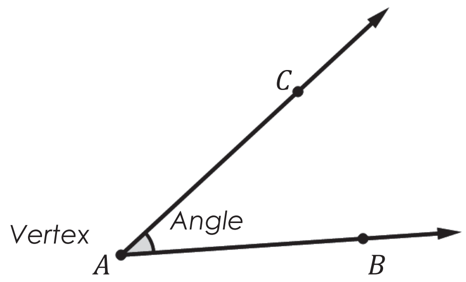
Section 3: Angles

The following Mathematics Florida Standards will be covered in this section:	
MAFS.912.G-CO.1.1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
MAFS.912.G-CO.1.2	Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not.
MAFS.912.G-CO.1.4	Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines and line segments.
MAFS.912.G-CO.1.5	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure.
MAFS.912.G-CO.3.9	Prove theorems about lines and angles; use theorems about lines and angles to solve problems. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
MAFS.912.G-CO.4.12	Make formal geometric constructions with a variety of tools and methods. Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

Topics in this Section

Section 3 – Topic 1
Introduction to Angles – Part 1

Consider the figure of angle A below.



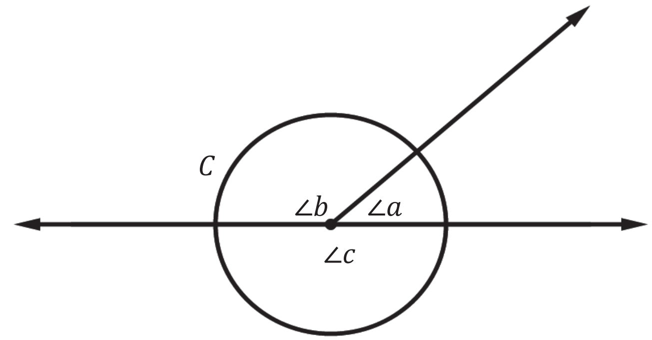
What observations can you make about angle A?

How else do you think we can name angle A?

Why do you think we draw an arc to show angle A?

Like circles, angles are measured in ______________ since they measure the amount of rotation around the center.

Consider the figure below.



Use the figure to answer the following questions.

What is the measure of circle C?

What is the measure of $\angle a + \angle b + \angle c$?

How many degrees is half of a circle?

What is the measure of $\angle a + \angle b$?

Two positive angles that form a straight line together are called ___________________ angles.

- When added together, the measures of these angles total ______________ degrees, forming a ___________ pair.

Draw an example of ***supplementary angles*** that form a ***linear pair***.

A quarter-circle is a _______________ angle.

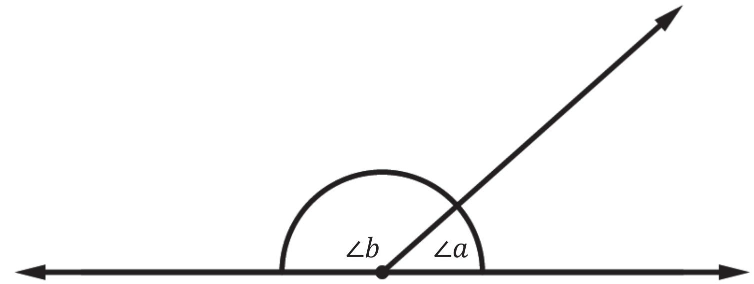
How many degrees are in a right angle?

Two positive angles that together form a right angle are called ___________________ angles.

Draw an example of ***complementary angles***.

Let's Practice!

1. In the figure below, $m\angle a = 7x + 5$ and $m\angle b = 28x$. The angles are supplementary.

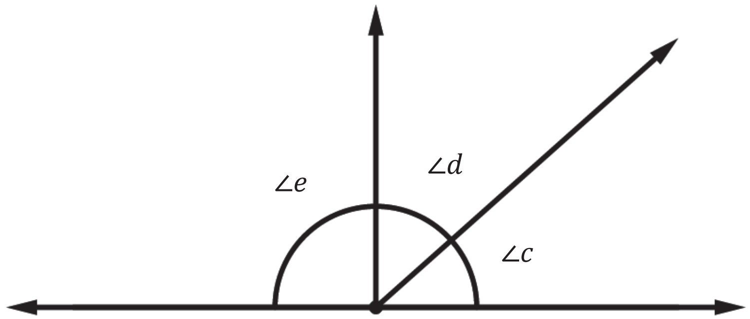


Find the value of x and the measure of $\angle a$ and $\angle b$ in degrees.

STUDY EDGE TIP

When we refer to the angle as $\angle ABC$, we mean the actual angle object. If we want to talk about the size or the measure of the angle in degrees, we often write it as $m\angle ABC$.

2. In the figure below, $m\angle c = 9x - 3$ and $m\angle d = 8x + 9$.



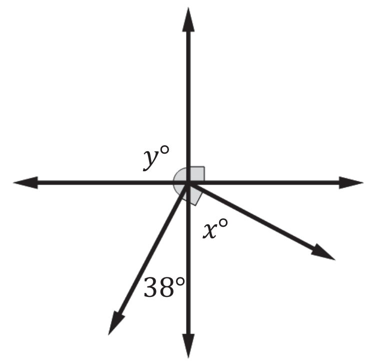
   a. If $x = 5$, are $\angle c$ and $\angle d$ complementary? Justify your answer.

   b. If $\angle c$, $\angle d$, and $\angle e$ form half a circle, then what is the measure of $\angle e$ in degrees?

Try It!

3. Angle A is 20 degrees larger than angle B. If A and B are complementary, what is the measure of angle A?

4. Consider the figure below.

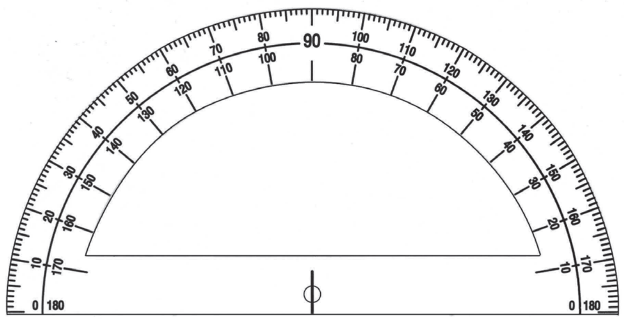


If the angle with value of $y°$ stretches from the positive y-axis to the ray that makes the 38° angle, set up and solve an appropriate equation for x and y.

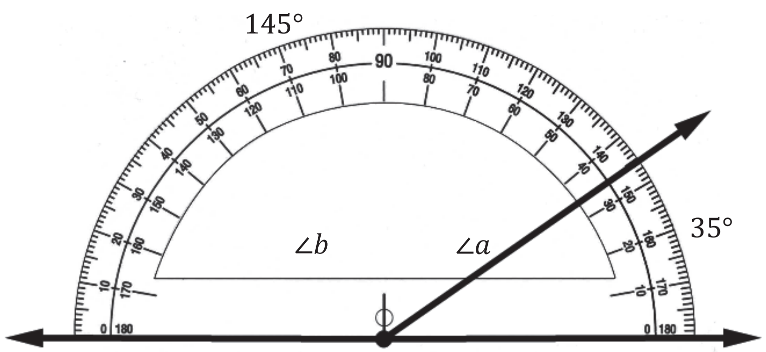
Section 3 – Topic 2
Introduction to Angles – Part 2

Measuring and classifying angles:

- We often use a ________________ to measure angles.



To measure an angle, we line up the central mark on the base of the ***protractor*** with the vertex of the angle we want to measure.

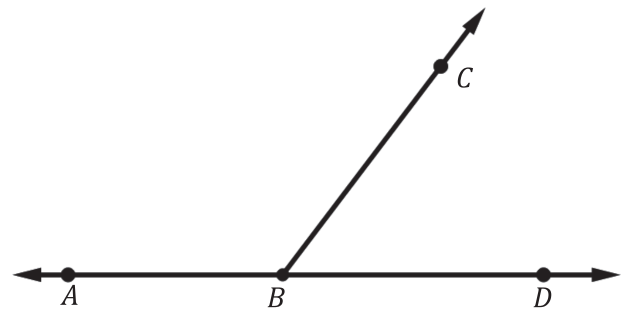


TAKE NOTE! *Postulates & Theorems*

The Protractor Postulate

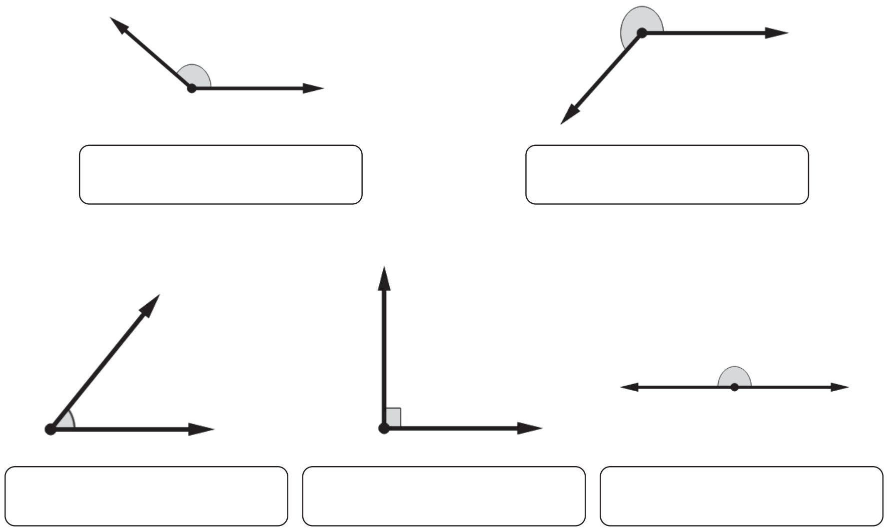
The measure of the angle is the absolute value of the difference of the real numbers paired with the sides of the angle, because the parts of angles formed by rays between the sides of a linear pair add to the whole, 180°.

Label and measure the angles in the following figure.



Match each of the following words to the most appropriate figure represented below. Write your answer in the space provided below each figure.

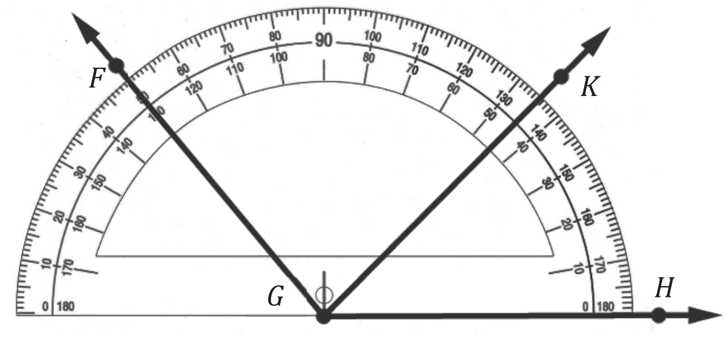
Acute	Obtuse	Right	Straight	Reflex



- An angle that measures less than $90°$ is _______________.
- An angle that measures greater than $90°$ but less than $180°$ is _______________.
- An angle that measures exactly $90°$ is _______________.
- An angle of exactly $180°$ is _______________.
- An angle greater than $180°$ is called a ____________ angle.

Let's Practice!

1. Use the figure below to fill in the blanks that define angles $\angle FGK$, $\angle FGH$, and $\angle KGH$ as acute, obtuse, right, or straight.



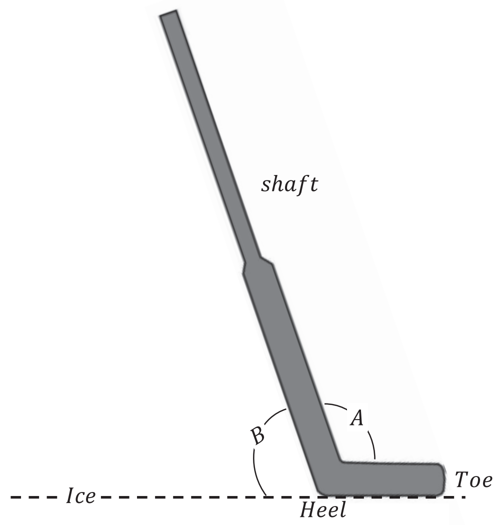
a. $\angle FGK$ is a(n) _______________ angle.

b. $\angle FGH$ is a(n) _______________ angle.

c. $\angle KGH$ is a(n) _______________ angle.

Try It!

2. A hockey stick comes into contact with the ice in such a way that the shaft makes an angle with the ice, labeled as angle B in the figure below. The angle between the shaft and the toe of the hockey stick is labeled as A.

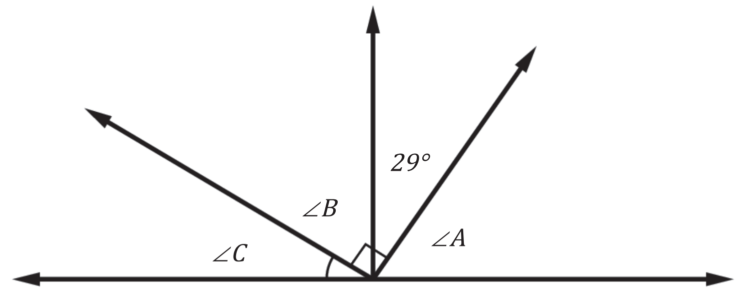


a. Determine the type of angle that is between the ice and the shaft. Is it acute, right, obtuse, or straight?

b. Determine the type of angle that is between the shaft and the toe. Is it acute, right, obtuse, or straight?

BEAT THE TEST!

1. Consider the figure below.



If $\angle B$ and $\angle C$ are complementary, then:

The measure of $\angle A$ is [ ] .

The sum of $m\angle A$ and $m\angle B$ is [ ] .

The sum of $m\angle A, m\angle B$, and $m\angle C$ is [ ] .

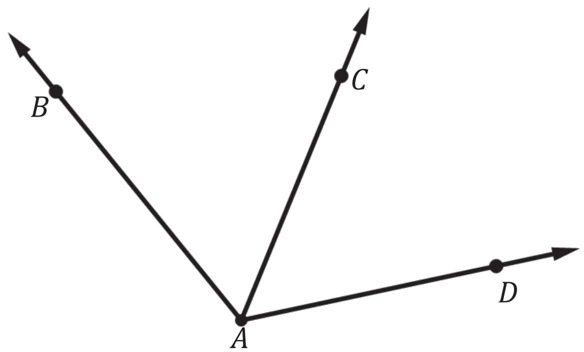
If $m\angle Z = m\angle A + m\angle C$, then $\angle Z$ is

- ○ acute
- ○ obtuse
- ○ right
- ○ straight

.

Section 3 – Topic 3
Angle Pairs – Part 1

Consider the following figure that presents an ***angle pair***.

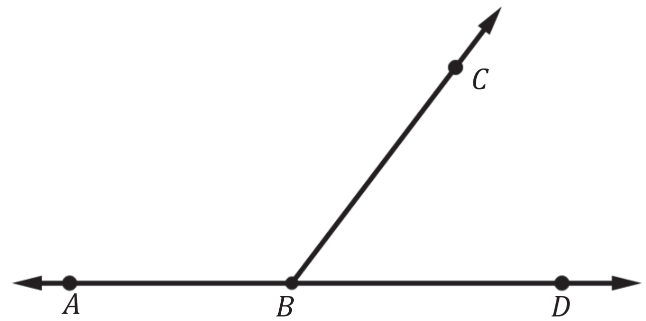


What common ray do $\angle BAC$ and $\angle CAD$ share?

Because these angle pairs share a ray, they are called _________________ angles.

Consider the following figure of ***adjacent angles***.

What observations can you make about the figure?



These adjacent angles are called a ____________ pair. Together, the angles form a ___________ angle.

What is the measure of a straight angle?

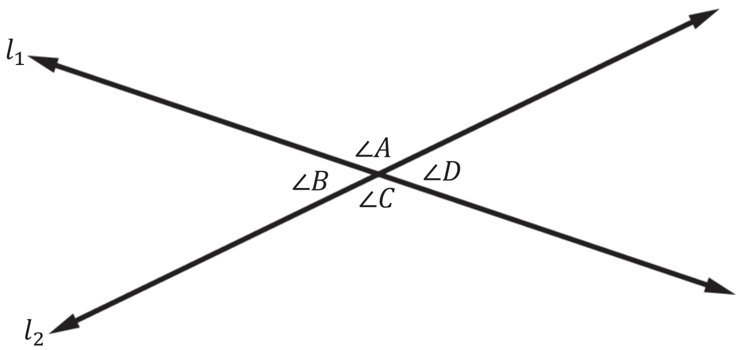
What is the measure of the sum of a ***linear pair****?*

TAKE NOTE! *Postulates & Theorems*

Linear Pair Postulate

If two positive angles form a linear pair, then they are supplementary.

Consider the figure below of angle pairs.



What observations can you make about $\angle A$ and $\angle C$?

What observations can you make about $\angle B$ and $\angle D$?

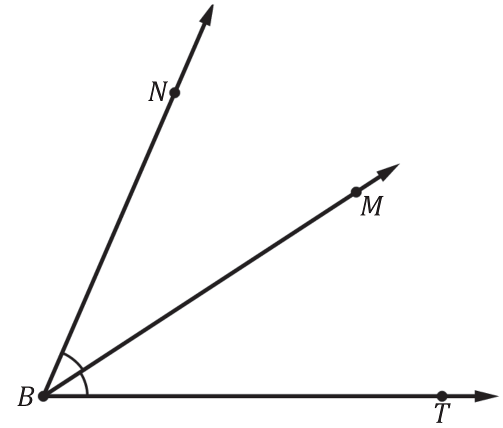
$\angle A$ and $\angle C$ form what we call a pair of ____________ angles.

What angle pairs form a set of ***vertical angles****?*

TAKE NOTE! *Postulates & Theorems*

Vertical Angles Theorem
If two angles are vertical angles, then they have equal measures.

Consider the figure below.



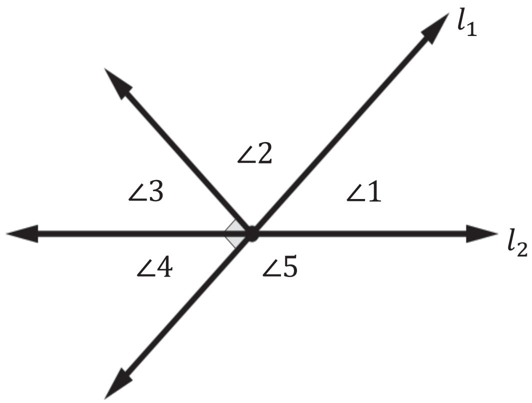
What observations can you make about the figure?

We call $\overrightarrow{BM}$ an ***angle bisector****.*

Make a conjecture as to why $\overrightarrow{BM}$ is called an angle bisector.

Let's Practice!

1. Consider the figure below.



Complete the following statements:

- $\angle 1$ and $\angle 4$ are ______________ angles.
- $\angle 1$ and $\angle 2$ are ______________ angles.
- $\angle 3$ and $\angle 4$ are ______________________ angles and ______________________ angles.
- $\angle 4$ and $\angle 5$ are ______________________ angles and ______________________ angles. They also form a ______________________.

2. If $\angle ACB$ and $\angle ACE$ are linear pairs, and $m\angle ACB = 5x + 25$ and $m\angle ACE = 2x + 29$, then

   a. Determine $m\angle ACB + m\angle ACE$.

   b. Determine the measures of $m\angle ACB$ and $m\angle ACE$.

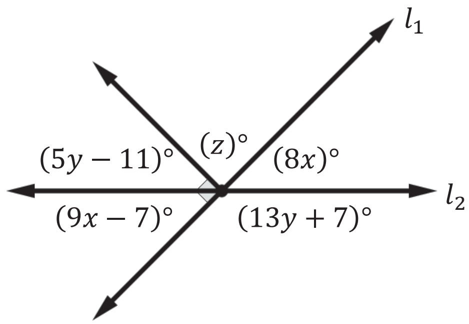
3. If $\angle MFG$ and $\angle EFN$ are vertical angles, and $m\angle MFG = 7x - 18$ and $m\angle EFN = 5x + 10$, then

   a. What can we say about $\angle MFG$ and $\angle EFN$ that will help us determine their measures?

   b. Determine the measures of $\angle MFG$ and $\angle EFN$.

Try It!

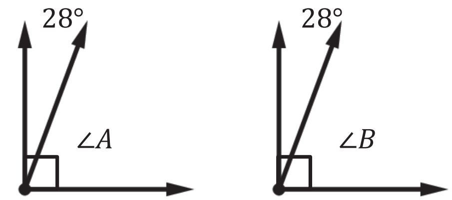
4. Consider the figure below.



Angle measures are represented by algebraic expressions. Find the value of x, y, and z.

Section 3 – Topic 4
Angle Pairs – Part 2

Consider the figure below.



What can you observe about $\angle A$ and $\angle B$?

TAKE NOTE! Postulates & Theorems

Congruent Complements Theorem

If $\angle A$ and $\angle B$ are complements of the same angle, then $\angle A$ and $\angle B$ are congruent.

Consider the figures below.

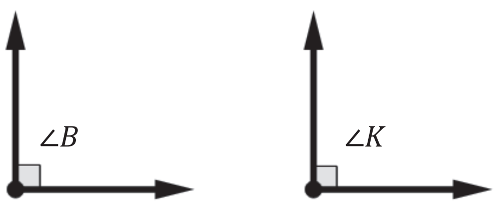
What can you observe about $\angle A$ and $\angle B$?

TAKE NOTE! Postulates & Theorems

Congruent Supplements Theorem

If $\angle A$ and $\angle B$ are supplements of the same angle, then $\angle A$ and $\angle B$ are congruent.

Consider the figure below.



What can you observe about $\angle B$ and $\angle K$?

TAKE NOTE! Postulates & Theorems

Right Angles Theorem

All right angles are congruent.

Let's Practice!

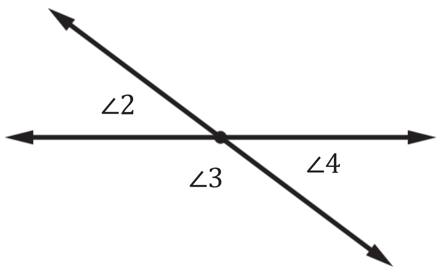
1. The measure of an angle is four times greater than its complement. What is the measure of the larger angle?

Try It!

2. $\angle X$ and $\angle Y$ are supplementary. One angle measures 5 times the other angle. What is the complement of the smaller angle?

Let's Practice!

3. Consider the figure below.



Given: $\angle 2$ and $\angle 3$ are a linear pair. $\angle 3$ and $\angle 4$ are a linear pair.

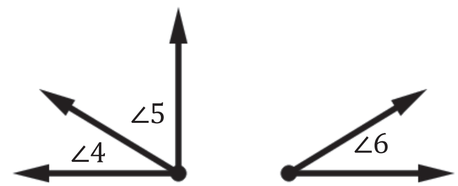
Prove: $\angle 2 \cong \angle 4$

Complete reasons 2 and 3 in the chart below.

Statements	Reasons
1. $\angle 2$ and $\angle 3$ are a linear pair. $\angle 3$ and $\angle 4$ are a linear pair.	**1.** Given
2. $\angle 2$ and $\angle 3$ are supplementary. $\angle 3$ and $\angle 4$ are supplementary.	**2.**
3. $\angle 2 \cong \angle 4$	**3.**

Try It!

4. Consider the figure below.



Given: $\angle 5$ and $\angle 6$ are complementary. $m\angle 4 + m\angle 5 = 90°$

Prove: $\angle 6 \cong \angle 4$

Complete the chart below.

Statements	Reasons
1.	**1.** Given
2.	**2.** Given
3. $\angle 4$ and $\angle 5$ are complementary	**3.**
4. $\angle 6 \cong \angle 4$	**4.**

BEAT THE TEST!

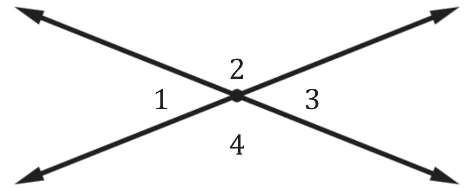
1. $\angle LMN$ and $\angle PML$ are linear pairs, $m\angle LMN = 7x - 3$ and $m\angle PML = 13x + 3$.

   Part A: $m\angle LMN =$

   Part B: $m\angle PML =$

   Part C: If $\angle PMR$ and $\angle LMN$ form a vertical pair and $m\angle PMR = 5y + 4$, find the value of y?

2. Consider the figure below.



Given: $\angle 1$ and $\angle 2$ form a linear pair.
$\angle 1$ and $\angle 4$ form a linear pair.

Prove: The Vertical Angle Theorem

Use the bank of reasons below to complete the table.

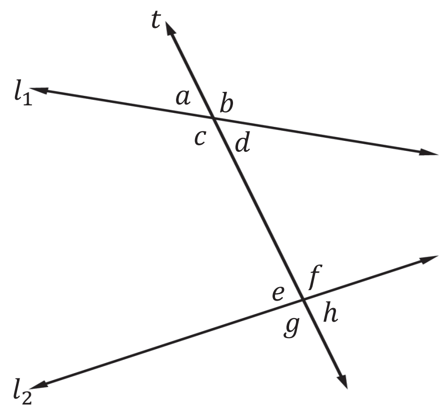
Congruent Supplement Theorem	Right Angles Theorem
Congruent Complement Theorem	Linear Pair Postulate

Statements	Reasons
1. $\angle 1$ and $\angle 2$ are linear pairs. $\angle 1$ and $\angle 4$ are linear pairs.	**1.** Given
2. $\angle 1$ and $\angle 2$ are supplementary. $\angle 1$ and $\angle 4$ are supplementary.	**2.**
3. $\angle 2 \cong \angle 4$	**3.**

Section 3 – Topic 5
Special Types of Angle Pairs Formed by Transversals and Non-Parallel Lines

Many geometry problems involve the intersection of three or more lines.

Consider the figure below.

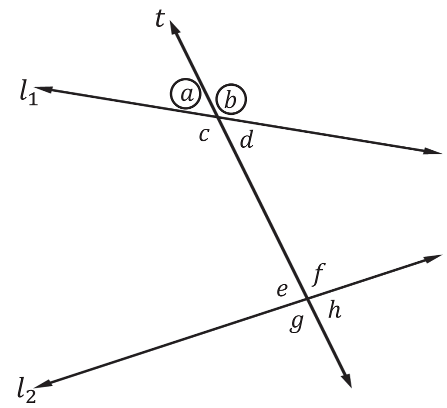


What observations can you make about the figure?

- Lines l_1 and l_2 are crossed by line t.
- Line t is called the ____________________, because it intersects two other lines (l_1 and l_2).
- The intersection of line t with l_1 and l_2 forms eight angles.

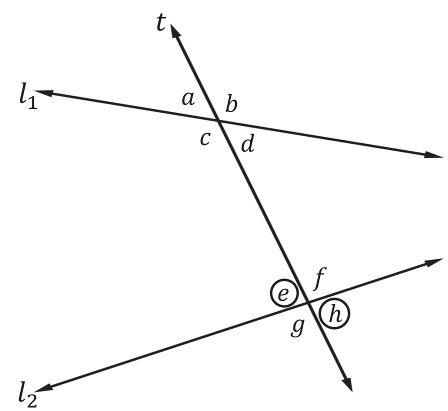
Identify angles made by transversals.

Consider the figure below. $\angle a$ and $\angle b$ form a ***linear pair***.



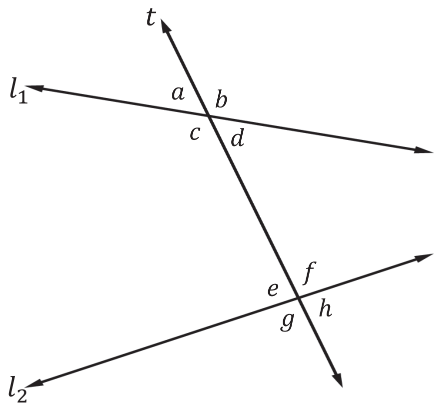
Box and name the other linear pairs in the figure.

Consider the figure below. $\angle e$ and $\angle h$ are ***vertical angles***.



Box and name the other pairs of vertical angles in the figure.

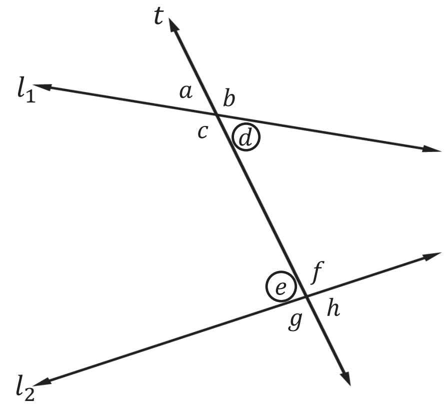
Consider the figure below.



Which part of the figure do you think would be considered the interior? Draw a circle around the interior angles in the figure. Justify your answer.

Which part of the figure do you think would be considered the exterior? Draw a box around the exterior angles in the figure. Justify your answer.

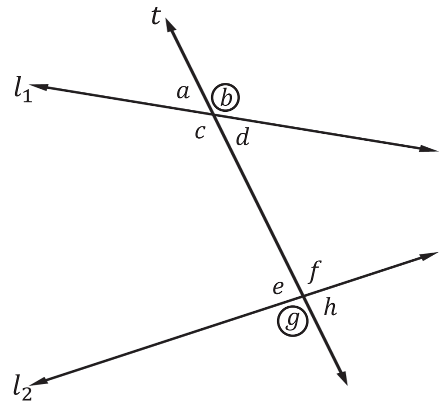
Consider the figure below. $\angle d$ and $\angle e$ are ***alternate interior angles***.



- The angles are in the interior region of the lines l_1 and l_2.
- The angles are on opposite sides of the transversal.

Draw a box around the other pair of alternate interior angles in the figure.

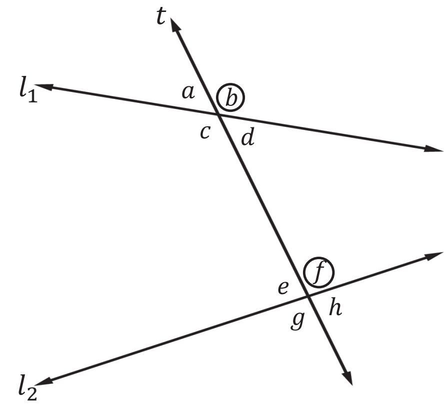
Consider the figure below. $\angle b$ and $\angle g$ are ***alternate exterior angles***.



- The angles are in the exterior region of lines l_1 and l_2.
- The angles are on opposite sides of the transversal.

Draw a box around the other pair of alternate exterior angles in the figure.

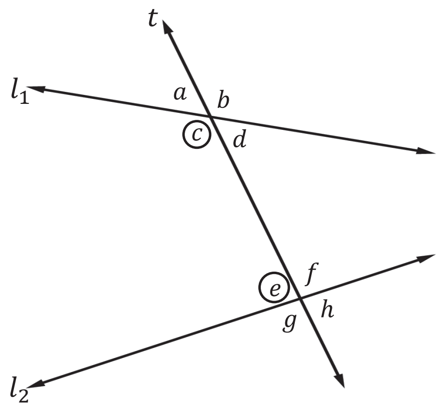
Consider the figure below. $\angle b$ and $\angle f$ are ***corresponding angles***.



- The angles have distinct vertex points.
- The angles lie on the same side of the transversal.
- One angle is in the interior region of lines l_1 and l_2. The other angle is in the exterior region of lines l_1 and l_2.

Draw a box around the other pairs of corresponding angles in the figure and name them below.

Consider the figure below. $\angle c$ and $\angle e$ are ***consecutive*** or ***same-side interior angles***.

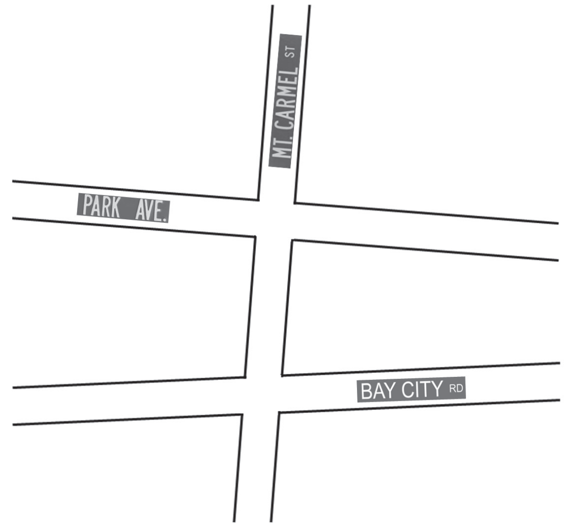


- The angles have distinct vertex points.
- The angles lie on the same side of the transversal.
- Both angles are in the interior region of lines l_1 and l_2.

Draw a box around the other pair of consecutive interior angles.

Let's Practice!

1. On the figure below, Park Ave. and Bay City Rd. are non-parallel lines crossed by transversal Mt. Carmel St.



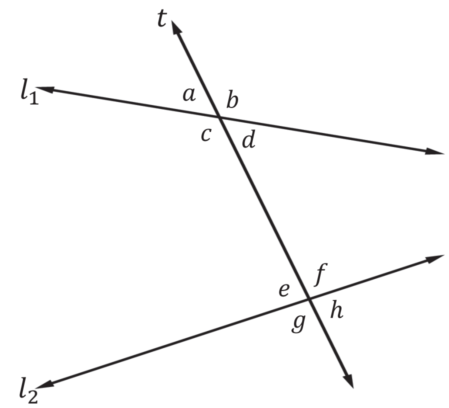
The city hired GeoNat Road Svc. to plan where certain buildings will be constructed and located on the map.

Position the buildings on the map by meeting the following conditions:

- ➢ The park and the city building form a linear pair.
- ➢ The city building and the police department are at vertical angles.
- ➢ The police department and the hospital are at alternate interior angles.
- ➢ The hospital and the fire department are at consecutive interior angles.
- ➢ The school is at a corresponding angle with the park and a consecutive interior angle to the police department.
- ➢ The library and the park are at alternate exterior angles.
- ➢ The church is at an exterior angle and it forms a linear pair with both the library and the school.

Try It!

2. Consider the figure below.



Which of the following statements is true?

Ⓐ If $\angle a$ and $\angle e$ lie on the same side of the transversal and one angle is interior and the other is exterior, then they are corresponding angles.

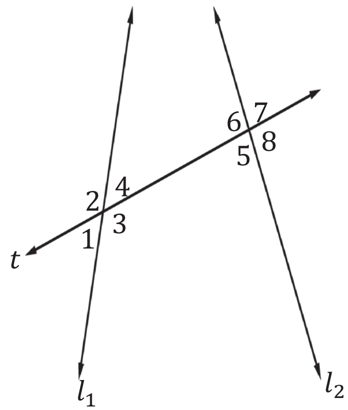
Ⓑ If $\angle b$ and $\angle h$ are on the exterior opposite sides of the transversal, then they are alternate exterior angles.

Ⓒ If $\angle b$ and $\angle c$ are adjacent angles lying on the same side of the transversal, then they are same-side/consecutive interior angles.

Ⓓ If $\angle b, \angle c, \angle f$ and $\angle g$ are between the non-parallel lines, then they are interior angles.

BEAT THE TEST!

1. Consider the figure below.

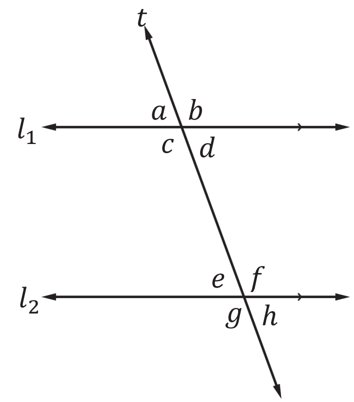


Match the angles on the left with their corresponding names on the right. Write the letter of the most appropriate answer beside each angle pair below.

_____	∠1 and ∠7	A.	Alternate Interior Angles
_____	∠5 and ∠6	B.	Consecutive Angles
_____	∠4 and ∠6	C.	Corresponding Angles
_____	∠5 and ∠7	D.	Vertical Angles
_____	∠4 and ∠5	E.	Alternate Exterior Angles
_____	∠3 and ∠8	F.	Linear Pair

Section 3 – Topic 6
Special Types of Angle Pairs Formed by Transversals and Parallel Lines – Part 1

Consider the following figure of a transversal crossing two parallel lines.



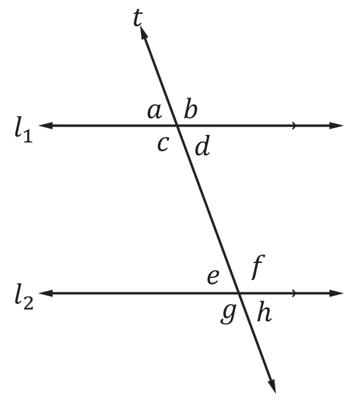
Name the acute angles in the above figure.

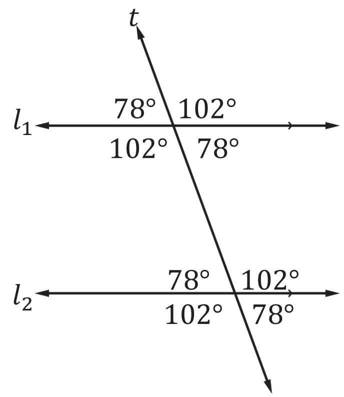
Name the obtuse angles in the above figure.

Which angles are congruent? Justify your answer.

Which angles are supplementary? Justify your answer.

Consider the following figures of transversal t crossing parallel lines, l_1 and l_2.

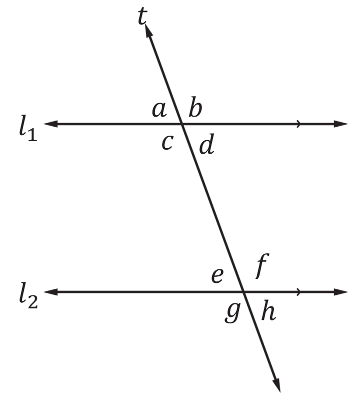


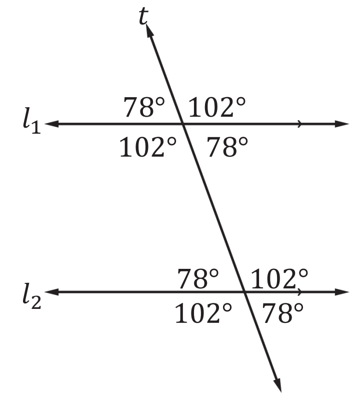


*Identify an example of the **Linear Pair Postulate**. Use the figure above to justify your answer.*

*Identify an example of the **Vertical Angles Theorem**. Use the figure above to justify your answer.*

Make a list of the interior and the exterior angles. What can you say about these angles?





*Identify each of the **alternate interior angles** in the above figures and determine the angles' measures.*

TAKE NOTE! Postulates & Theorems

Alternate Interior Angles Theorem
If two parallel lines are cut by a transversal, the alternate interior angles are congruent.

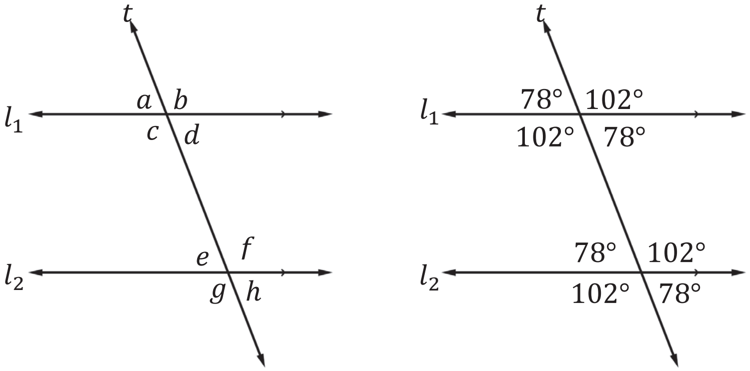
Converse of the Alternate Interior Angles Theorem
If two lines are cut by a transversal and the alternate interior angles are congruent, the lines are parallel.

*Identify the **alternate exterior angles** in the above figures and determine the angles' measures.*

TAKE NOTE! Postulates & Theorems

Alternate Exterior Angles Theorem
If two parallel lines are cut by a transversal, the alternate exterior angles are congruent.

Converse of the Alternate Exterior Angles Theorem
If two lines are cut by a transversal and the alternate exterior angles are congruent, the lines are parallel.



*Identify the **corresponding angles** in the above figures. What does each angle measure?*

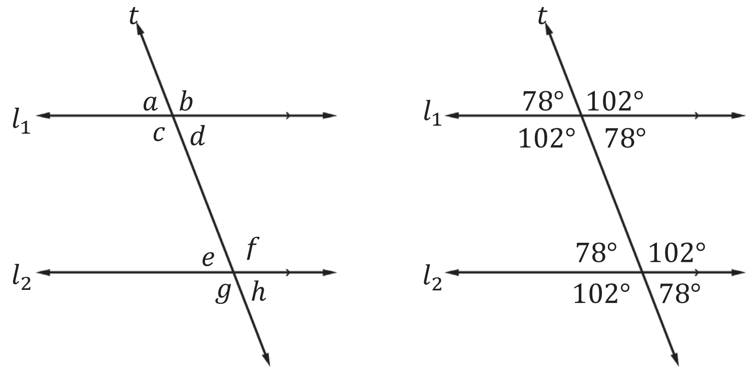
TAKE NOTE! *Postulates & Theorems*

Corresponding Angles Theorem

If two parallel lines are cut by a transversal, the corresponding angles are congruent.

Converse of the Corresponding Angles Theorem

If two lines are cut by a transversal and the corresponding angles are congruent, the lines are parallel.



*Identify the **same-side/consecutive angles** in the above figures. What does each angle measure?*

TAKE NOTE! *Postulates & Theorems*

Same-side Consecutive Angles Theorem

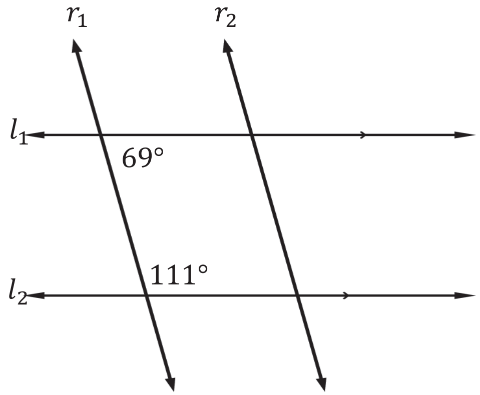
If two parallel lines are cut by a transversal, the interior angles on the same side of the transversal are supplementary.

Converse of the Same-side Consecutive Angles Theorem

If two lines are cut by a transversal and the interior angles on the same side of the transversal are supplementary, the lines are parallel.

Let's Practice!

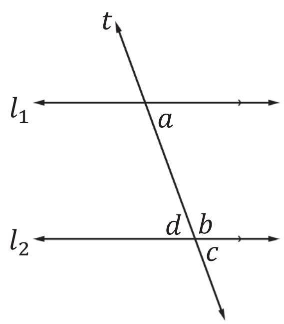
1. Which lines of the following segments are parallel? Circle the appropriate answer, and justify your answer.



Ⓐ r_1 and r_2
Ⓑ l_1 and l_2
Ⓒ r_1 and l_2
Ⓓ l_1 and r_2

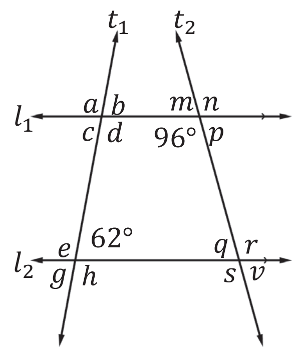
2. Which of the following is a condition for the figure below that will ***not*** prove $l_1 \parallel l_2$?

Ⓐ $\angle a \cong \angle c$
Ⓑ $\angle b + \angle d = 180$
Ⓒ $\angle a \cong \angle d$
Ⓓ $\angle a + \angle b = 180$



Try It!

3. Consider the figure below, where l_1 and l_2 are parallel and cut by transversals t_1 and t_2. Find the values of a, b and v.



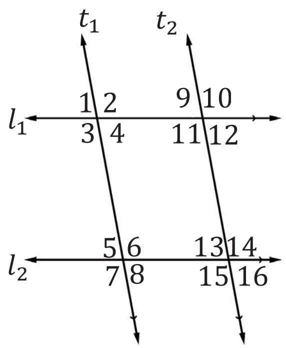
Section 3 – Topic 7
Special Types of Angle Pairs Formed by Transversals and Parallel Lines – Part 2

Let's Practice!

1. Complete the chart below using the following information.

Given:
$\angle 4$ and $\angle 7$ are supplementary. $\angle 8$ and $\angle 16$ are congruent.

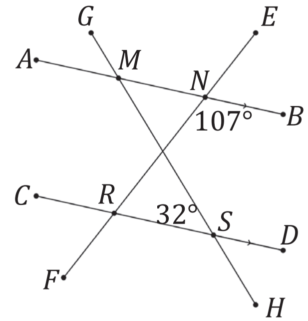
Prove: $l_1 \parallel l_2$ and $t_1 \parallel t_2$



Statements	Reasons
1.	**1.** Given
2.	**2.** Given
3. $\angle 7 \cong \angle 6$; $\angle 13 \cong \angle 16$	**3.**
4.	**4.** Substitution
5. $l_1 \parallel l_2$	**5.**
6. $t_1 \parallel t_2$	**6.**

Try It!

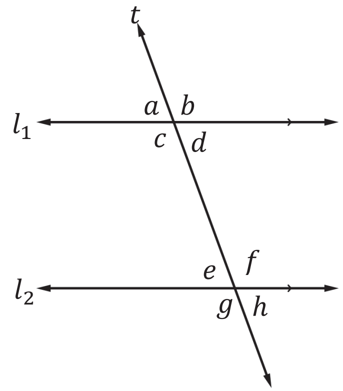
2. Consider the figure below. Find the measures of $\angle AMS$ and $\angle CRF$, and justify your answers.



3. Complete the chart below using the following information.

Given: $l_1 \parallel l_2$

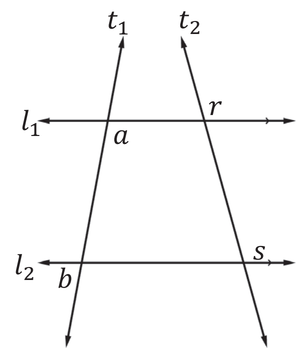
Prove: $m\angle a + m\angle g = 180°$



Statements	Reasons
1. $l_1 \parallel l_2$	**1.**
2.	**2.** Linear Pair Postulate
3.	**3.** Definition of Supplementary
4. $\angle c \cong \angle g$	**4.**
5.	**5.** Definition of Congruent
6. $m\angle a + m\angle g = 180°$	**6.**

BEAT THE TEST!

1. Consider the figure below in which $l_1 \parallel l_2$, $m\angle a = 13y$, $m\angle b = 31y + 4$, $m\angle r = 30x + 40$, and $m\angle s = 130x - 160$.

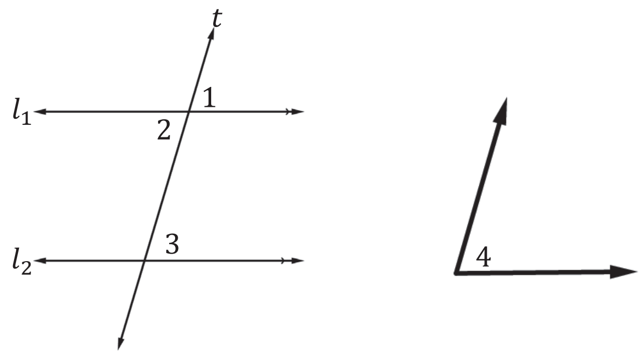


What are the values of $\angle a, \angle b, \angle r$, and $\angle s$?

$\angle a =$ ______ $\angle b =$ ______

$\angle r =$ ______ $\angle s =$ ______

2. Consider the figure below.



Given: $l_1 \parallel l_2$; $\angle 2 \cong \angle 4$

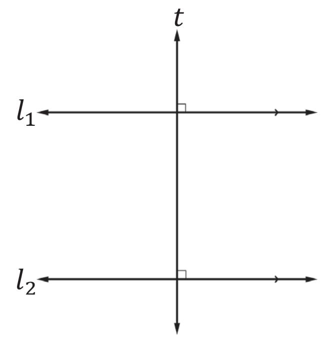
Prove: $\angle 1 \cong \angle 4$ and $\angle 4 \cong \angle 3$

Complete the following chart.

Statements	**Reasons**
1. $l_1 \parallel l_2$; $\angle 2 \cong \angle 4$	**1.** Given
2. $\angle 1 \cong \angle 2$	**2.** Vertical Angles Theorem
3. $\angle 1 \cong \angle 4$	**3.**
4. $\angle 1 \cong \angle 3$	**4.**
5. $\angle 4 \cong \angle 3$	**5.** Transitive Property of Congruence

Section 3 – Topic 8
Perpendicular Transversals

Consider the following figure of a transversal cutting parallel lines l_1 and l_2.

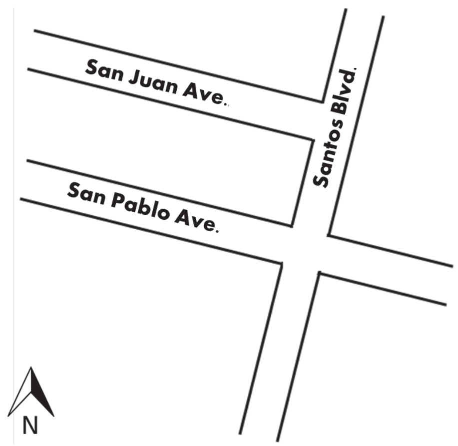


What observations can you make about the figure?

A transversal that cuts two parallel lines forming right angles is called a _________________ transversal.

> *TAKE NOTE!* *Postulates & Theorems*
>
> **Perpendicular Transversal Theorem**
> In a plane, if a line is perpendicular to one of two parallel lines, then it is perpendicular to the other line also.
>
> **Perpendicular Transversal Theorem Corollary**
> If two lines are both perpendicular to a transversal, then the lines are parallel.

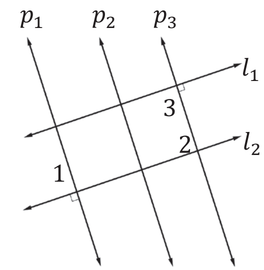
Consider the figure below. San Pablo Ave. and Santos Blvd. are perpendicular to one another. San Juan Ave. was constructed later and is parallel to San Pablo Ave.



Using the **Perpendicular Transversal Theorem**, what can you conclude about the relationship between San Juan Ave. and Santos Blvd.?

Let's Practice!

1. Consider the following information.



Given: $p_1 \parallel p_2$, $p_2 \parallel p_3$, $l_2 \perp p_1$, and $l_1 \perp p_3$

Prove: $l_1 \parallel l_2$

Complete the following paragraph proof.

Because it is given that $p_1 \parallel p_2$ and $p_2 \parallel p_3$, then $p_1 \parallel p_3$ by the____.

This means that $\angle 1 \cong \angle$ _____, because they are corresponding angles.

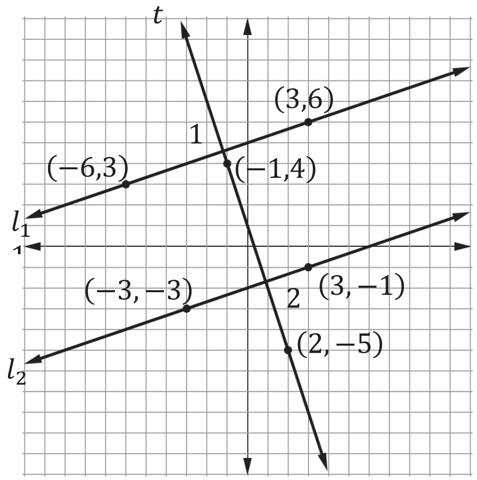
If $l_2 \perp p_1$, then $m\angle 1 = 90°$. Thus, $m\angle 2 =$ ______________.

This means $p_3 \perp l_2$, based in the definition of perpendicular lines.

It is given that $l_1 \perp p_3$, so $l_1 \parallel l_2$, based on the corollary that states ____.

Try It!

2. Consider the lines and the transversal drawn in the coordinate plane below.

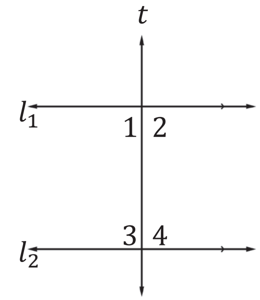


a. Prove that $\angle 1 \cong \angle 2$. Justify your work.

b. Prove that $m\angle 1 = m\angle 2 = 90°$. Justify your work.

BEAT THE TEST!

1. Consider the figure to the right, and correct the proof of the Perpendicular Transversal Theorem.



Given: $\angle 1 \cong \angle 4$ and $l_1 \perp t$ at $\angle 2$.
Prove: $l_2 \perp t$

Two of the reasons in the chart below do not correspond to the correct statement. Circle those two reasons.

Statements	Reasons
1. $\angle 1 \cong \angle 4$; $l_1 \perp t$ at $\angle 2$	**1.** Given
2. $l_1 \parallel l_2$	**2.** Consecutive Angles Theorem
3. $\angle 2$ is a right angle	**3.** Definition of perpendicular lines
4. $m\angle 2 = 90°$	**4.** Definition of right angle
5. $m\angle 2 + m\angle 4 = 180°$	**5.** Converse of Alternate Interior Angles Theorem
6. $90° + m\angle 4 = 180°$	**6.** Substitution property
7. $m\angle 4 = 90°$	**7.** Subtraction property of equality
8. $l_2 \perp t$	**8.** Definition of Perpendicular Lines

Section 3 – Topic 9
Angle-Preserving Transformations

Consider the figures below. The lines l_1 and l_2 are parallel, and Figure B is a rotation of Figure A.

Figure A

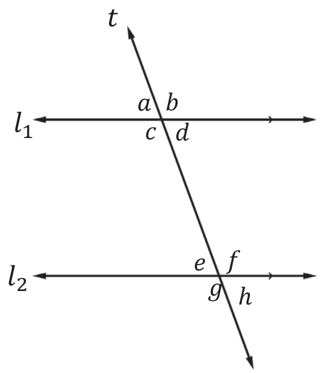
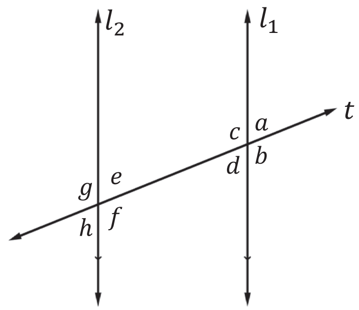


Figure B



The figures above represent an ***angle-preserving transformation***.

What do you think it means for something to be an angle-preserving transformation?

Does the transformation preserve parallelism? Justify your answer.

Angle-preserving transformations refer to reflection, translation, rotation and dilation that preserve ___________________ ______________ and ______________after the transformation.

The conditions for parallelism of two lines cut by a transversal are:

- Corresponding angles
- Alternate interior angles
- Alternate exterior angles

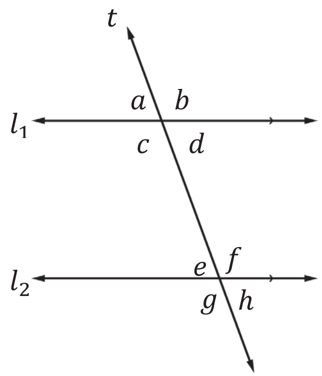
$\left.\begin{array}{l}\\ \\ \\ \end{array}\right\}$ are _______________

- Same-side/consecutive angles are _________________

Since the transformations preserve angle measures, these conditions are also preserved. Therefore, parallel lines will remain parallel after any of these four transformations.

Let's Practice!

1. Consider the figure below in which $l_1 \parallel l_2$.

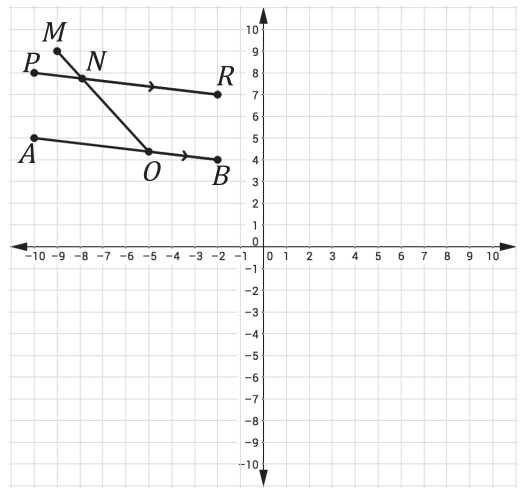


   a. Determine the angles that are congruent with $\angle a$ after a translation of $\angle a$: ________________

   b. Determine the angles that are supplementary with $\angle a$ and $\angle e$ after a 180° rotation of $\angle a$: _______________

Try It!

2. Consider the following figure.

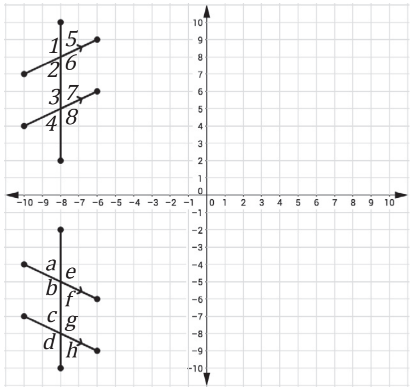


   a. Reflect the above image across $y = x$.

   b. If $\angle MOB = 117°$ and $\angle M'N'P' = 63°$, prove that after the reflection, $\angle M'O'B' = 117°$.

Let's Practice!

3. The figure in Quadrant III of the coordinate plane below is a transformation of the figure in Quadrant II.

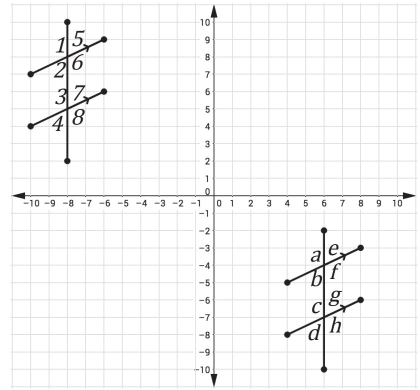


   a. What type of transformation is shown above? Justify your answer.

   b. Write a paragraph proof to prove that $\angle 6$ and $\angle f$ are supplementary.

Try It!

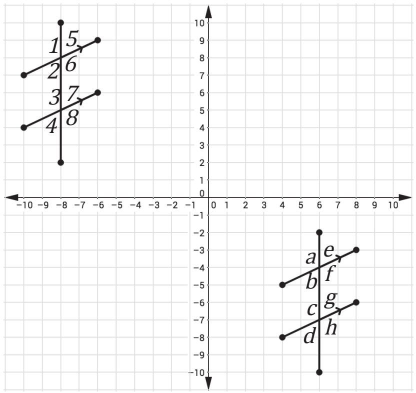
4. The figure in Quadrant IV of the coordinate plane below is a transformation of the figure in Quadrant II.



   a. What type of transformation is shown above? Justify your answer.

   b. Write a paragraph proof to prove that $\angle 4$ and $\angle e$ are congruent.

5. Consider the figure again and the statements below.



$m\angle 2 = 4x - 11$

$m\angle 4 = 4y + 1$

$m\angle g = 3x + 3$

$m\angle h = 13y - 8$

Find the following values. Explain how you found the answers.

$x =$

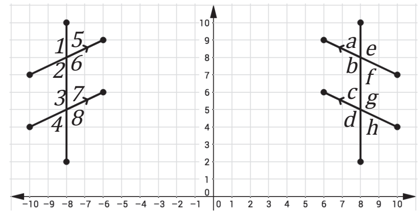
$y =$

$m\angle 1 =$

$m\angle d =$

BEAT THE TEST!

1. The figure in the second quadrant of the coordinate plane below was transformed into the figure in the first quadrant. Mark the most appropriate answer in each shape below.



Part A: The figure was

- o dilated
- o rotated
- o reflected
- o translated

- o across the x-axis
- o across the y-axis
- o around the origin
- o by a scale factor of -1
- o sixteen units to the right

Part B: $\angle 1$ is

- o complementary
- o congruent
- o corresponding
- o supplementary

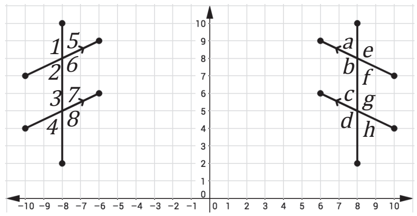
to $\angle c$.

Part C: $\angle 6$ is

- o complementary
- o congruent
- o corresponding
- o supplementary

to $\angle g$.

2. Consider the same figure as in the previous question.

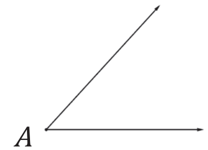


If $m\angle 7 = 2x - 4$ and $m\angle b = 6x$, then $m\angle 8 =$ _____ and $m\angle h =$ _____.

Section 3 – Topic 10
Constructions of Angles, Perpendicular Lines, and Parallel Lines

We are going to learn how to do three constructions in this video.

Copy $\angle A$:



Step 1. Draw a ray that will become one of the two rays of the new angle. Label the ray $\overrightarrow{DE}$.

Step 2. Place your compass at the vertex of $\angle A$. Create an arc that intersects both rays of $\angle A$.

Step 3. Using this same measurement, do the same thing beginning at the endpoint of $\overrightarrow{DE}$.

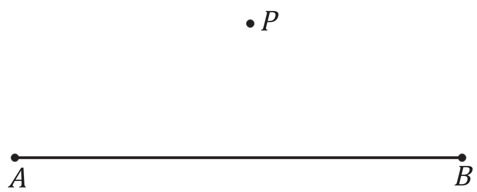
Step 4. Set your compass to the distance from one of the intersection points of the arc and $\angle A$. Then, set your compass to the other intersection point of the arc and $\angle A$.

Step 5. Using the same setting, place the compass on the intersection point of the $\overrightarrow{DE}$ and the arc drawn in step 2. Draw an arc that intersects the other side.

Step 6. This will create two arcs that intersect at a point. Label the point F. Draw the line going from this point to the endpoint of the ray to complete the copy of the angle.

Let's Practice!

1. Construct a line through P perpendicular to the given line segment $\overline{AB}$:



Step 1. Place the point of the compass on point P, and draw an arc that crosses $\overline{AB}$ twice. Label the two points of intersection C and D.

Step 2. Place the compass on point C and make an arc above $\overline{AB}$ that goes through P, and a similar arc below $\overline{AB}$.

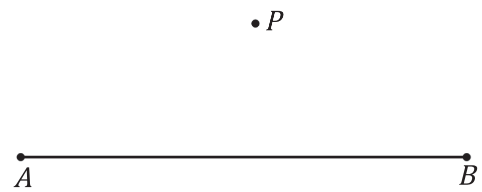
Step 3. Keeping the compass at the same width as in step 2, place the compass on point D, and repeat step 2.

Step 4. Draw a point where the arcs drawn in Step 2 and Step 3 intersect. Label that point R.

Step 5. Draw a line segment through points P and R, making $\overline{PR}$ perpendicular to $\overline{AB}$.

Try It!

2. Construct a line segment through P parallel to the given line segment $\overline{AB}$:



Step 1. Draw a point on AB and label the point C. Create a ray from point C through point P to create an angle. Label the angle $\angle R$.

Step 2. Set the width of the compass to about half the distance between P and C, place the point on C, and draw an arc across both lines.

Step 3. Using the same setting, place the compass on point P, and draw a similar arc to one in step 2.

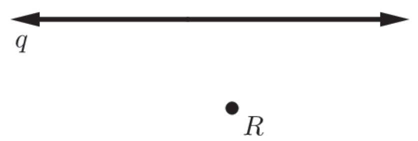
Step 4. Set the width of the compass to the distance where the lower arc crosses the two lines, move the compass to where the upper arc crosses ray CP and draw an arc across the upper arc, forming point Q.

Step 5. Draw a line segment through points P and Q, creating line segment PQ that is parallel to $\overline{AB}$.

Which theorem relating to parallel lines can we use to prove that our construction is correct? ____________________________

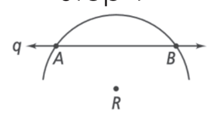
BEAT THE TEST!

1. Consider the figure below.

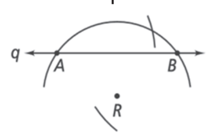


Celine attempted to construct a line through point R that is perpendicular to line. In which step did she make a mistake? Mark the most appropriate answer below. Justify your answer.

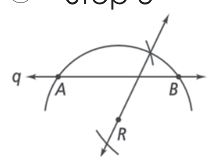
Ⓐ Step 1



Ⓑ Step 2



Ⓒ Step 3



Ⓓ All the steps are correct. She did not make a mistake.