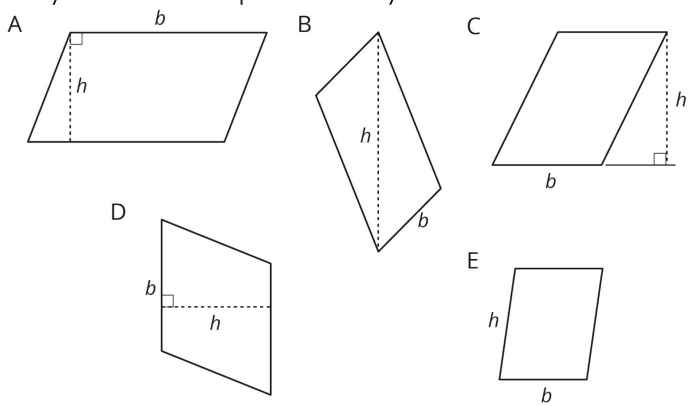
   Non-examples: The dashed segment in each drawing does *not* represent the corresponding height for the given base.



Select **all** statements that are true about bases and heights in a parallelogram.

- ☐ Only a horizontal side of a parallelogram can be base.
- ☐ Any side of a parallelogram can be a base.
- ☐ A height can be drawn at any angle to the side chosen as the base.
- ☐ A base and its corresponding height must be perpendicular to each o
- ☐ A height can only be drawn inside a parallelogram.
- ☐ A height can be drawn outside of the parallelogram, as long as it is drawn at a 90-degree angle to the base.
- ☐ A base cannot be extended to meet a height

2. Five students labeled a base b and a corresponding height h for each of these parallelograms. Are all drawings correctly labeled? Explain how you know.

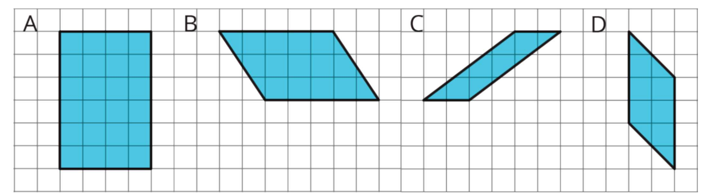


1.5.3a Exploration Activity: Finding the Formula for Area of Parallelograms

For each parallelogram:

- Identify a base and a corresponding height, and record their lengths in the table that follows.
- Find the area and record it in the right-most colu

In the last row, write an expression using b an h for the are any parallelo



parallelogram	base (units)	height (units)	area (sq units)
A			
B			
C			
D			
any parallelogram	b	h	

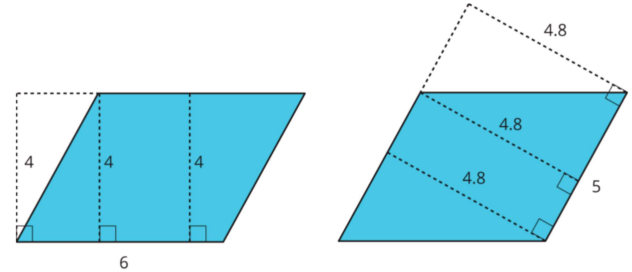
1.5.3b Exploration Extension: Are you ready for more?

1. What happens to the area of a parallelogram if the height doubles, but the base is unchanged? If the height triples? If the height is 100 times the original?

2. What happens to the area if both the base and the height double? Both triple? Both are 100 times their original lengths?

Lesson 5 Summary

- We can choose any of the four sides of a parallelogram as the **base**. Both the side (the segment) and its length (the measurement) are called the base.
- If we draw any perpendicular segment from a point on the base to the opposite side of the parallelogram, that segment will always have the same length. We call that value the **height**. There are infinitely many line segments that can represent the h

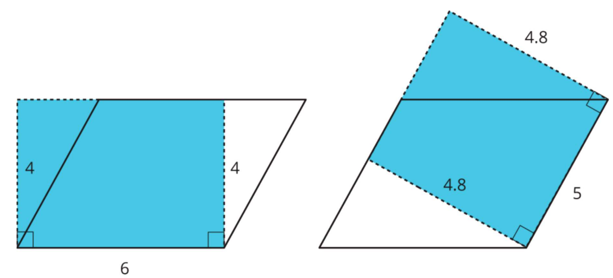


Here are two copies of the same parallelogram. On the left, the side that is the base is 6 units long. Its corresponding height is 4 units. On the right, the side that is the base is 5 units long. Its height is 4.8 units. For both, three different segments are shown to represent the height. We could draw in many more

No matter which side is chosen as the base, the area of the parallelogram is the product of that base and its corresponding height. We can check it:

$4 \times 6 = 24$ a $\quad 4.8 \times 5 = 24$

We can see why this is true by decomposing and rearran the parallelograms into rectangles.

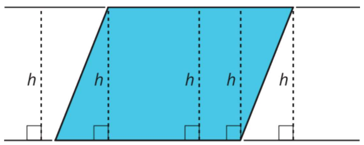


Notice that the side lengths of each rectangle are the base and height of the parallelogram. Even though the two rectangles have different side lengths, the products of the side lengths are equal, so they have the same area! And both rectangles have the same area as the parallelogram

We often use letters to stand for numbers. If b is base of a parallelogram (in units), and h is the corresponding height (in units), then the area of the parallelogram (in square units) is the product of these two numbers. $b \cdot h$

Notice that we write the multiplication symbol with a small dot instead of a × symbol. This is so that we don't get confused about whether × means multiply, or whether the letter x is standing in for a number.

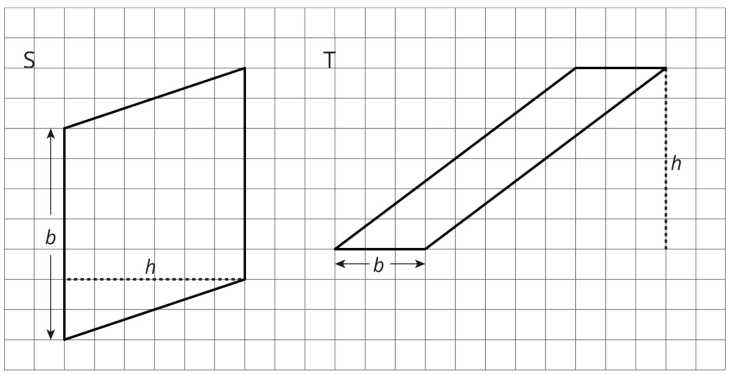
In high school, you will be able to prove that a perpendicula segment from a point on one side of a parallelogram to the opposite side will always have the same length.



You can see this most easily when you draw a parallelogram on graph paper. For now, we will just use th

1.5.4 Cool-Down

Parallelograms S and T are each labeled with a base and a corresponding height

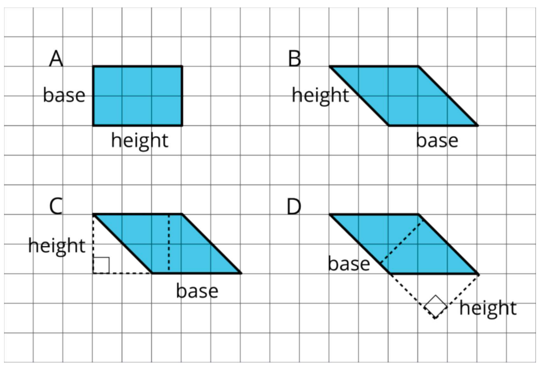


1. What are the values of b and h for each parallelogram?
   a. Parallelogram S: b = _________, h = _______
   b. Parallelogram T: b = _________, h = _________

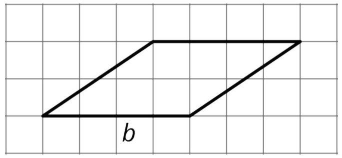
2. Use the values of b and h to find the area of each parallelogram.
   a. Area of Parallelogram S:
   b. Area of Parallelogram T:

1.5.5 Practice Problems

1. Select **all** parallelograms that have a correct height labeled for the given base.

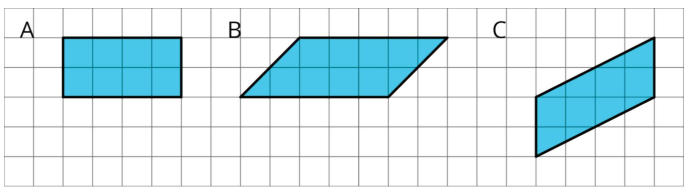


2. The side labeled b has been chosen as the base for this parallelogram.

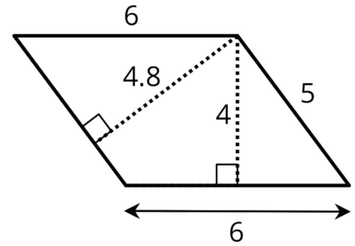


Draw a segment showing the height corresponding to that base.

3. Find the area of each par

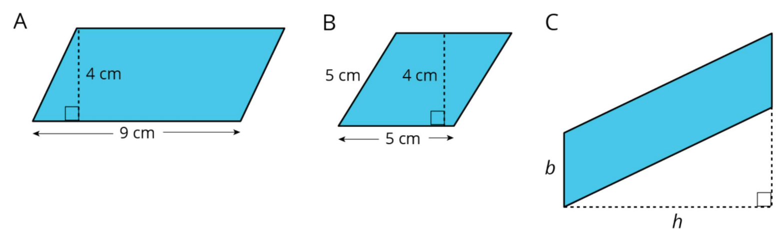


4. If the side that is 6 units long is the base of this parallelogram, what is its corresponding height?



A. 6 units
B. 4.8 units
C. 4 units
D. 5 units

5. Find the area of each parallelogram.



6. Do you agree with each of these statements? Explain your reasoning.

   a. A parallelogram has six sides.

   b. Opposite sides of a parallelogram are parallel.

   c. A parallelogram can have one pair or two pairs of parallel sides.

   d. All sides of a parallelogram have the same length.

   e. All angles of a parallelogram have the same measure.

7. A square with an area of 1 square meter is decomposed into 9 identical small squares. Each small square is decomposed into two identical triangles.

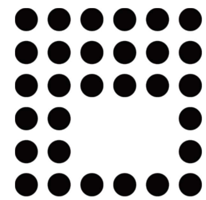
   a. What is the area, in square meters, of 6 triangles? If you get stuck, draw a diagram.

   b. How many triangles are needed to compose a region that is $1\frac{1}{2}$ square meters?

Unit 1, Lesson 6: Area of Parallelograms

Let's practice finding the area of parallelograms.

1.6.1 Warm-Up: Missing Dots

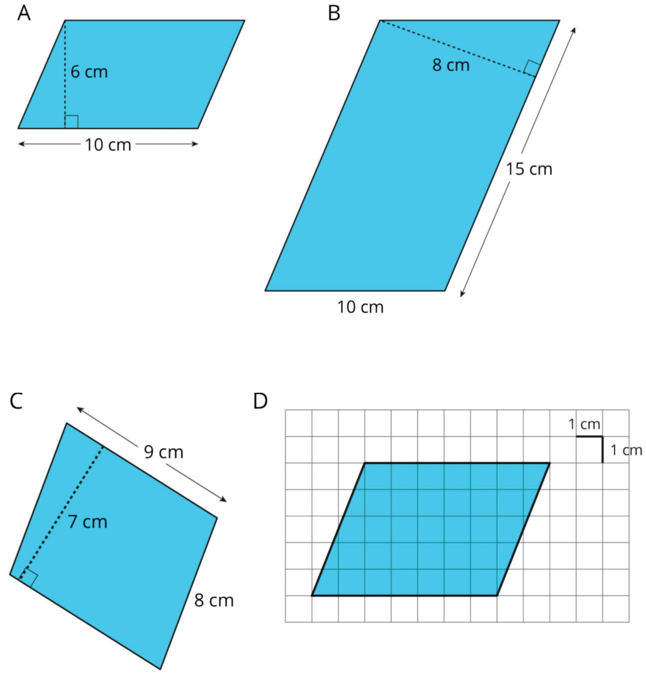


How many dots are in the image?

How do you see them?

1.6.2a Exploration Activity: More Areas of Parallelograms

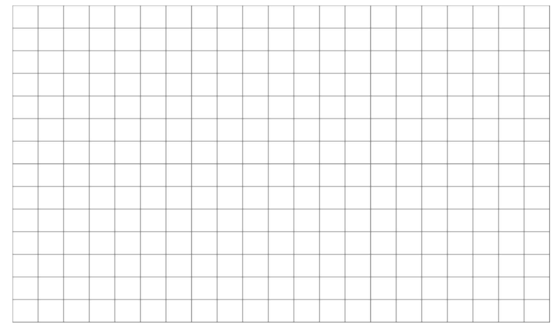
1. Find the area of each parallelogram. Show your reasoning.



2. In Parallelogram B of the first question, what is the corresponding height for the base that is 10 cm long? Explain or show your reasoning.

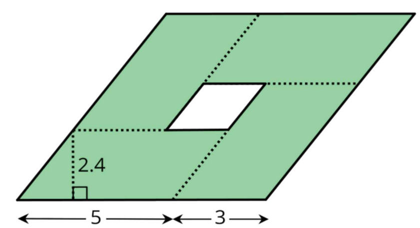
3. Two different parallelograms P and Q both have an area of 20 square units. Neither of the parallelograms are rectangles.

   On the grid, draw two parallelograms that could be P and Q.



1.6.2b Exploration Extension: Are you ready for more?

Here is a parallelogram composed of smaller parallelograms. The shaded region is composed of four identical parallelograms. All lengths are in inches.

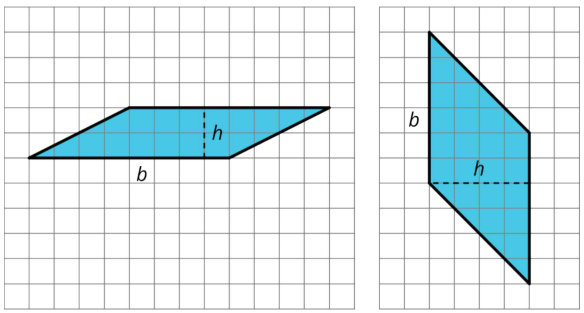


What is the area of the unshaded parallelogram in the middle? Explain or show your reasoning.

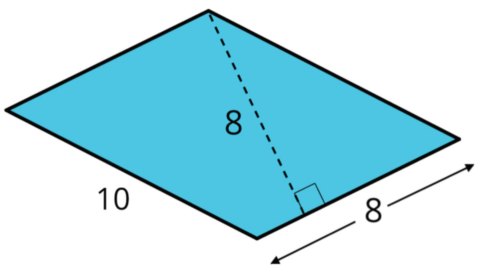
Lesson 6 Summary

Any pair of base and corresponding height can help us find the area of a parallelogram, but some base-height pairs are more easily identified than others.

- When a parallelogram is drawn on a grid and has *horizontal* sides, we can use a horizontal side as the base. When it has *vertical* sides, we can use a vertical side as the base. The grid can help us find (or estimate) the lengths of the base and of the corresponding height.

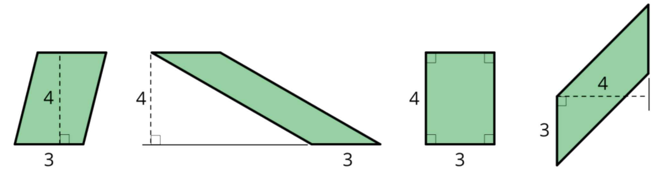


- When a parallelogram is *not* drawn on a grid, we can still find its area if a base and a corresponding height are known.

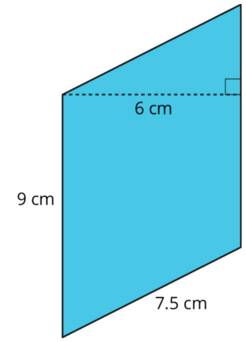


In this parallelogram, the corresponding height for the side that is 10 units long is not given, but the height for the side that is 8 units long is given. This base-height pair can help us find the area.

Regardless of their shape, parallelograms that have the same base and the same height will have the same area; the product of the base and height will be equal. Here are some parallelograms with the same pair of base-height measurements.



1.6.3 Cool-Down



1. Find the area of the parallelogram. Explain or show your reasoning.

2. Was there a length measurement you did not use to find the area? If so, explain why it was not used.