

## Section 15: Additional Modeling with Geometry

**The following Mathematics Florida Standards will be covered in this section:**

MAFS.912.G-GMD.1.3	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
MAFS.912.G-MG.1.1	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
MAFS.912.G-MG.1.2	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
MAFS.912.G-MG.1.3	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).
MAFS.912.G-GPE.2.7	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.
MAFS.912.G-SRT.3.6	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

MAFS.912.G-SRT.3.8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. Explain and use the relationship between the sine and cosine of complementary angles.
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### Topics in this Section

- Topic 1: Density
- Topic 2: Minimizing and Maximizing
- Topic 3: Angles of Elevation and Depression
- Topic 4: Typographic Grid Systems Based on Ratios
- Topic 5: Areas in Real-World Contexts
- Topic 6: Volume in Real-World Contexts



## Section 15 – Topic 1

### Density

What are some examples of density in the real-world?

Let's use geometry to understand density.

- Density is a ratio of two measurements of an object, \_\_\_\_\_ and \_\_\_\_\_.
- The formula for density is \_\_\_\_\_.

Density is used to calculate things like the mass and volume of metals, energy and British Thermal Units (BTUs), and food or liquids in different types of containers.

#### Let's Practice!

1. Kids in Cuba's rural areas often play baseball with aluminum balls, each with a diameter of  $7.26\text{ cm}$ . If aluminum has a density of  $2.7\text{ g/cm}^3$ , what is the mass of each ball?
2. Homogenized milk has a density of  $1.032\text{ kg/L}$  and heavy cream has a density of  $1.005\text{ kg/L}$  at  $10^\circ\text{C}$ . At  $10^\circ\text{C}$ , a container holding 8 liters of liquid weighs 8.04 kilograms. Does the container hold homogenized milk or heavy cream? How do you know?

#### Try It!

3. An American Airlines policy states that carry-on luggage should not weigh more than 22.68 kilograms and have dimensions of no more than  $23 \times 36 \times 56$  centimeters, including handle and wheels. A passenger keeps track of the density of her bag, but she does not keep track of its mass. If the volume of her bag is  $44,785\text{ cm}^3$  and density is  $0.0015\text{ kg/cm}^3$ , then does her piece of luggage meet the criteria for American Airlines?

One of the most widely used density-related measures is the density of an area of land, such as a city.

This is determined by the ratio of the number of \_\_\_\_\_ to area of land. This concept is called \_\_\_\_\_.

#### Let's Practice!

4. Use the table below to answer each of the following questions.

State	Population	Land Area (sq. mi.)
Florida	18,801,310	53,927
North Carolina	9,535,483	48,711
Georgia	9,687,653	57,906
Texas	25,145,561	261,797

- a. Which state listed in the table has the greatest population density? Round your answer to the nearest person per square mile.



- b. Explain why the population density of Georgia is lower than that of North Carolina despite Georgia's larger population and larger land area. How many more people does Georgia need in order to have a greater population density than North Carolina?

**Try It!**

5. The local theater has an area of 2,121 square feet. At the last banquet, the fire department surveyed the theater to make sure the event was not overcrowded. The fire department inspectors found the population density to be 0.1428 people per square foot. How many people attended the theater on that day? Justify your answer.

**BEAT THE TEST!**

1. Mr. De Leon's Geometry class has 34 students. After lunch everyone returns to the classroom. Within an hour, each person produces 800 BTUs of heat. The room is 25 feet by 32 feet by 12 feet. How many BTUs per cubic foot were produced?

BTUs per cubic foot.

2. Consider the table below that lists some metals and their densities.

<b>Metal</b>	<b>Density (<math>g/cm^3</math>)</b>
Gold	19.32
Silver	10.50
Copper	8.96
Platinum	21.45
Bronze	9.87

A necklace found at a pawn shop is 192 grams and has a volume of 19.5 cubic centimeters. What metal is the necklace made of?



## Section 15 – Topic 2 Minimizing and Maximizing

A special case of geometry problems involves maximizing and minimizing.

For example:

- Maximizing or minimizing some dimensions, area, or volume
- Minimizing costs or maximizing profits

*In what real-world scenario do we need to maximize area or volume?*

*In what real-world scenario do we need to minimize area or volume?*

*In what real-world scenario do we need to minimize costs using geometry?*

*In what real-world scenario do we need to maximize profits using geometry?*

### **Let's Practice!**

1. Coolmore Ashford™ Stud is a farm with one of the largest breeding operations of thoroughbred racehorses in the world. The director of operations of Coolmore Ashford™ needs to enclose a rectangular area for yearlings that have been purchased and are ready for pick-up.

The director wants to find the largest possible area he can enclose with 2,400 meters of fencing and he hires you to design the area.

- a. What is your equation for perimeter?
- b. What would be your area equation in terms of width?
- c. Once you have your area equation, how do you find the maximum?
- d. What is the largest possible area that can be enclosed with 2,400 meters of fencing?
- e. What would be the dimensions of the enclosed rectangular pen with the maximum possible area?



### Try It!

- It is little Alex's birthday party! Alex's parents plan to rent 1,600 feet of fencing for a small petting zoo. They will form two paddocks with one shared fence running down the middle; one for donkeys and the other for goats. What is the maximum area that Alex's parents can obtain for the entire zoo? What are the dimensions of each of the two paddocks?
- Each *Dubble Bubble*™ gumball measures an inch in diameter. The packaging company manager has three different options for packaging the gumballs. The table below summarizes his options.

Box Name	Dimensions
Box 1	$1.5 \times 2 \times 4$ inches
Box 2	$2.4 \times 2 \times 5$ inches
Box 3	$1.25 \times 4 \times 4.8$ inches

The store will sell the boxes of gumballs at the same price no matter what type of box is used; however, the manager wants to use a box that holds the least amount of gumballs to maximize profits. Determine which box the manager should use.

### BEAT THE TEST!

- The leaders of an afterschool program are creating a rectangular garden against the back of their building with a fence around it so that instructors can teach gardening principles to their students. They only have 120 feet of fencing available for the project.

What would the dimensions of the garden be if the builders attached one side of the fence to the building in order to make the area of the garden as large as possible?

- (A)  $30 \times 60$  ft
  - (B)  $40 \times 40$  ft
  - (C)  $50 \times 25$  ft
  - (D)  $70 \times 30$  ft
- Manufacturers sell cylindrical cans by the dozen. Workers pack the cans in rectangular containers that are available in different sizes, depending on how the cans are packed or arranged. Cans can be packed in different ways.
    - $2 \times 6$  cans
    - $3 \times 4$  cans
    - $1 \times 12$  cans
    - $2 \times 3 \times 2$  cans

A cost-effective rectangular container is the one that can be made with the least amount of material and still hold the dozen cylindrical cans. A can of tomato sauce is 4 inches tall with a diameter of 3 inches. Which packing configuration is the most cost-effective?

- (A)  $1 \times 12$  cans
- (B)  $2 \times 3 \times 2$  cans
- (C)  $2 \times 6$  cans
- (D)  $3 \times 4$  cans



**Section 15 – Topic 3**  
**Angles of Elevation and Depression**

Angles of elevation are angles \_\_\_\_\_ the horizon.

Angles of depression are angles \_\_\_\_\_ the horizon.

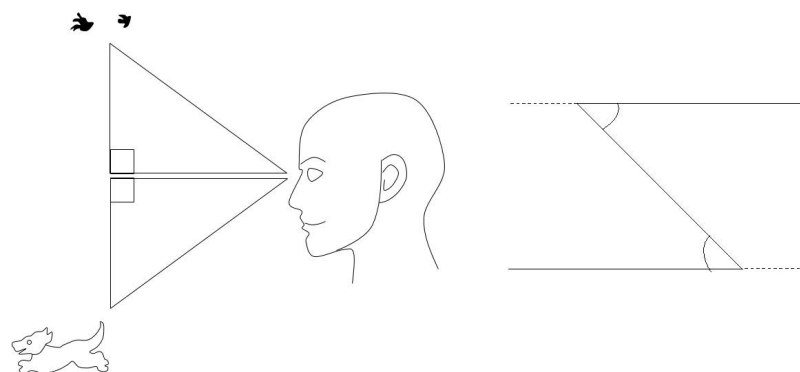
Explain angles of elevation and angles of depression in your own words.

Explain how a right triangle fits into angles of elevation or depression.

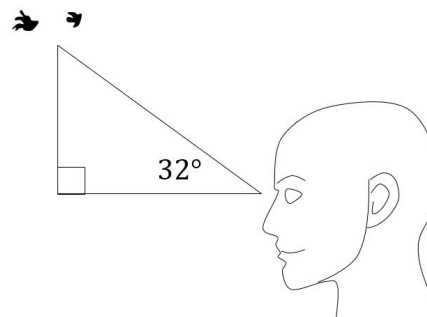
Describe a situation where you would deal with angles of elevation.

Describe a situation where you would deal with angles of depression.

Label the angles of elevation (E) and depression (D) in the appropriate spaces provided.



Suppose that you see a flock of birds at an angle of elevation of  $32^\circ$ .



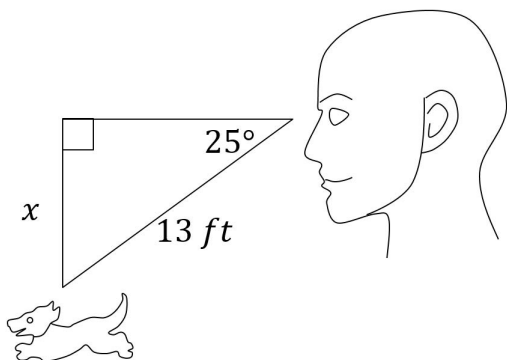
If the birds are flying at an altitude of  $12,000 \text{ ft}$ , then what does this flight pattern mean with regards to angles of elevation or depression?

If your eye level is  $6\text{ ft}$  above the ground, then what is the vertical distance from your eyes to the bird?

How can you use this information to find your horizontal distance from the birds?

### Let's Practice!

1. Suppose that an airplane is currently flying at an altitude of  $39,000\text{ ft}$  and will be landing on a tarmac  $128\text{ mi}$  away. Find the average angle at which the airplane must descend for landing. Round your answer to the nearest tenth of a unit.
2. Consider the diagram below that represents someone's eye level as he looks at his dog. Find the value of  $x$ , and round to the nearest hundredth of a foot.



### Try It!

3. If Lionel has an eye level of 5 feet and standing 40 feet from a flagpole that is 32 feet tall, then what is the angle of elevation?
4. Suppose that you are standing on a 59.5-foot hill looking down on a lake at an angle of depression of  $48^\circ$ . How far are you from the lake? (Round your answer to the nearest foot.)

### BEAT THE TEST!

1. A man is 6 feet 3 inches tall. The tip of his shadow touches a fire hydrant that is 13 feet 6 inches away. What is the angle of elevation from the end of the base of the fire hydrant to the top of the man's head? (Round to the nearest tenth of a degree.)  

(A)  $24.8^\circ$   
(B)  $34.5^\circ$   
(C)  $42.6^\circ$   
(D)  $65.2^\circ$

## Section 15 – Topic 4

### Typographic Grid Systems Based on Ratios

A \_\_\_\_\_ is a two-dimensional framework made up of a pattern of intersecting straight or curved lines.

This structure can be used to organize shapes in a logical manner.

The city of Oakland, California received a grant from the *Fédération Internationale de Football Association (FIFA™)* to design and build a new soccer complex for local residents. A  $700 \times 400$  ft rectangular tract of land is available in the city suburbs.

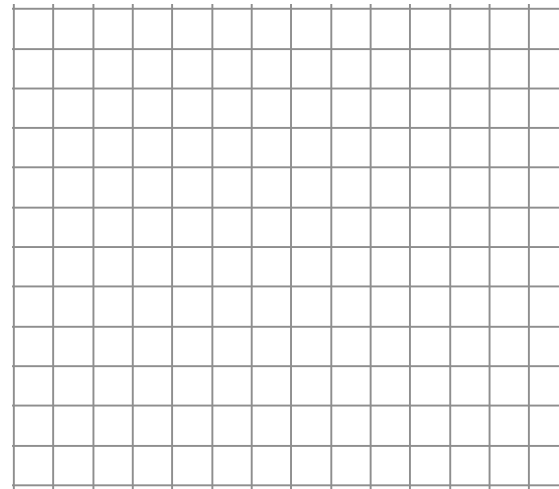
If each full-size soccer field measures  $120 \times 80$  yds, then how many full-size soccer fields can fit in this site?

If each medium-size soccer field measures  $60 \times 40$  yds, then how many medium size soccer fields can fit in this site?

### **Let's Practice!**

1. The complex in Oakland, California, will contain two full-size soccer fields and a multipurpose building. The complex will meet the following specifications:
  - Each full-size soccer field must measure 120 by 80 yards.
  - Each field must be at least 30 feet from the boundaries of the tract.
  - At least 15 yards must separate each field.
  - The multipurpose building must be only one-story tall. It must be a trapezoid shape, and it must be at least 50 feet away from any of the fields.

Sketch the design on the grid below, maximizing the size of the multipurpose building. The distance between gridlines is 50 feet.

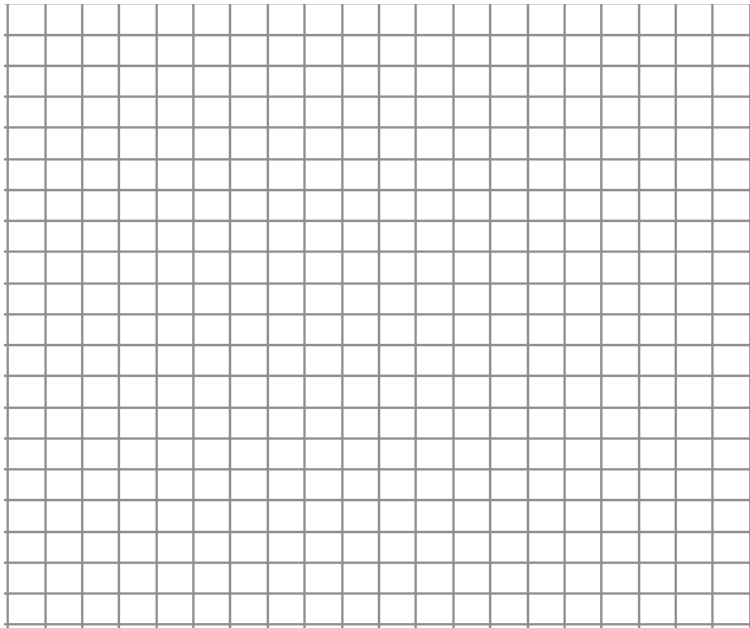




**Try It!**

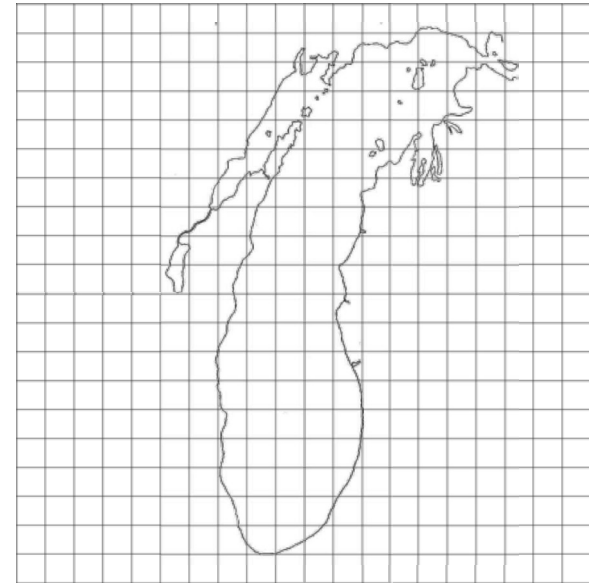
2. Ben is selling half of his property. He owns a tract of land in the shape of an isosceles trapezoid with bases measuring 900 feet and 700 feet with a perpendicular distance of 300 feet between the bases. Ben wants to subdivide his lot into two regions of equal areas by installing a fence between them.

Draw a diagram below that models this problem. The distance between gridlines is 50 feet. Label all given and calculated lengths. How much land is he selling?



**BEAT THE TEST!**

1. Using the following map, estimate the area of Lake Michigan. The distance between gridlines is 18 miles.

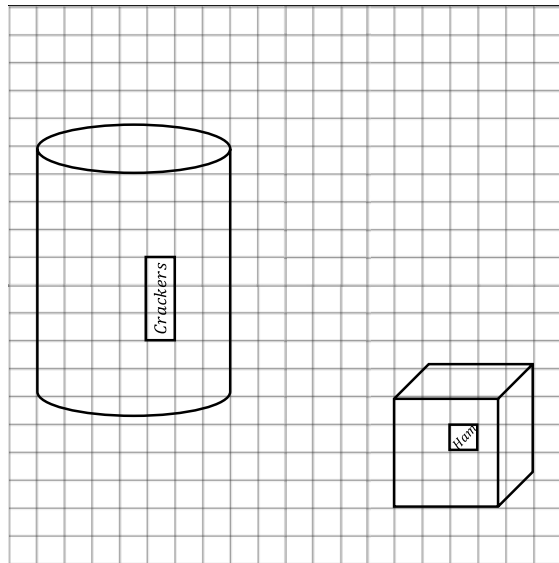


Which of the following is the best estimate of the surface area of the lake?

- (A)  $9,940 \text{ mi}^2$
- (B)  $18,124 \text{ mi}^2$
- (C)  $22,394 \text{ mi}^2$
- (D)  $28,012 \text{ mi}^2$

2. At a food packaging company, workers pack soda crackers into big cans and ham slices into boxes. Use the diagram below to estimate the amount of soda crackers workers can fit into a can and the amount of ham slices workers can fit into a box.

Each grid on the graph below represents a square inch. Each soda cracker is an eighth of an inch thick and each ham slice is a fourth of an inch thick. The crackers and ham are stacked on top of each other in their respective containers.

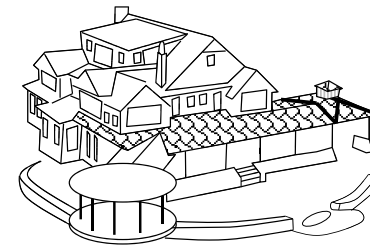
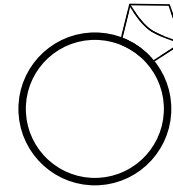


Workers can pack  ham slices into a box and pack  soda crackers into a can.

## Section 15 – Topic 5

### Areas in Real-World Contexts

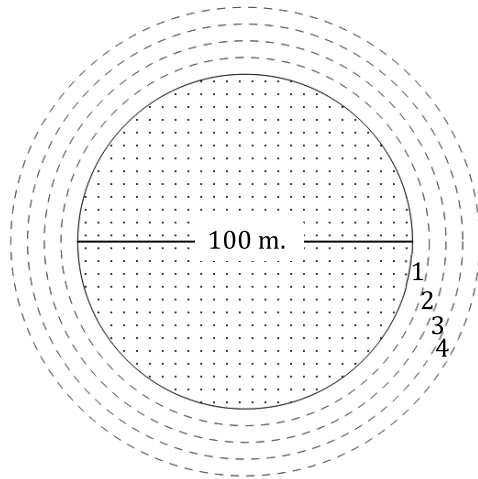
Use geometric shapes to describe these objects found in the real-world.



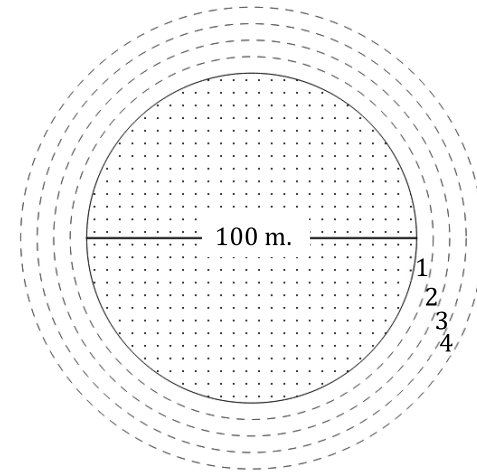
We can use measures of geometric shapes to find the area, volume, surface area, perimeter, or circumference of a shape found in the real-world.

**Let's Practice!**

1. Four friends run at a circular track every evening after work. As shown below, the track has four lanes and each lane is 5 meters wide.



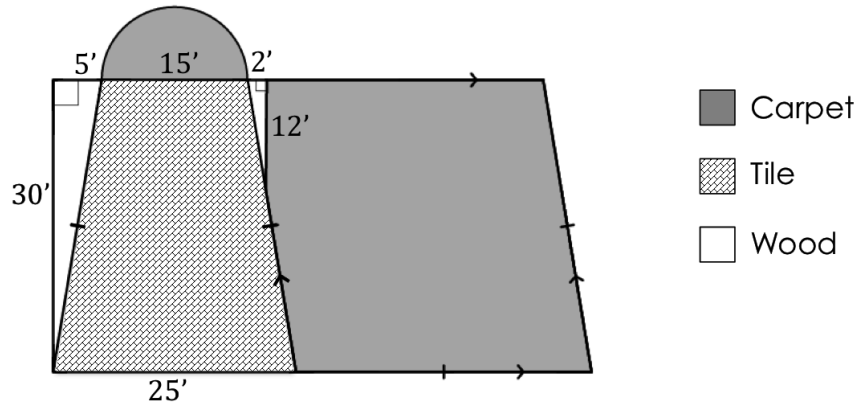
- a. How much of the circumference does each lane occupy?



- b. If the four friends want to race each other, then would the competition be fair for the runners on the outer lanes if they all started side by side? Why or why not?
- c. How much longer would the race be for the runners in lanes 2, 3, and 4 than for the runner in lane 1?

**Try It!**

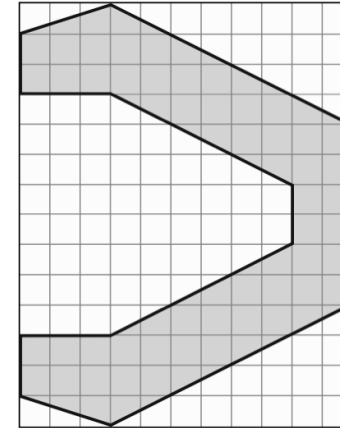
2. The new floor plan for the interior of a home is below. Marisa wants to cover various parts of the floors with carpet, tile, or wood.



If carpet costs  $\$1.14/sq. ft$ , tile costs  $\$1.29/sq. ft$ , and wood flooring costs  $\$1.99/sq. ft$ , then how much will this project cost?

**BEAT THE TEST!**

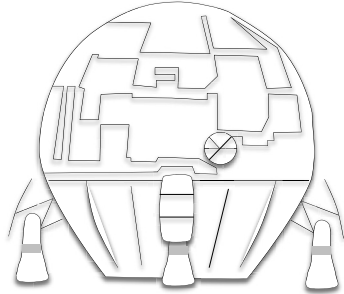
1. Your company, *Putting Around*, creates miniature golf courses. The diagram below shows the project design of one course.



The diameter of the golf hole on a putting green is 4.25". The miniature golf course planned in this project will have 3 holes. How many square feet of turf will be needed to cover the putting green if one square represents 1.5 square feet? Justify your answer.

**Section 15 – Topic 6**  
**Volume in Real-World Contexts**

The company *SpaceY* is designing a commercial spherical spaceship and plans to start selling one-week trips to people who want to orbit around the Earth. *SpaceY* allocates  $1,000 \text{ ft}^3$  per person, plus an additional  $3,067.5 \text{ ft}^3$  for various necessary machinery. The diameter of the ship is 53.6 ft.



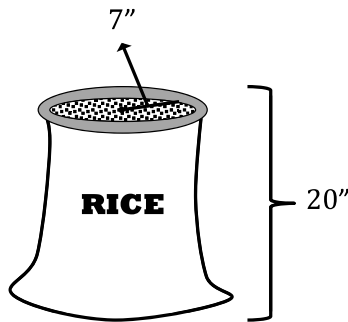
What is the maximum number of people who would be able to ride in the ship per trip?

A competitor, *Why Space?*, designs another spaceship, more accessible in terms of price, but smaller by a scale factor of  $\frac{1}{3}$  with respect to the original dimensions.

What is the maximum number of people who would be able to ride in this ship?

### Let's Practice!

1. At your local supermarket, there is a sack full of rice in a shape of a cylinder with height of 20" and radius of 7".



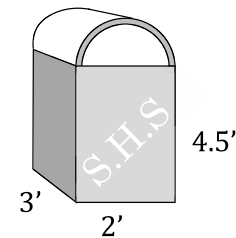
- a. If there are approximately 50 grains of rice in a cubic inch, then approximate how many grains of rice are in this sack?
- b. There is also a 2.5-kilogram bag of rice that is selling for the same price as the above cylinder sack. If there 64 grains of rice weigh 1 gram total, then is the cylinder sack or the bag of rice a better deal? Justify your answer.

### Try It!

2. The local recreation center is building a new Olympic-size pool to be 164 feet long, 82 feet wide, and 12 feet deep.
- a. Approximately how much water will the pool hold?
- b. The excavated dirt for the pool described above will be hauled away by wheelbarrow and dumped into a truck. If the wheelbarrow holds 9 cubic feet of dirt, then how many wheelbarrows of dirt must be hauled away and dumped into the truck?

### BEAT THE TEST!

1. Janitors at Strickland High School empty 25 full trash cans every day. The design of each trash can is shown below.



Based on the information and diagram above, what is the total volume of trash that the janitors empty every day?

$ft^3$ .