Section 10: Modeling with Geometry

The following Mathematics Florida Standards will be covered in this section:	
MAFS.912.G-MG.1.1	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
MAFS.912.G-MG.1.2	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
MAFS.912.G-MG.1.3	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).
MAFS.912.G-SRT.3.6	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
MAFS.912.G-SRT.3.7	Explain and use the relationship between the sine and cosine of complementary angles.
MAFS.912.G-SRT.3.8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. Understand that by similarity, side

	ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. Explain and use the relationship between the sine and cosine of complementary angles.

Videos in this Section

Video 1:   Density
Video 2:   Minimizing and Maximizing
Video 3:   Angles of Elevation and Depression
Video 4:   Typographic Grid Systems Based on Ratios
Video 5:   Areas in Real World Contexts
Video 6:   Volume in Real World Contexts

Section 10 – Video 1
Density

What are some examples of density in the real world?

Let's examine how we can use geometry to understand density.

➤ Density is a ratio of two measurements of an object, _____ and _____.

➤ The formula for density is _____.

Density is used to calculate mass and volume of metals, problems related to energy and British Thermal Units (BTUs), and food or liquids in different types of containers.

Let's Practice!

Kids in Cuba's rural areas often play baseball with aluminum balls, each with a diameter of $7.26\ cm$. If aluminum has a density of $2.7\ g/cm^3$, what is the mass of each ball?

At $10°C$, homogenized milk has a density of $1.032\ kg/L$, while heavy cream has a density of $1.005\ kg/L$.

At $10°C$, a container holding 8 liters of liquid weighs 8.04 kilograms. Does the container hold homogenized milk or heavy cream? How do you know?

Try It!

An American Airlines policy states that carry-on luggage should not weigh more than 22.68 kilograms and have dimensions of no more than $23 \times 36 \times 56$ centimeters, including handle and wheels. A passenger keeps track of the density of her bag, but she does not keep track of its mass. The volume of her bag is 44,785 cm^3, and density is 0.0015 kg/cm^3. Does this piece of luggage fulfill the criteria of American Airlines? Justify your answer.

One of the most widely used density-related measures is the density of an area of land such as a city.

➤ This is basically the ratio of the number of _____ to area of land. This concept is called

_____ _____.

$$population\ density = \frac{\rule{4cm}{0.4pt}}{}$$

Let's Practice!

Use the table below to answer each of the following questions.

State	Population	Land Area (sq. mi.)
Florida	18,801,310	53,927
North Carolina	9,535,483	48,711
Georgia	9,687,653	57,906
Texas	25,145,561	261,797

Which of the states listed in the table has the greatest population density and what is that density? Round your answer to the nearest person per square mile.

Why is the population density of Georgia smaller than that of North Carolina despite Georgia's larger population and larger land area? How many more people does Georgia need to have a greater population density than North Carolina?

Try It!

The local theater has an area of 2,121 square feet. At the last banquet, the fire department surveyed the theater to make sure the event was not overcrowded. Fire department inspectors found the population density to be 0.1428 people per square foot. This ratio included all people in the building, standing and seated. How many total people attended the theater on that day? Justify your answer.

BEAT THE TEST!

1. Mr. De Leon's Geometry class has 34 students. After lunch everyone returns to the classroom. Within an hour, each person produces 800 BTUs. The room is 25 feet by 32 feet by 12 feet. How many BTUs per cubic foot were produced?

| | BTUs per cubic foot.

2. Consider the table below that lists some metals and their densities.

Metal	Density (g/cm^3)
Gold	19.32
Silver	10.50
Copper	8.96
Platinum	21.45
Bronze	9.87

Your necklace is 192 grams, and it has a volume of 19.5 cubic centimeters. What metal is your necklace made out of?

3. The density of gold is 19.32 g/cm^3. Jimmy Jackson built a pyramid made out of pure gold about 2.13 $m.$ tall with a mass of 298.7 kg. If the base of the pyramid is a square, what are the dimensions of the base of the pyramid?

Section 10 – Video 2
Minimizing and Maximizing

A special case of geometry problems involves having to maximize or minimize.

For example,

➢ Maximizing or minimizing some dimensions, area, or volume

  In what real world scenario do we need to maximize area or volume?

  In what real world scenario do we need to minimize area or volume?

➢ Minimizing costs or maximizing profits

  In what real world scenario do we need to minimize costs using geometry?

  In what real world scenario do we need to maximize profits using geometry?

Let's Practice!

Coolmore Ashford Stud is a farm with one of the largest breeding operations of thoroughbred racehorses in the world. The director of operations of Coolmore wants to enclose a rectangular area for yearlings that have been purchased and are ready for pick-up.

The director wants to find the largest possible area he can enclose with 2,400 meters of fencing, and he hires you to design the area.

What is your equation for perimeter?

What would be your area equation in terms of width?

Once you have you your area equation, how do you find the maximum?

Then, what is the largest possible area company employees can enclose with 2,400 meters of fencing?

What would be the dimensions of the enclosed rectangular pen with the maximum possible area?

Try It!

It is little Alex's birthday party! Alex's parents plan to rent 1600 feet of fencing for a small petting zoo. They will form two paddocks with one shared fence running down the middle; one for donkeys and the other for goats. What is the maximum area that Alex's parents can obtain, and what are the dimensions of each of the two paddocks?

Let's Practice!

Each *DubbleBubble* gumball is an inch diameter. The packaging company manager has three different options for packaging the gumballs.

Box Name	Dimensions
Optimum	$1.5 \times 2 \times 4 \; inches$
Classic	$2.4 \times 2 \times 5 \; inches$
Modern	$1.25 \times 4 \times 4.8 \; inches$

The store will sell the boxes of gumballs at the same price no matter what type of box is used. However, the manager wants to use a box that requires the least amount of gumballs to maximize profits. Which box should the manager use?

BEAT THE TEST!

1. The leaders of an afterschool program are creating a rectangular garden against the back of their building with a fence around it, so that instructors can teach gardening principles to their students. They only have 120 feet of fencing available for the project.

   What would the dimensions of the garden be if the builders attached one side of it to the afterschool building to make the area of the garden as large as possible?

   A $30 \times 60 \ ft.$
   B $40 \times 40 \ ft.$
   C $50 \times 25 \ ft.$
   D $70 \times 30 \ ft.$

2. Manufacturers sell cylindrical cans by the dozen. Workers pack the cans in rectangular containers that are available in the following sizes:

- ➢ 2 × 6,
- ➢ 3 × 4,
- ➢ 1 × 12, and
- ➢ 2 × 3 × 2.

A can of tomato sauce is 4 inches tall with a diameter of 3 inches. Which rectangular container is requiring the least volume (and will be the most cost effective)?

A 1 × 12
B 2 × 3 × 2
C 2 × 6
D 3 × 4

Section 10 – Video 3
Angles of Elevation and Depression

Angles of elevation are angles _____ the horizontal.

Angles of depression are angles _____ the horizontal.

Explain angles of elevation and angles of depression in your own words.

How does a right triangle fit into this topic?

Describe a situation where you would deal with angles of elevation.

Describe a situation where you would deal with angles of depression.

Consider the diagram below. Label the angles of elevation and depression in the appropriate spaces provided.
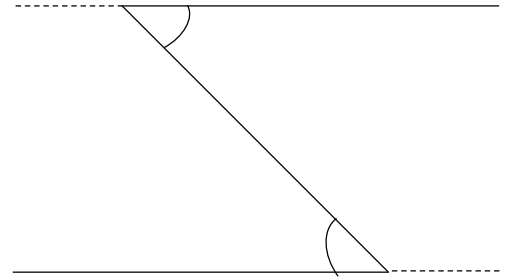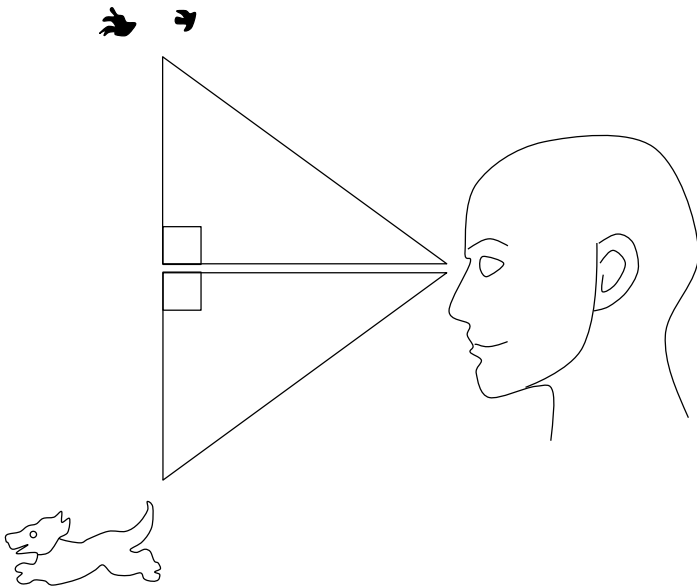
Consider the diagram below. Imagine that you see a flock of birds at an angle of elevation of 32°.
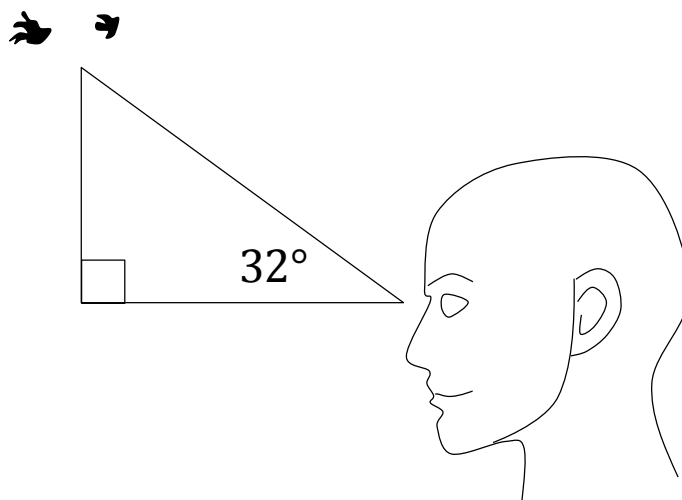
If the birds are flying at an altitude of $12{,}000\ ft$.; what does this flight pattern mean in regard to angles of elevation or depression?

If your eye level is $6\ ft$. above the ground, what is the vertical distance from your eyes to the bird?

How can you use this information to find your horizontal distance from the birds?

Let's Practice!

An airplane is flying at an altitude of 39,000 ft. The airplane will be landing on a tarmac 128 mi. away. Find the average angle at which the airplane must descend for landing. Round your answer to the nearest tenth of a unit.

Consider the diagram below that represents someone's eye level as he looks at his dog. Find the value of x, and round to the nearest hundredth foot.

25°

x

13 $ft.$
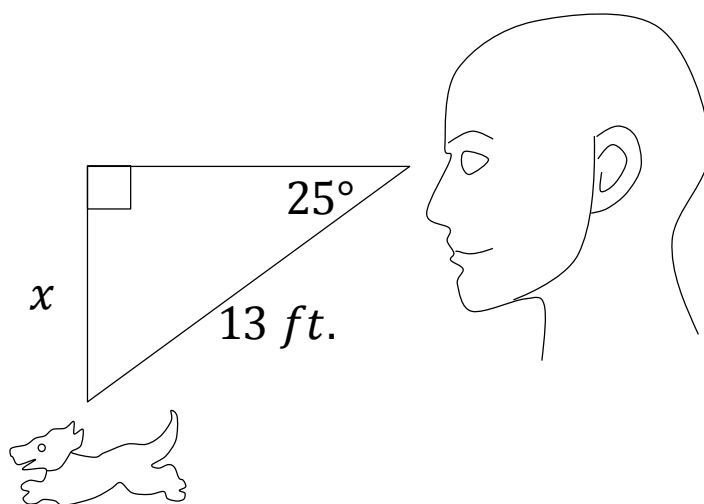
Try It!

Lionel is standing 40 ft. from a flagpole. The flagpole is 32 ft. tall, and Lionel's eye level is 5 ft. from the ground. What is the angle of elevation?

You are standing on a 59.5 foot high hill looking down on a lake at an angle of depression of 48°. How far are you from the lake? (Round your answer to the nearest foot.)

BEAT THE TEST!

1. A man is 6 feet 3 inches tall. The tip of his shadow touches a fire hydrant that is 13 feet 6 inches away from the man. What is the angle of elevation from the end of the base of the fire hydrant to the top of the man's head? (Round your answer to the nearest tenth of a degree.)

> A 24.8
> B 34.5
> C 42.6
> D 65.2

Section 10 – Video 4
Typographic Grid Systems Based on Ratios

➢ A _____ is a two-dimensional framework made up of a pattern of intersecting straight or curved lines.

➢ This structure can be used to organize shapes in a logical manner.

➢ Let's consider a real world example of a typographic grid system based on ratios.

The city of Oakland, California received a grant from the *Fédération Internationale de Football Association* (FIFA) to design and build a new soccer complex for local residents. A $700 \times 400 \, ft.$ rectangular tract of land is available in the city suburbs.

How many full size soccer fields fit in this site? Each full-size soccer field measures $120 \times 80 \, yds.$

How many medium size soccer fields fit in this site? Each medium-size soccer field measures $60 \times 40 \, yds.$

Let's Practice!

The complex in Oakland, California, will contain two full-size soccer fields and a multipurpose building that houses locker rooms, offices, an equipment room, a weight room, a conference room, and a refreshment stand. The complex will meet the following specifications:

> Each full-size soccer field must measure 120 by 80 yards.
> Each field must be at least 30 feet from the boundary of the tract.
> At least 15 yards must separate each field.
> The multipurpose building must be only one-story tall. It has a trapezoid shape, and it must be at least 50 feet from any of the fields.

Sketch the design on the grid below, maximizing the size of the multipurpose building. The distance between gridlines is 50 feet.
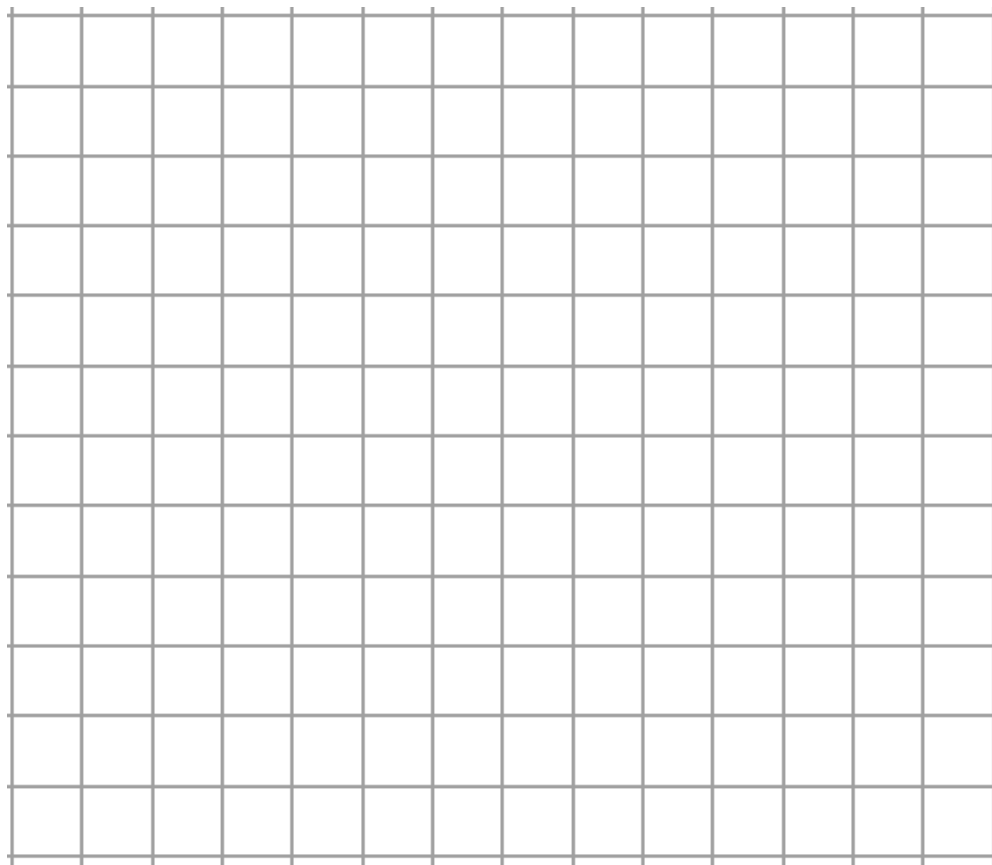
Try it!

Ben is selling half of his property. He owns a track of land in the shape of an isosceles trapezoid with bases measuring 900 feet and 700 feet with a perpendicular distance of 300 feet between the bases. Ben wants to subdivide his lot into two regions of equal areas by installing a fence between them.

Draw a diagram below that models this problem. The distance between gridlines is 50 feet. Label all given and calculated lengths.
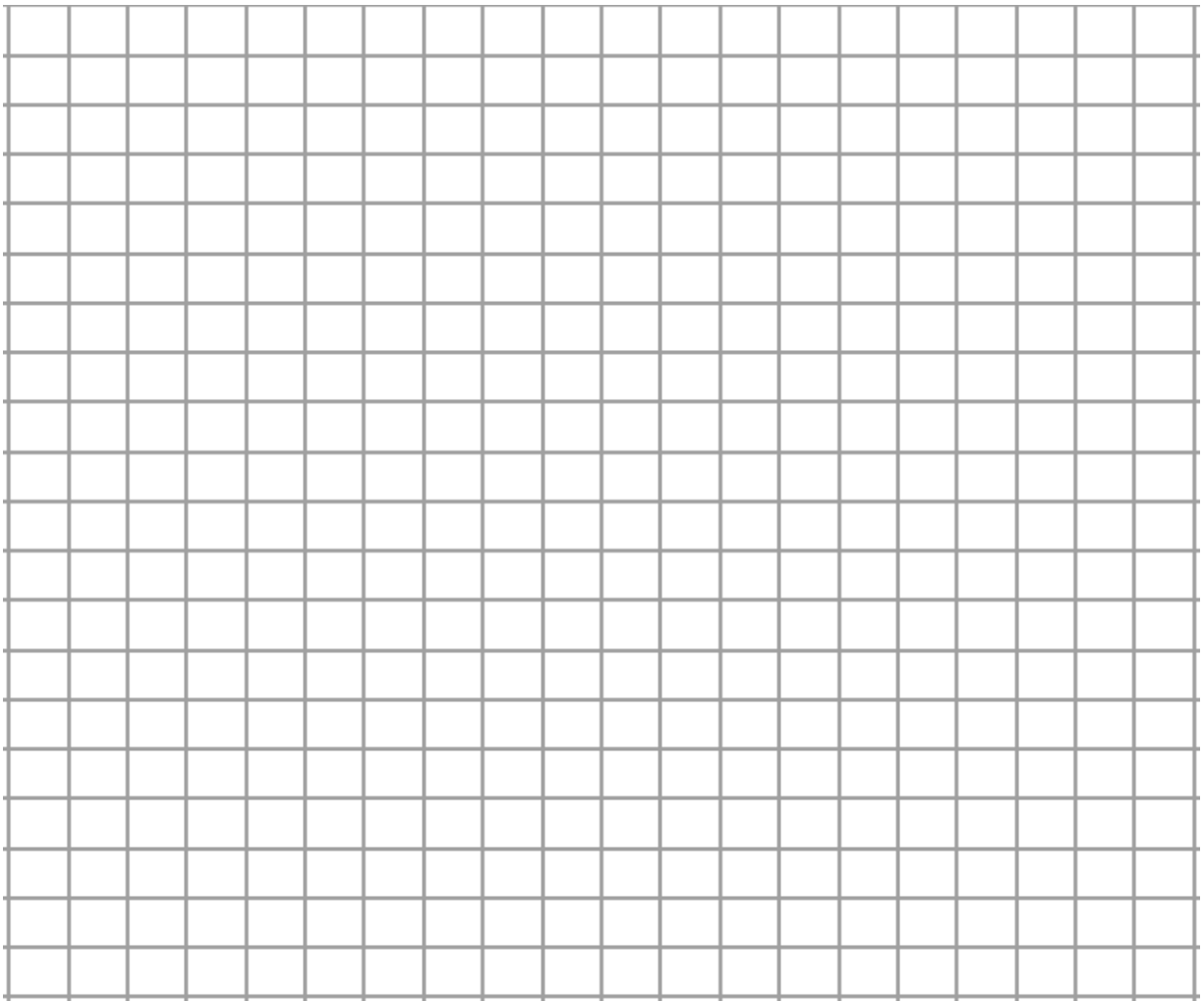
What is the length of the fence?

How much area of land he is selling.

BEAT THE TEST!

1. Using following map, estimate the area of Lake Michigan. The distance between gridlines is 18 miles.
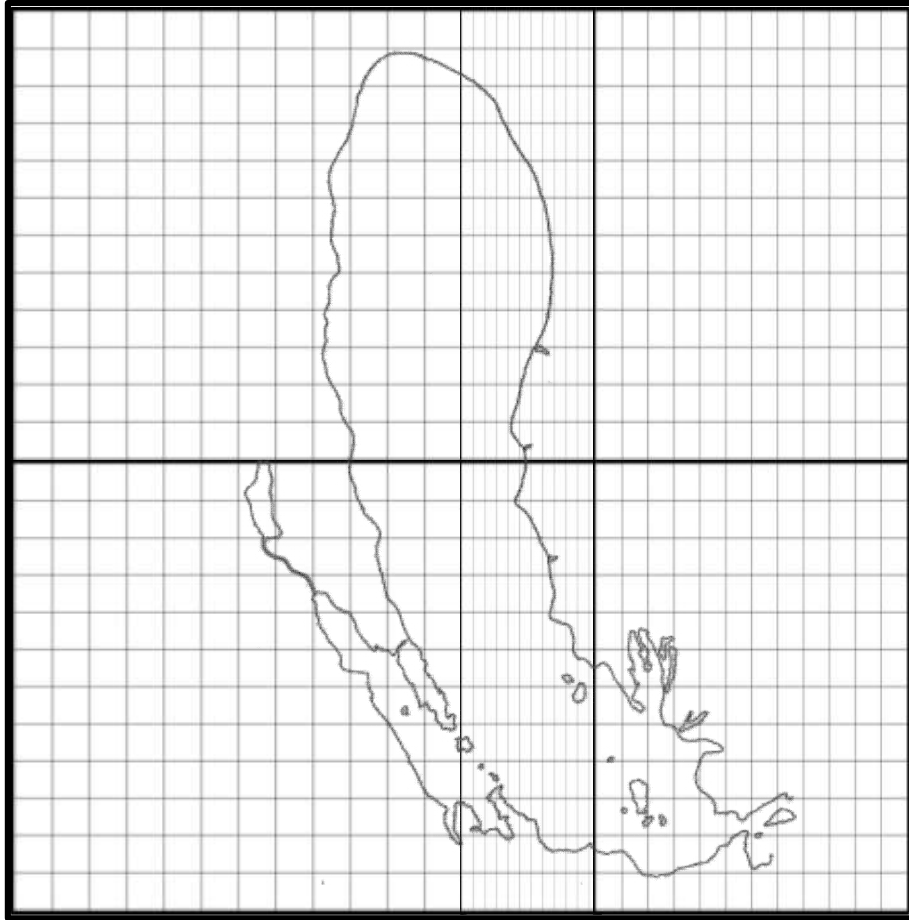
Which of the following is the best estimate of the surface area of the lake?

    A 9,940 mi^2
    B 18,124 mi^2
    C 22,394 mi^2
    D 28,012 mi^2

2. You work for a food packaging company, and your job is to pack soda crackers in a big can and ham slices in a box. Use the diagram below to estimate the amount of soda crackers you can fit into the can and the amount of ham slices you can fit into the box.

Each grid on the graph below represents a square inch. Each soda cracker is an eighth of an inch thick and each ham slice is a fourth of an inch thick. The crackers and ham are stacked on top of each other in their respective containers.

hams.

crackers.
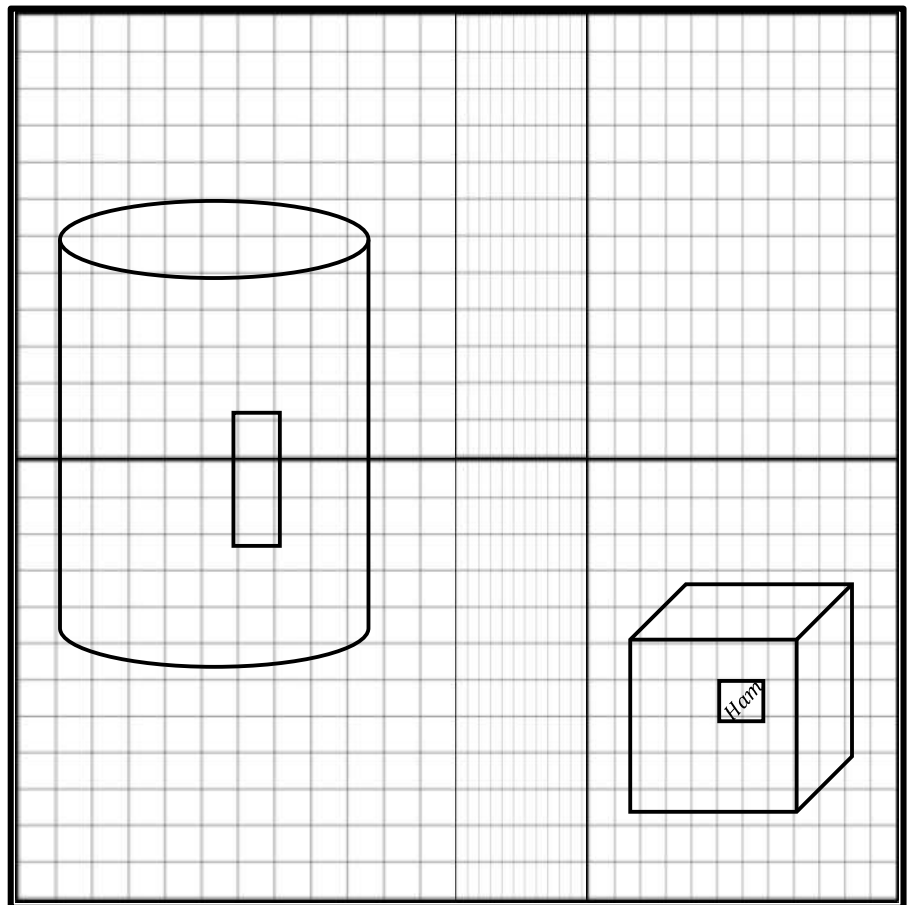
Section 10 – Video 5
## Areas in Real World Contexts

Describe these objects found in the real world using geometric shapes.
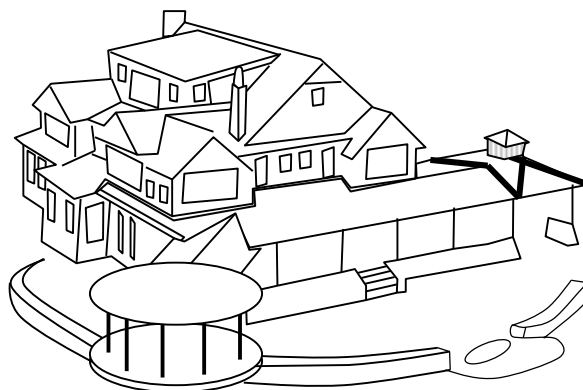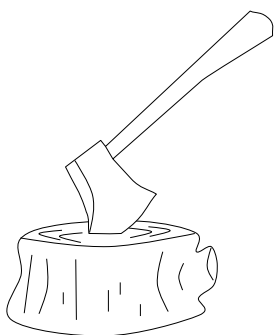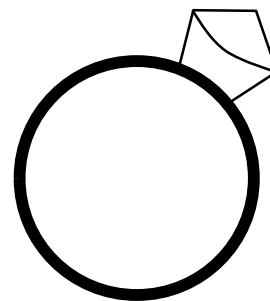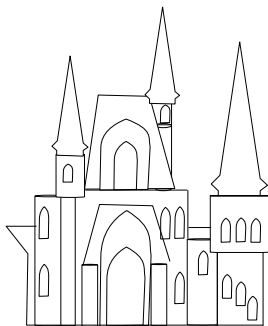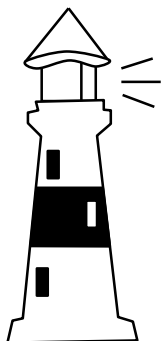
We can use measures of geometric shapes to find the area, volume, surface area, perimeter, or circumference of a shape found in the real world.

Let's Practice!

Four *amigos* run at a circular track every evening after work. Consider the diagram, below which represents the track that they use. As shown, the track has four lanes, and each lane is 5 meters wide.
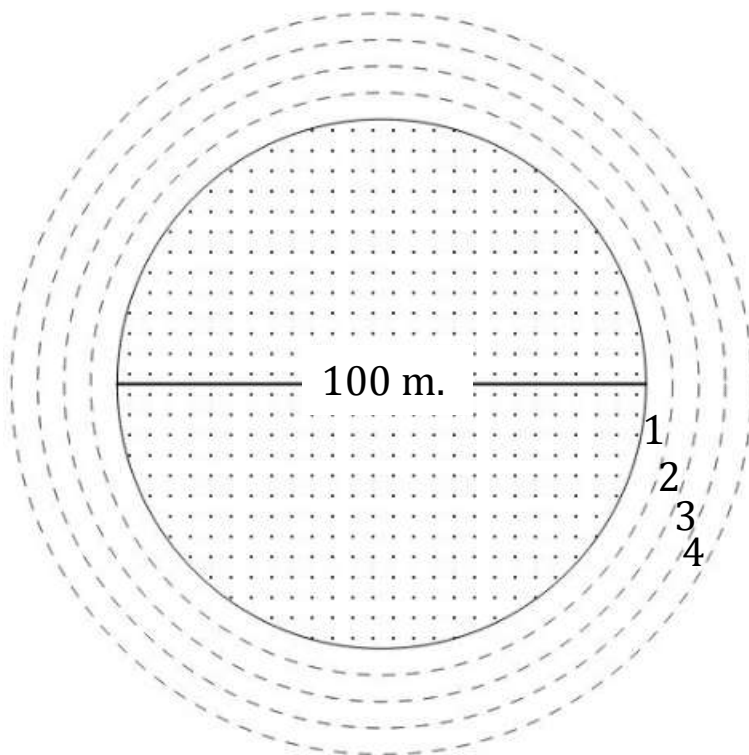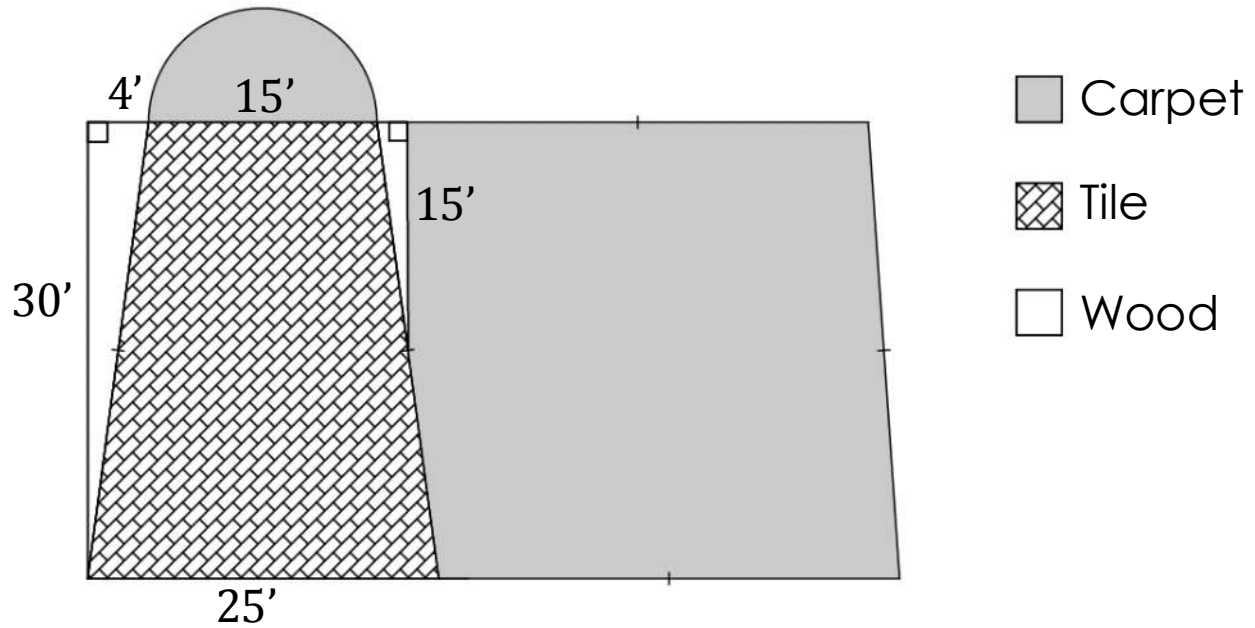

How much area does each lane occupy in the above diagram?

100 m.

1
2
3
4
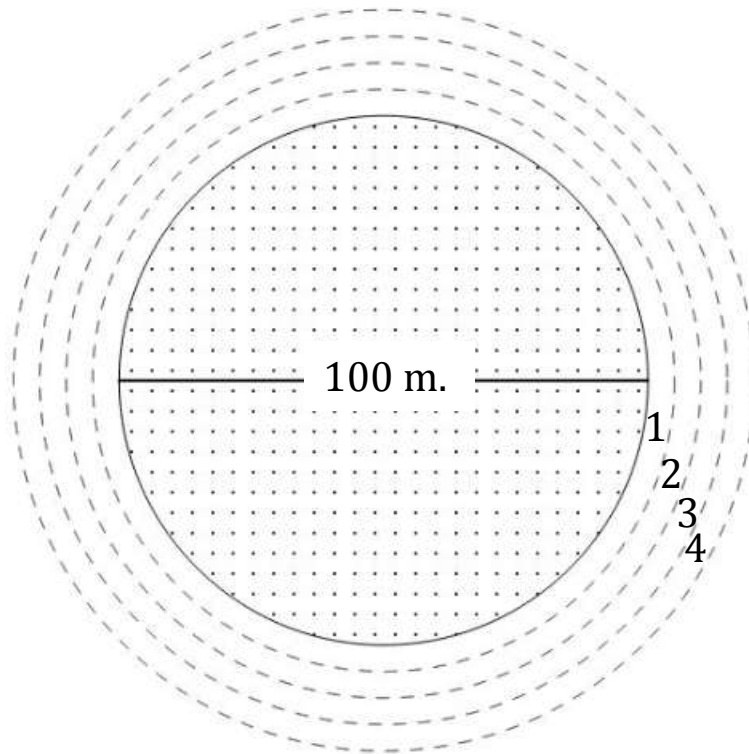
If the four amigos want to race each other, would the competition be fair for the runners on the outer lanes if they all started side by side? Why or why not?

How much longer would the race be for the runners in lanes 2, 3, and 4 than for the runner in lane 1?
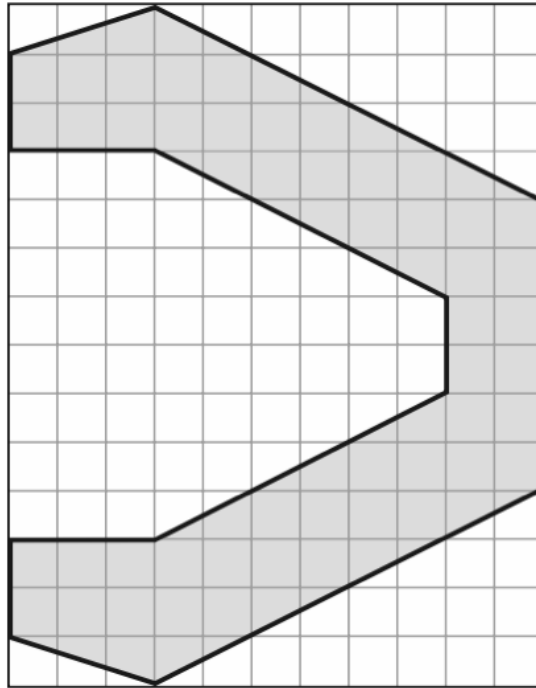
The new floor plan for the interior of a home is below.

Carpet cost $1.14/sq.ft. Tile cost $1.29/sq.ft. Wood Flooring cost $1.99/sq.ft. How much will this project cost?

BEAT THE TEST!

1. Your company will have its own miniature golf course. Consider the diagram, below that shows the project design.


The diameter of the golf hole on a putting green is 4.25". The miniature golf course planned in this project will have 3 holes. How many square feet of turf will be needed to cover the putting green if one square represents 1.5 square feet? Justify your answer.

Section 10 – Video 6
Volume in Real World Contexts

Suppose *NASA* is designing a commercial spherical spaceship to start selling one-week trips for an orbit around the Earth. *NASA* allocates 1,000 ft^3 for each person, plus an additional 3,067.5 ft^3 for various necessary machines and other items such as furniture, bathroom supplies, kitchenettes, etc. The diameter of the ship is 53.6 ft.
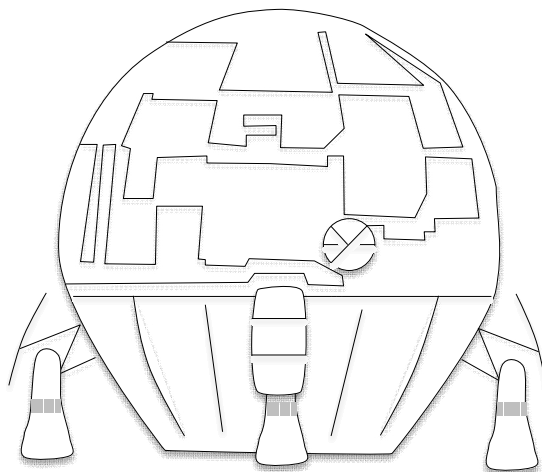
What is the maximum number of people that would be able to ride in the ship per trip?

A contractor designs another spaceship, more accessible in terms of price, but smaller at a scale factor of $\frac{1}{3}$ with respect to the original dimensions.

What is the maximum number of people that would be able to ride the contractor's ship?

Let's Practice!

At your local supermarket, there is a sack full of rice in a shape of a cylinder with height of 20"and radius of 7".

7"

20"
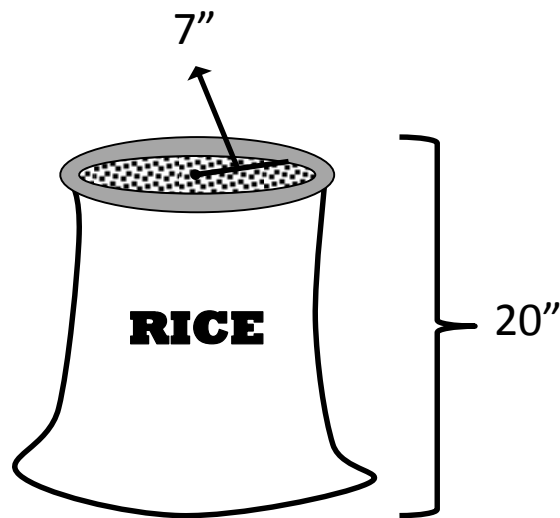
RICE

If there are approximately 50 grains of rice in a cubic inch, approximately how many grains of rice are in this sack?

There is also a 2.5-kilogram bag of rice at the same price as the above cylinder sack. If there are 64 grains of rice in 1 gram of rice, which one has more rice, the cylinder sack or the bag? Justify your answer.

Try It!

The local recreation center is building a new Olympic-size pool to be 164 feet long, 82 feet wide, and 12 feet deep.

Based on the above information, approximately how much water will the pool hold?

The excavated dirt for the pool described above will be hauled away by wheelbarrow and dumped into a truck. If the wheelbarrow holds 9 cubic feet of dirt, how many wheelbarrows of dirt must be hauled away and dumped into the truck?

BEAT THE TEST!

1. The Strickland High School janitors empty 25 full trash cans every day. The design of each trash can is shown below.

4.5'

3'

2'

S.H.S
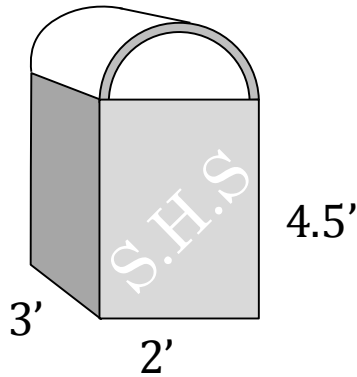
Based on the information and diagram above, what is the total volume of trash that the janitors empty?

_____ ft^3.