Section 2: Introduction to Geometry – Basic Transformations

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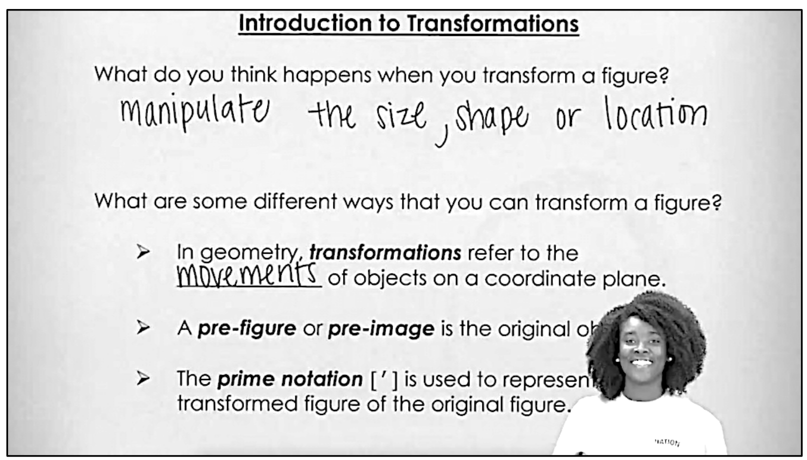


The following Mathematics Florida Standards will be covered in this section:
G-CO.1.2 - Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not.
G-CO.1.4 - Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
G-CO.1.5 - Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure. Specify a sequence of transformations that will carry a given figure onto itself.

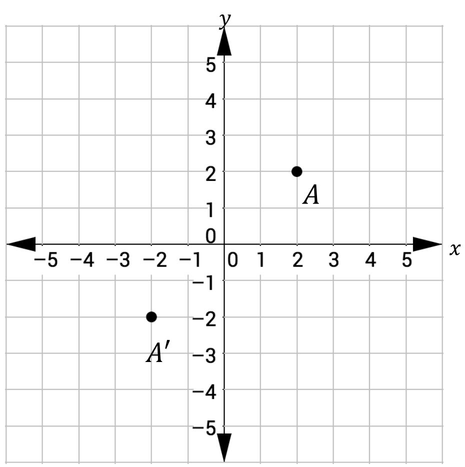
Section 2: Introduction to Geometry – Basic Transformations
Section 2 – Topic 1
Introduction to Transformations

What do you think happens when you transform a figure?

What are some different ways that you can transform a figure?

- In geometry, ***transformations*** refer to the _____________ of objects on a coordinate plane.
- A ***pre-figure*** or ***pre-image*** is the original object.
- The ***prime notation*** (′) is used to represent a transformed figure of the original figure.

Consider the graph below, circle the pre-image and box the transformed image. Describe the transformation.



There are two main categories of transformations: ***rigid*** and ***non-rigid***.

- A _________________ transformation changes the size of the pre-image.
- A _________________ transformation does not change the size of the pre-image.

Write a real-world example of a rigid transformation.

Write a real-world example of a non-rigid transformation.

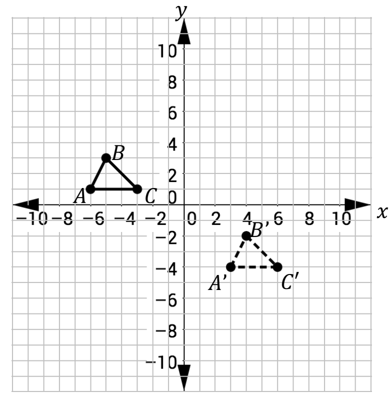
There are four common types of transformations:

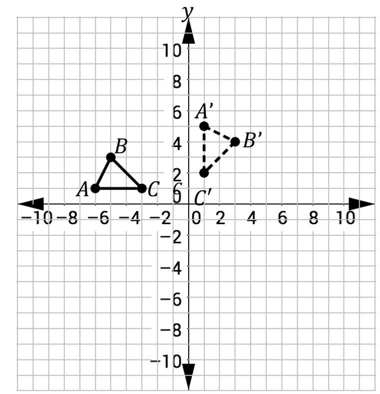
- A ***rotation*** turns the shape around a center point.
- A ***translation*** slides the shape in any direction.
- A ***dilation*** changes the size of an object through an enlargement or a reduction.
- A ***reflection*** flips the object over a line (as in a mirror image).

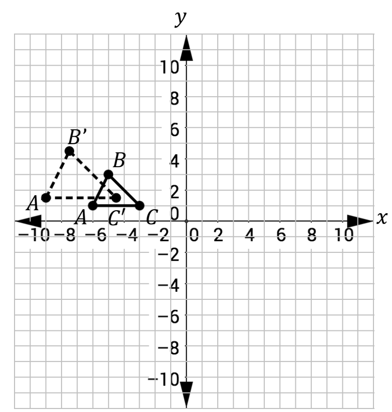
In the table below, indicate whether the transformation is rigid or non-rigid and justify your answer.

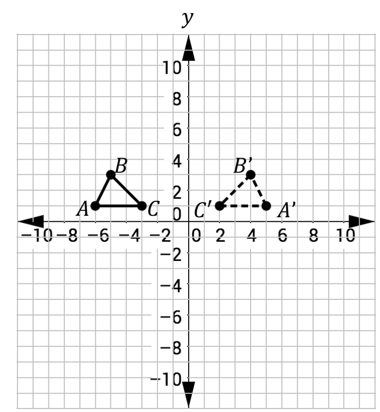
Transformation	Rigid/Non-Rigid	Justification
Translation	○ Rigid ○ Non-Rigid	
Reflection	○ Rigid ○ Non-Rigid	
Rotation	○ Rigid ○ Non-Rigid	
Dilation	○ Rigid ○ Non-Rigid	

Now, identify the transformations shown in the following graphs and write the names of the transformations in the corresponding boxes under each graph.





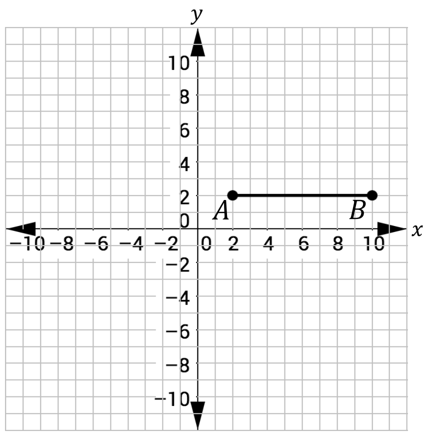




Let's Practice!

1. Consider $\overline{AB}$ in the coordinate plane below.

    a. Write the coordinates of each endpoint, the length of the segment, and the midpoint of the segment.



A(_____,_____)

B(_____,_____)

Length: ______ units

Midpoint: (_____,_____)

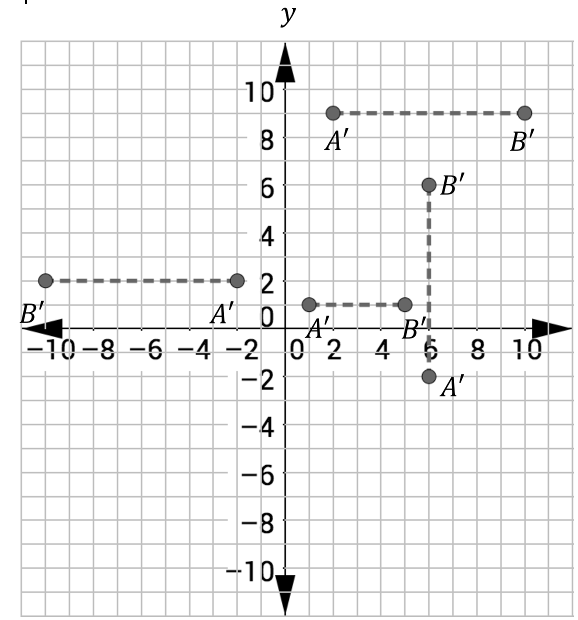
    b. Write the coordinates of A′ and B′ after the following transformations.

Transformations	A'	B'
$\overline{AB}$ is translated 5 units up and 3 units to the left.		
$\overline{AB}$ is rotated 180° clockwise about the origin.		

Try It!

2. Consider the transformations of $\overline{AB}$ in the previous problem.

    a. Trace the lines and identify the transformations on the graph.

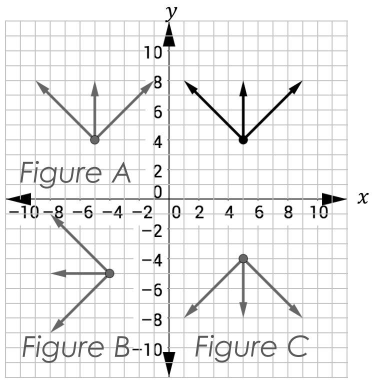


    b. What are the A' and B' coordinates for each transformation below? Fill in the length and midpoint of each segment indicated in the chart.

Transformation	Coordinates	Length	Midpoint
Translation	A'(___,___); B'(___,___)		
Dilation	A'(___,___); B'(___,___)		
Rotation	A'(___,___); B'(___,___)		
Reflection	A'(___,___); B'(___,___)		

BEAT THE TEST!

1. Three rays share the same vertex $(5, 4)$ as shown in the coordinate plane below.



Part A: Which figure represents a reflection across the y-axis?

Part B: Which of the following statements are true about the figure? Select all that apply.

- ☐ A rotation of $360°$ will carry the object onto itself.
- ☐ A reflection of the figure along the x-axis carries the figure to Quadrant II.
- ☐ In Figure A, $(x', y') = (x + 10, y)$.
- ☐ If the vertex of Figure A is translated $(x + 1, y - 9)$, it will carry onto the vertex of Figure B.
- ☐ Figure C is a reflection on the x-axis of Figure A.

Section 2 – Topic 2
Examining and Using Translations

A ***translation*** "slides" an object a fixed distance in a given direction, preserving the same __________ and _________.

Suppose a geometric figure is translated h units along the x-axis and k units along the y-axis. We use the following notation to represent the transformation:

$$T_{h,k}(x, y) = (x + h, y + k) \text{ or } (x, y) \rightarrow (x + h, y + k)$$

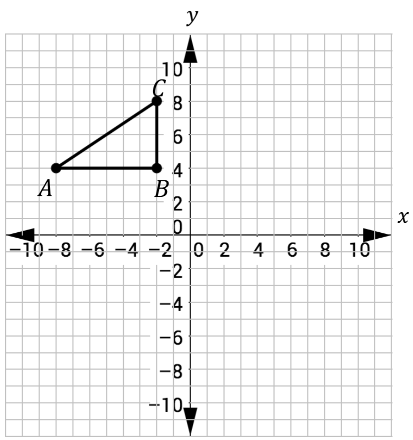
- $(x, y) \rightarrow (x + 2, y - 5)$ translates the point (x, y) 2 units ______ and 5 units _______.

- What is the algebraic description for a transformation that translates the point (x, y) 2 units to the left and 3 units upward?

- What is the algebraic description for a transformation that translates the point (x, y) 3 units to the right and 2 units downward?

Let's Practice!

1. Transform triangle ABC according to $(x, y) \rightarrow (x + 3, y - 2)$. Write the coordinates for triangle $A'B'C'$.



A'(_____,_____)

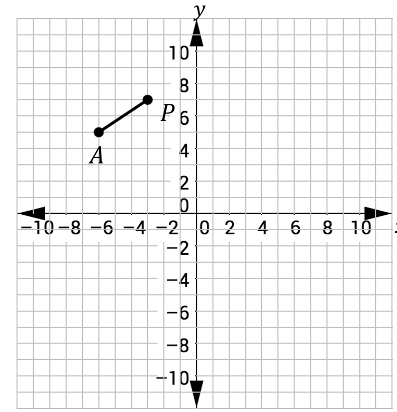
B'(_____,_____)

C'(_____,_____)

2. When the transformation $(x, y) \rightarrow (x + 10, y + 5)$ is performed on point A, its image, point A', is on the origin. What are the coordinates of A? Justify your answer.

Try It!

3. $\overline{AP}$ undergoes the translation $T_{h,k}(x, y)$, such that $A'(1, 1)$ and $P'(4, 3)$.



a. What are the values of h and k?

$h =$ ________ units

$k =$ ________ units

b. Which of the following statements is true?

Ⓐ $\overline{AP}$ and $\overline{A'P'}$ have different locations.
Ⓑ $\overline{AP}$ and $\overline{A'P'}$ have different shapes.
Ⓒ $\overline{AP}$ and $\overline{A'P'}$ have different sizes.
Ⓓ $\overline{AP}$ and $\overline{A'P'}$ have different directions.

BEAT THE TEST!

1. When the transformation $(x, y) \rightarrow (x - 4, y + 7)$ is performed on point P, its image is point $P'(-3, 4)$. What are the coordinates of P?

   Ⓐ $(-7, 11)$
   Ⓑ $(-1, 3)$
   Ⓒ $(1, -3)$
   Ⓓ $(7, -11)$

2. Consider the following points.

   $$R(-6, 5) \text{ and } U(5, -6)$$

   $\overline{RU}$ undergoes the translation $(x, y) \rightarrow (x + h, y + k)$, such that $R'(5, 1)$ and $U'(16, -10)$.

   Part A: Complete the following algebraic description.

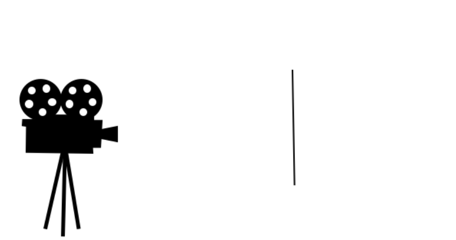
   $(x, y) \rightarrow (x + \boxed{}, y + \boxed{})$

   Part B: What is the difference between $\overline{RU}$ and $\overline{R'U'}$?

Section 2 – Topic 3
Examining and Using Dilations – Part 1

Dilation stretches or shrinks the original figure.

Consider the following figure.



What is making the projected image shrink or grow?

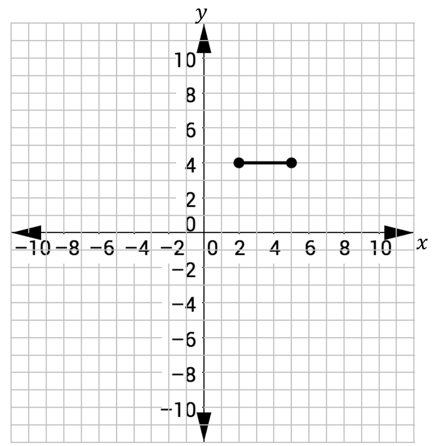
The ***center of dilation*** is a fixed point in the plane about which all points are expanded or contracted.

How different is one line from the other in the above figure?

The ***scale factor*** refers to how much the figure grows or shrinks, and it is denoted by k.

- If $0 < k < 1$, the image gets smaller and closer to the center of dilation.
- If $k > 1$, the image gets larger and farther from the center of dilation.

Consider the following graph.

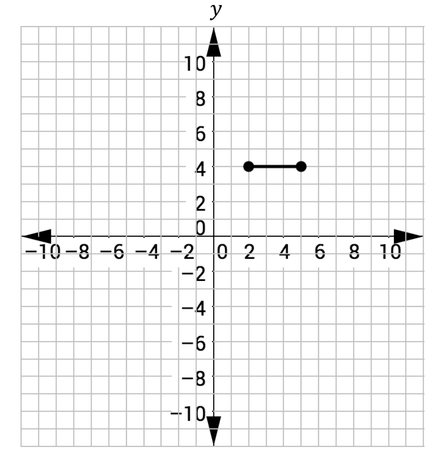


How do you dilate the line segment on the above graph centered at a point on the same line?

Use $(2, 4)$ as the center of dilation and complete the following:

- If $k = 2$, then the dilated line segment will have coordinates: ____________ and ____________.
- If $k = \frac{1}{2}$, then the dilated line segment will have coordinates: ____________ and ____________.
- When dilating a line that passes through the center of dilation, the line is _____________.

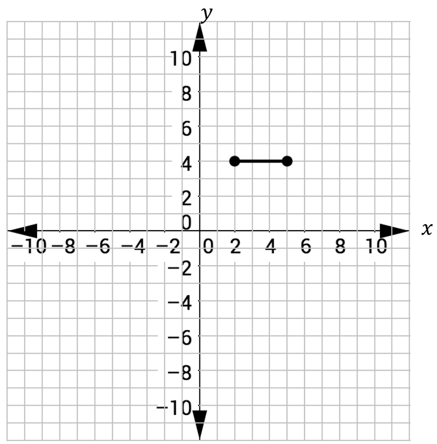
Consider the following graph.



How do you dilate the line segment on the above graph centered at the origin?

- If $k = 2$, then the dilated line segment will have coordinates: ____________ and ____________.
- If $k = \frac{1}{2}$, then the dilated line segment will have coordinates: ____________ and ____________.
- When dilating a line that does not pass through the center of dilation, the dilated line is ______________ to the original.
- $(x, y) \rightarrow (kx, ky)$ changes the size of the figure by a factor of k when the center of dilation is the origin.

Consider the following graph.



Use $(9, 6)$ as the center of dilation and complete the following statements:

- If $k = 4$, then the dilated line segment will have the coordinates ____________ and ____________.
- If $k = \frac{1}{4}$, then the dilated line segment will have the coordinates ____________ and ____________.

In conclusion,

- A dilation produces an image that is the same ________ as the original, but is a different ________.
- When dilating a line segment, the dilated line segment is longer or shorter with respect to the ___________ _______________.

Section 2 – Topic 4
Examining and Using Dilations – Part 2

Let's Practice!

1. $\overline{AB}$ has coordinates $A(-3, 9)$ and $B(6, -12)$. $\overline{PQ}$ has coordinates $P(3, -6)$ and $Q(3, 9)$.

   a. Find the coordinates of $\overline{A'B'}$ after a dilation with a scale factor of $\frac{2}{3}$ centered at the origin.

   b. Find the coordinates of $\overline{P'Q'}$ after a dilation with a scale factor of $\frac{1}{5}$ centered at $(3, -1)$.

2. Line l is mapped onto the line t by a dilation centered at the origin with a scale factor of 3. The equation of line l is $2x - y = 7$. What is the equation for line t?

   Ⓐ $6x - 3y = 21$
   Ⓑ $\frac{1}{6}x - y = \frac{1}{21}$
   Ⓒ $y = 2x - 21$
   Ⓓ $y = 6x - 21$

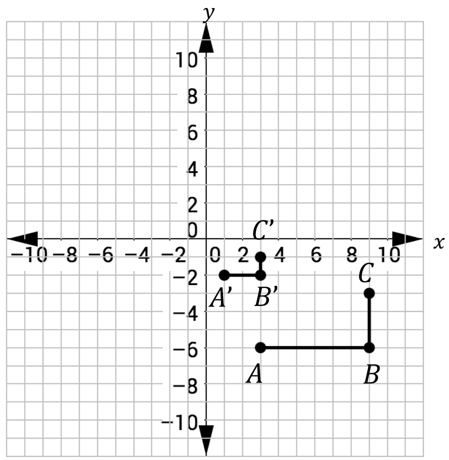
3. Suppose the line l represented by $f(x) = 2x - 1$ is transformed into $g(x) = 2\big(f(x + 1)\big) - 7$.

   a. Describe the transformation from $f(x)$ to $g(x)$.

   b. What is the y-coordinate of $g(0)$?

Try It!

4. What is the scale factor for the dilation of ABC into $A'B'C'$?



$k =$

5. $\overline{CD}$ has coordinates $C(-8,-2)$ and $D(-4,-12)$.

   a. Determine the coordinates of $\overline{C'D'}$ if $(x,y) \rightarrow (3x,3y)$.

   b. Find the coordinates of $\overline{C'D'}$ after a dilation with a scale factor of 2 centered at $(2,2)$.

BEAT THE TEST!

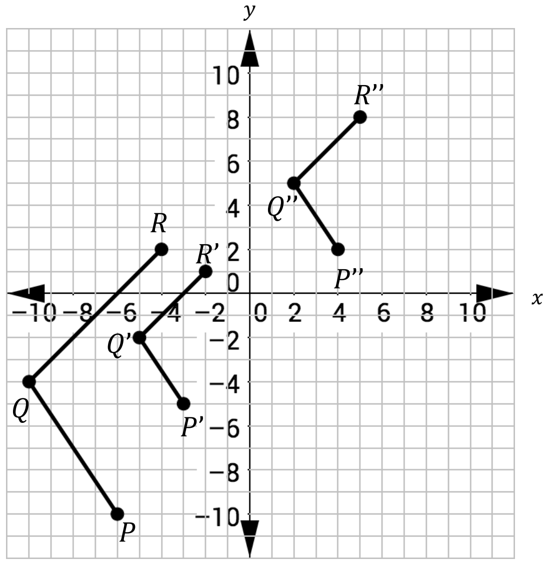
1. $\overline{M'T'}$ has coordinates $M'(-8,10)$ and $T'(2,-4)$, and it is the result of the dilation of $\overline{MT}$ centered at the origin. The coordinates of $\overline{MT}$ are $M(-4,5)$ and $T(1,-2)$. Complete the following algebraic description so that it represents the transformation of $\overline{MT}$.

   $(x,y) \rightarrow (\boxed{}\,x, \boxed{}\,y)$

2. Line l is mapped onto line m by a dilation centered at the origin with a scale factor of $\frac{4}{5}$. Line m is represented by $y = 3x + 8$ and it passes through the point which coordinates are $(-4,-4)$. Which of the following is true about line l?

   Ⓐ Line l is parallel to line m.
   Ⓑ Line l is perpendicular to line m.
   Ⓒ Line l passes through the origin.
   Ⓓ Line l is the same as line m.

3. $P(-6,-10)$, $Q(-10,-4)$ and $R(-4,2)$ form figure PQR.



Part A: Gladys transformed figure PQR into $P'Q'R'$. Which of the following represents her transformation?

Ⓐ $(x,y) \to (\frac{1}{2}x, \frac{1}{2}y)$
Ⓑ $(x,y) \to (2x, 2y)$
Ⓒ $(x,y) \to (x+3, y+5)$
Ⓓ $(x,y) \to (x-3, y-5)$

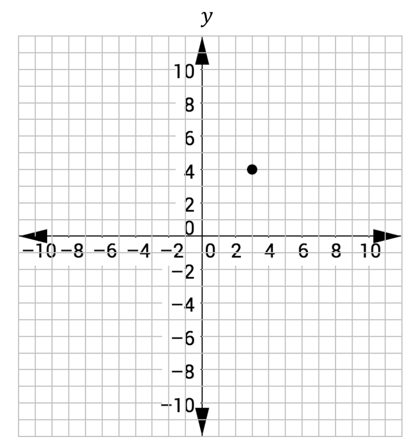
Part B: She then transformed $P'Q'R'$ into $P''Q''R''$. What is the transformation?

$(x,y) \to$ (_______________, _______________)

Section 2 – Topic 5
Examining and Using Rotations

A ***rotation*** changes the ____________ of a figure by moving it around a fixed point to the right (clockwise) or to the left (counterclockwise).

Let's consider the following graph.



Use the graph to help you determine the coordinates for (x', y') after the following rotations about the origin.

Degree	Counterclockwise	Clockwise
90° Rotation:	$R_{90°}(3,4) =$ ______	$R_{-90°}(3,4) =$ ______
180° Rotation:	$R_{180°}(3,4) =$ ______	$R_{-180°}(3,4) =$ ______
270° Rotation:	$R_{270°}(3,4) =$ ______	$R_{-270°}(3,4) =$ ______

The function $R_t(x,y)$ rotates the point (x,y) $t°$ ________________ around the origin.

The function $R_{-t}(x,y)$ rotates the point (x,y) $t°$ ________________ around the origin.

Make generalizations about rotations around the origin to complete the following table.

Degree	Counterclockwise	Clockwise
$90°$ Rotation:	$(x, y) \rightarrow$ ________	$(x, y) \rightarrow$ ________
$180°$ Rotation:	$(x, y) \rightarrow$ ________	$(x, y) \rightarrow$ ________
$270°$ Rotation:	$(x, y) \rightarrow$ ________	$(x, y) \rightarrow$ ________

What happens if the rotation is $360°$ clockwise or counterclockwise?

What happens if the center of rotation is not the origin?

What happens if the degree of rotation is a degree other than $90°, 180°, 270°$, and $360°$?

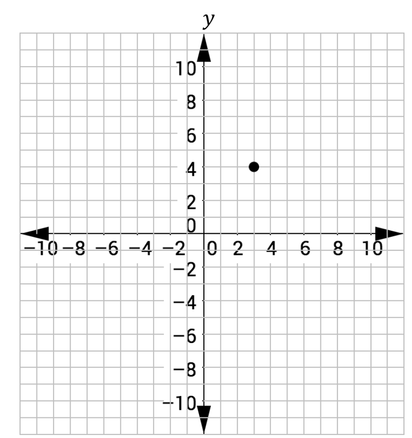
Let's Practice!

1. $\overline{RT}$ has endpoints $R(0,3)$ and $T(4,1)$. Rotate $\overline{RT}$ clockwise $90°$ about the origin.

   a. Write an algebraic description of the transformation of $\overline{RT}$.

   b. What are the endpoints of the new line segment?

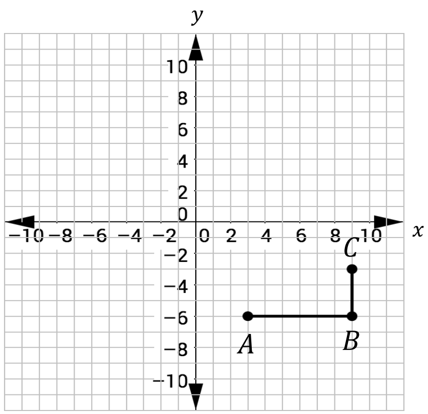
2. Let's consider the following graph.



   Rotate $(3,4)$ counterclockwise $90°$ about $(5,7)$.

Try It!

3. Consider the following graph.



a. Rotate figure ABC $270°$ counterclockwise about the origin. Graph the new figure on the coordinate plane, and complete each blank below with the appropriate coordinates.

A (___, ___) → A'(___, ___)
B (___, ___) → B'(___, ___)
C (___, ___) → C'(___, ___)

b. Rotate figure ABC $90°$ clockwise about the origin. Graph the new figure on the coordinate plane and complete each blank below with the appropriate coordinates.

A (___, ___) → A'(___, ___)
B (___, ___) → B'(___, ___)
C (___, ___) → C'(___, ___)

BEAT THE TEST!

1. $\overline{PQ}$ has endpoints $P(-2,-1)$ and $Q(6,-5)$. Consider the transformation $(x,y) \rightarrow (y,-x)$ for $\overline{PQ}$. What kind of transformation is this? What are the coordinates of $\overline{P'Q'}$?

2. Point $K(10,-3)$ is rotated $90°$ clockwise. Which of the following is the y −coordinate of K'?

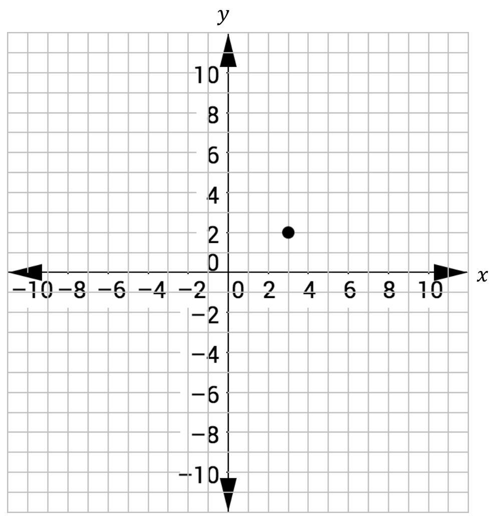
Ⓐ -10
Ⓑ -3
Ⓒ 3
Ⓓ 10

Section 2 – Topic 6
Examining and Using Reflections

A ***reflection*** is a mirrored version of an object. The image does not change __________, but the figure itself reverses.

The function $r_{line}(x, y)$ reflects the point (x, y) over the given line. For instance, $r_{x-axis}(3, 2)$ reflects the point $(3, 2)$ over the x-axis.

Let's examine the line reflections of the point $(3, 2)$ over the x-axis, y-axis, $y = x$, and $y = -x$.



Reflection over	Notation	New coordinates
x-axis	$r_{x-axis}(3, 2)$	
y-axis	$r_{y-axis}(3, 2)$	
$y = x$	$r_{y=x}(3, 2)$	
$y = -x$	$r_{y=-x}(3, 2)$	

Make generalizations about reflections to complete the following table.

Reflection over	Notation	New coordinates
x-axis	$r_{x-axis}(x, y)$	
y-axis	$r_{y-axis}(x, y)$	
$y = x$	$r_{y=x}(x, y)$	
$y = -x$	$r_{y=-x}(x, y)$	

Let's Practice!

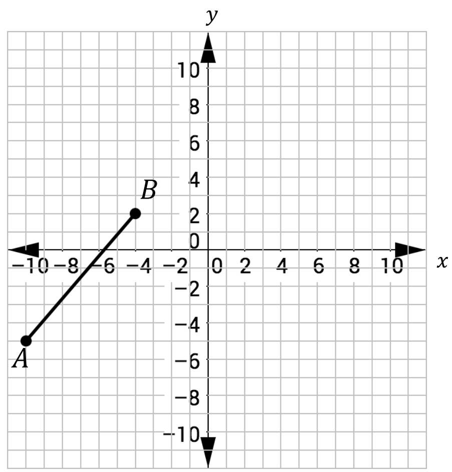
1. Suppose the line segment with endpoints $C(1, 3)$ and $D(5, 2)$ is reflected over the y-axis, and then reflected again over $y = x$. What are the coordinates C'' and D''?

Try It!

2. Suppose a line segment with endpoints are $A(-10,-5)$ and $B(-4,2)$ is reflected over $y=-x$.

   a. What are the coordinates of A'(___,___) and B'(___,___)?

   b. Graph $\overline{A'B'}$ on the coordinate plane below.



BEAT THE TEST!

1. Consider the following points.

   $F(-3,-10)$ and $E(10,-3)$

   Let $\overline{F'E'}$ be the image of $\overline{FE}$ after a reflection across line l. Suppose that F' is located at $(-3,10)$ and E' is located at $(10,3)$. Which of the following is true about line l?

   Ⓐ Line l is represented by $y=-x$.
   Ⓑ Line l is represented by $y=x$.
   Ⓒ Line l is represented by the x-axis.
   Ⓓ Line l is represented by the y-axis.

2. Suppose a line segment whose endpoints are $G(8,2)$ and $H(14,-8)$ is reflected over $y=x$.

   What are the coordinates of G'(___,___) and H'(___,___)?

Test Yourself! Practice Tool

Great job! You have reached the end of this section. Now it's time to try the "Test Yourself! Practice Tool," where you can practice all the skills and concepts you learned in this section. Log in to Math Nation and try out the "Test Yourself! Practice Tool" so you can see how well you know these topics!