Section 4: Introduction to Polygons – Part 1

The following Mathematics Florida Standards will be covered in this section:	
MAFS.912.G-CO.1.1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
MAFS.912.G-CO.1.2	Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not.
MAFS.912.G-CO.1.4	Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
MAFS.912.G-CO.1.5	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure. Specify a sequence of transformations that will carry a given figure onto itself.
MAFS.912.G-CO.3.10	Prove theorems about triangles. Use theorems about triangles to solve problems: measures of interior angles of a triangle sum to 180.
MAFS.912.G-MG.1.1	Use geometric shapes, their measures, and their properties to describe objects.
MAFS.912.G-SRT.1.1	Verify experimentally the properties of dilations given by a center and a scale factor. A dilation takes a line not passing through the center of the dilation to a parallel line and leaves a line passing through the center unchanged. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

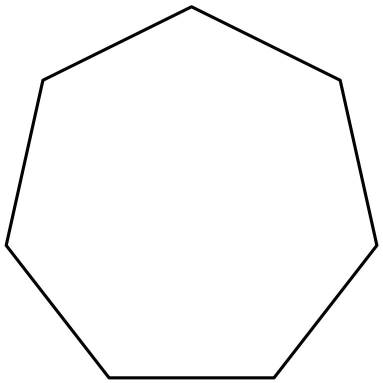
Topics in this Section

Section 4 – Topic 1
Introduction to Polygons – Part 1

The word polygon can be split into two parts:

- "poly-" means _____________.
- "gon" means ______________.

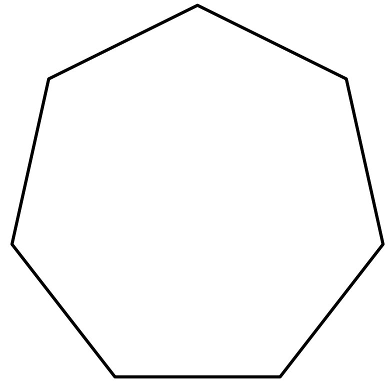
Consider the polygon below.



Are polygons closed or open figures? Explain your answer.

How many sides does a polygon have?

Consider the polygon again.



The ***interior angles of a polygon*** are the angles on the inside of the polygon formed by each pair of adjacent sides.

Use I to label the interior angles of the polygon above.

An ***exterior angle of a polygon*** is an angle that forms a linear pair with one of the interior angles of the polygon.

Use E to label the exterior angels of the polygon above.

Draw a representation of each of the polygons below.

Name	Definition	Representation
Regular	All angles and sides of this polygon are congruent.	
Irregular	All angles and sides of this polygon are not congruent.	
Convex	This polygon has no angles pointing inwards. That is, no interior angles can be greater than $180°$.	
Concave	This polygon has an interior angle greater than $180°$.	
Simple	This polygon has one boundary and doesn't cross over itself.	

We can also classify a polygon by the number of sides.

# Sides	Names	# Sides	Names
3		8	
4		9	nonagon
5	pentagon	10	
6		11	hendecagon
7	heptagon	12	dodecagon

Let's discover some facts about angle measures of polygons.

How many sides does a triangle have?

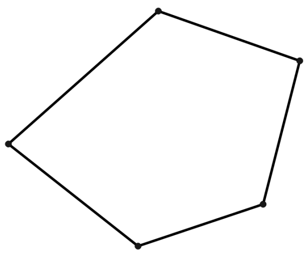
What is the sum of the interior angles of a triangle?

Consider the quadrilateral below.

How many sides does a quadrilateral have?

Use your knowledge of triangles to find the sum of the interior angles of the quadrilateral.

Consider the pentagon below.

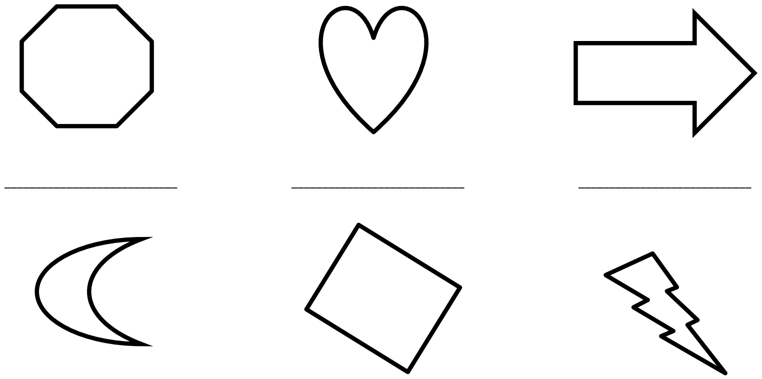


How many sides does a pentagon have?

Use your knowledge of triangles to find the sum of the interior angles of the pentagon.

Let's Practice!

1. Determine whether the following shapes are polygons. Classify the polygons by their number of sides.



Try It!

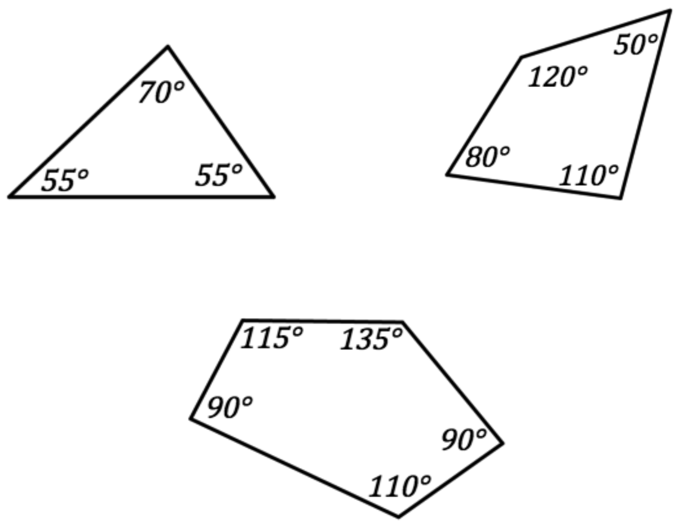
2. Complete the table below.

Number of sides	Sum of interior angles
3	
4	
5	
6	
7	
n	

Section 4 – Topic 2
Introduction to Polygons – Part 2

Let's Practice!

1. Consider each of the following polygons. Find the sum of the exterior angles in each polygon below.



2. The sum of the exterior angles of any polygon equals _______.

Try It!

3. A convex pentagon has exterior angles with measures 66°, 77°, 82° and 62°.

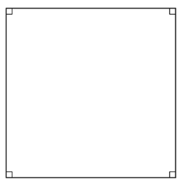
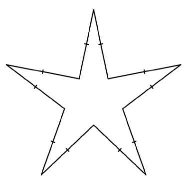
   a. Draw a representation of the pentagon.

   b. What is the measure of an exterior angle of the pentagon at the fifth vertex?

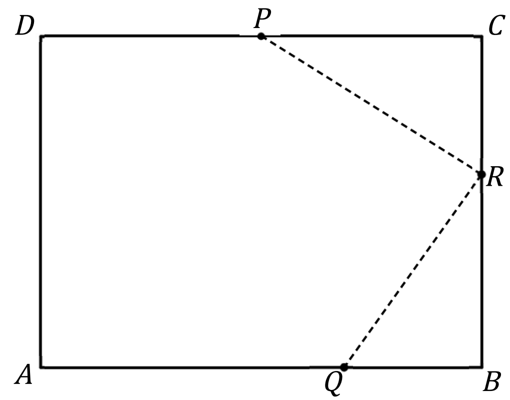
   c. Classify the pentagon as regular or irregular. Justify your answer.

BEAT THE TEST!

1. Eddie has a square piece of paper that he cuts into different shapes. The table below shows the piece of paper in the first row and then, each figure he made in the other rows. Classify each figure as regular, concave, and/or convex by marking the appropriate box. Name each type of polygon represented by filling in each blank provided.

Figure	Regular	Concave	Convex	Name the Polygon
	☐	☐	☐	____________
	☐	☐	☐	____________
	☐	☐	☐	____________

2. Consider the figure below. Moses cut rectangle $ABCD$ into pentagon $AQRPD$.



If $m\angle PRQ = 71°$ and $m\angle PRC \cong m\angle QRB$, verify the sum of the interior angles of pentagon $AQRPD$ using two different methods. Justify your answers.

Section 4 – Topic 3
Angles of Polygons

In the previous video, you learned the formula to find the sum of the angles of a polygon.

How can you use the sum of interior angles formula to find the number of sides of a polygon?

How can you use the sum of interior angles formula to find the measure one angle of a regular polygon?

Can the same process be used to find the measure of one angle of an irregular polygon? Explain your reasoning.

Let's Practice!

1. What are the measures of each interior angle and each exterior angle of regular hexagon $MARLON$?

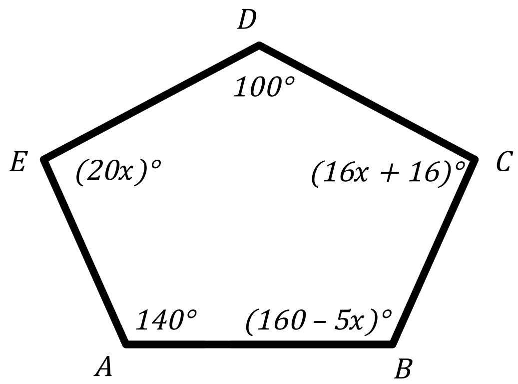
2. The sum of the interior angles of a regular polygon is $1080°$.

   a. Classify the polygon by the number of sides.

   b. What is the measure of one interior angle of the polygon?

   c. What is the measure of one exterior angle of the polygon?

3. Consider pentagon $ABCDE$.



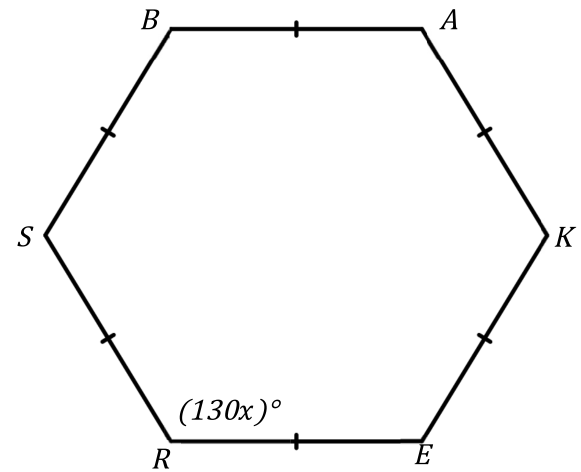
a. Find the value of x.

b. Find the value of each $\angle A$, $\angle B$, $\angle C$, $\angle D$, and $\angle E$.

c. Find the value of each exterior angle.

Try It!

4. Consider the regular hexagon below.



a. Find the value of x.

b. Find the value of each interior angle.

c. Find the value of each exterior angle.

5. If the measure of an exterior angle of a regular polygon is 24°, how many sides does the polygon have?

6. Given a regular decagon and a regular dodecagon, which one has a greater exterior angle? By how much is the angle greater?

BEAT THE TEST!

1. A teacher showed the following exit ticket on the projector.

> 1. What is the sum of the interior angle measures of a regular 24-gon?
> 2. Pentagon ABCDE has interior angles that measure 60° and 160° and another pair of interior angles that measure 130° each. What is the measure of an interior angle at the fifth vertex?

A student completed the following exit ticket.

1) $\text{Sum} = \frac{(n-2)180}{n}$

$= \frac{(24-2)180}{24}$

$= \frac{(22)180}{24}$

$= 165$

2) $130 + 60 + 160 + x = (5-2)180$

$350 + x = 540$

$x = 190$

Which of the following statements is true?

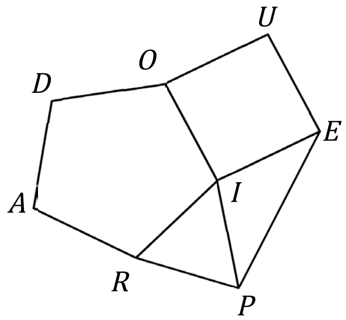
Ⓐ Both answers are correct.

Ⓑ Answer #1 is incorrect. The student found the individual angle, not the sum of the angles. The answer should be 3960°. Answer #2 is correct.

Ⓒ Answer #1 is correct. Answer #2 is incorrect. There are two angles measuring 130°, but only one was counted in the sum. The answer should be 60°.

Ⓓ Both answers are incorrect. In #1 the student found the individual angle, not the sum of the angles. The answer is 3960°. In #2 there are two angles measuring 130°, but only one was counted in the sum. The answer should be 60°.

2. Consider the figure below.



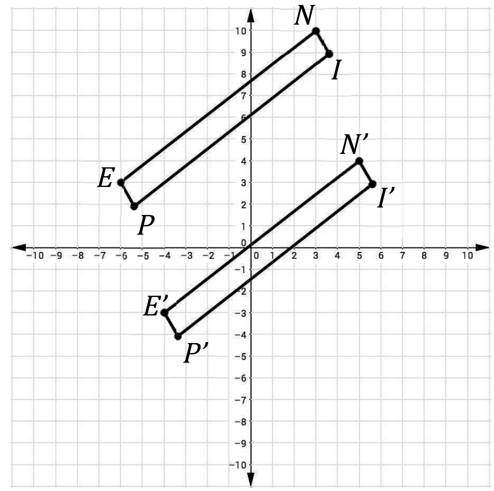
$DARIO$ is a regular pentagon, RIP is an equilateral triangle, and $EIOU$ is a square.

Part A: What is the measure of $\angle IPE$?

Part B: What is the measure of exterior angle $\angle DOU$?

Section 4 – Topic 4
Translations of Polygons

Describe the translation of rectangle $PINE$.

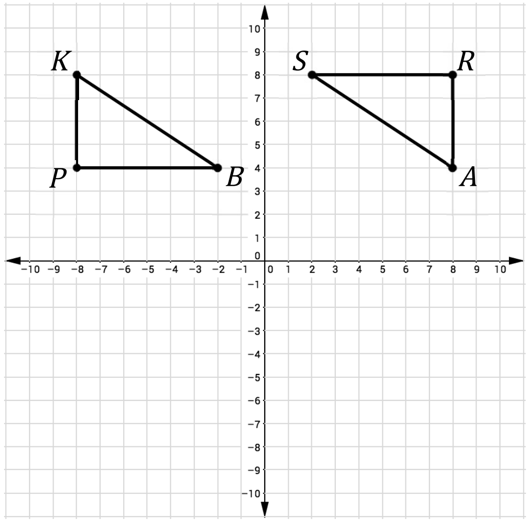


- The original object and its image are ________________.
- In other words, the two objects are identical in every respect except for their ______________.

Draw line segments linking a vertex in the original image to the corresponding vertex in the translated image. Make observations about the line segments.

Let's Practice!

1. Consider the two right triangles below.



Rectangle $PARK$ is formed when right triangles PBK and RSA are translated. $PARK$ has vertices at $P(-3,4)$, $A(3,4)$, $R(3,8)$, and $K(-3,8)$.

Describe how rectangle $PARK$'s location on the coordinate plane is possible with only one translation for PBK and one translation for RSA.

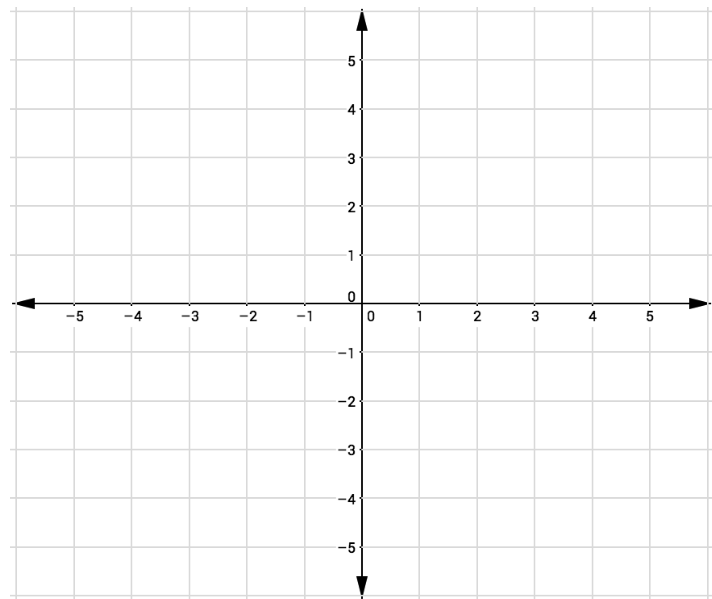
Try It!

2. $C(0,-1)$, $A(-2,2)$, $M(2,4)$, $P(3,0)$ is transformed by $(x,y) \rightarrow (x-2, y-1)$.

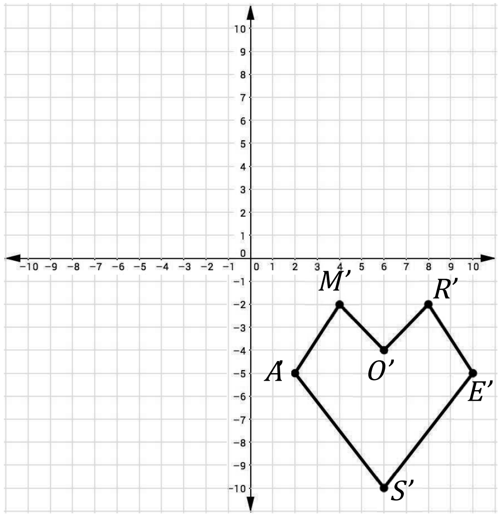
   a. What is the x – coordinate of A'?

   b. What is the y – coordinate of P'?

   c. Show the translation on the coordinate plane below.



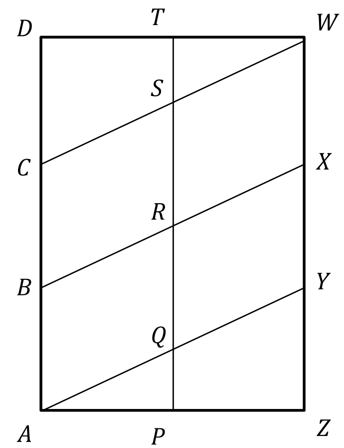
3. Polygon $A'M'O'R'E'S'$ is the image of polygon $AMORES$ after a translation $(x+8, y-7)$.



What are the original coordinates of each point of polygon $AMORES$?

BEAT THE TEST!

1. Consider the figure below.



If R is the image of A after a translation, then which point is the image of Q under the same translation?

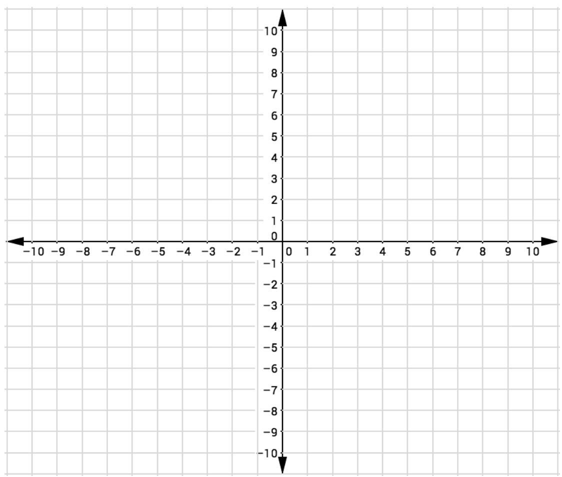
Section 4 – Topic 5
Reflections of Polygons

Think back to what you know about reflections to answer the questions below.

What are the mirror lines? Draw a representation of a mirror line.

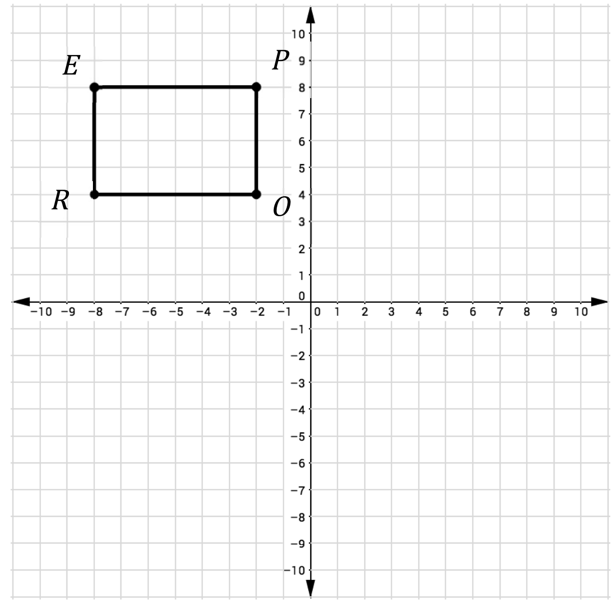
What is a mirror point? Draw representation of a mirror point.

What are the most common mirror point(s) and line(s)?



Let's Practice!

1. Consider rectangle $ROPE$ on the coordinate plane.

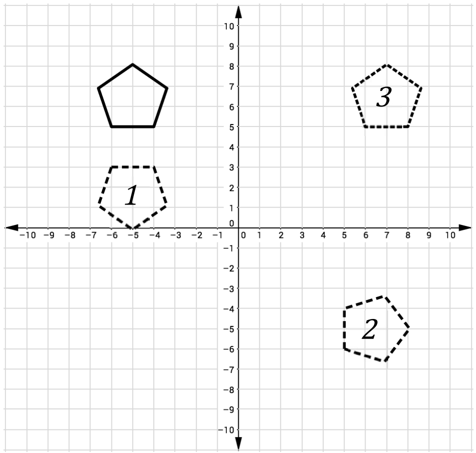


   a. Draw a reflection over the x–axis. Write down the coordinates of the reflected figure.

   b. Draw a reflection over the y–axis. Write down the coordinates of the reflected figure.

Try It!

2. The pentagon below was reflected three different times and results in the dashed pentagons labeled as $1, 2,$ and 3.

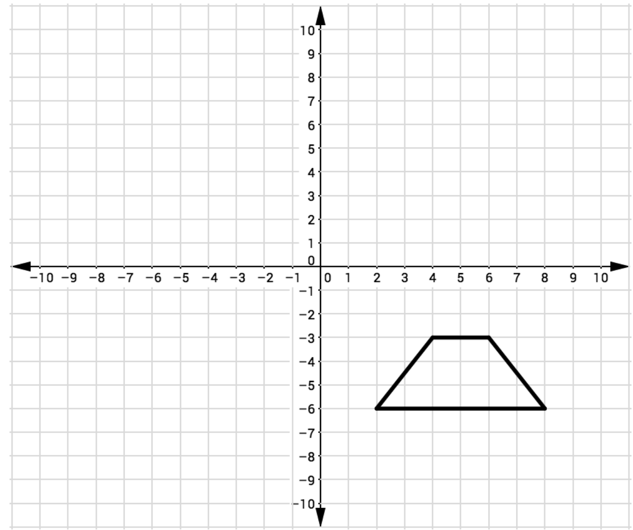


Describe each reflection in the table below.

Reflection 1	**Reflection 2**	**Reflection 3**

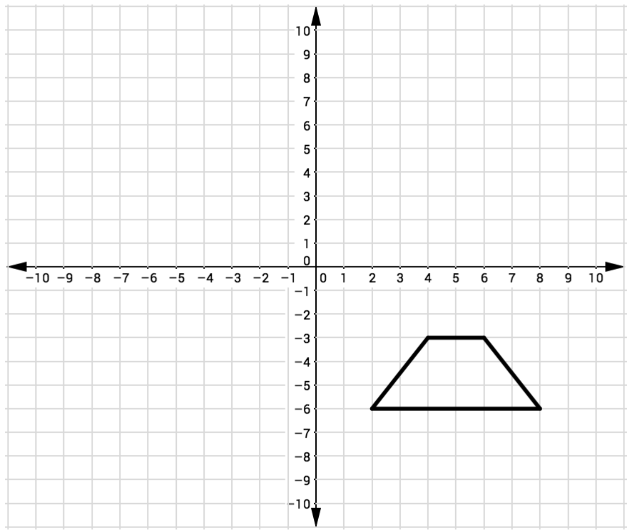
Let's Practice!

3. Reflect the following image over $y = x$.



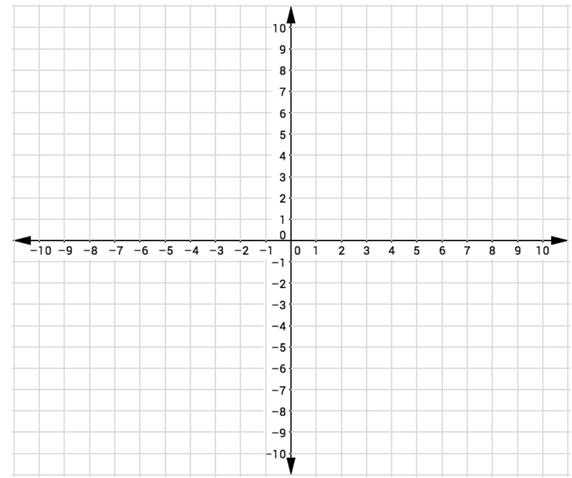
Try It!

4. Reflect the following image over $y = -x$.



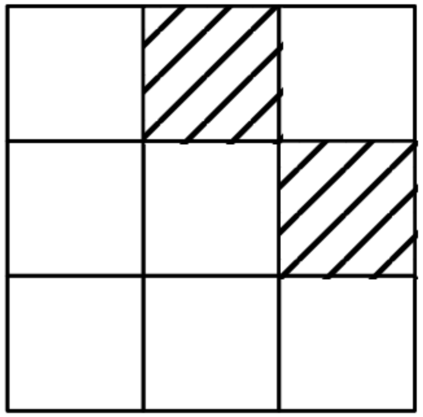
What are the similarities and differences between reflecting over the axis and reflecting over other linear functions?

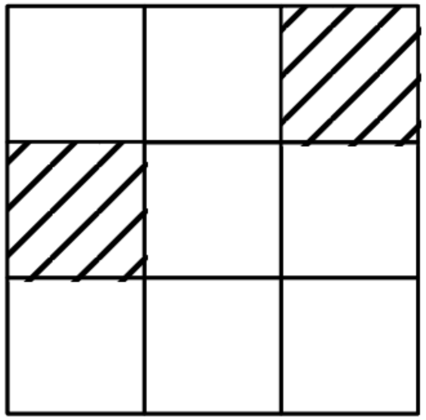
5. Pentagon $CALOR$ is the result of a reflection of pentagon $FRISA$ over $y = x$. $CALOR$ has vertices at $C(2,-2), A(0,-4)$, $L(1,-6)$, $O(3,-6)$, and $R(4,-4)$. In which quadrant was pentagon $FRISA$ located before being reflected to create $CALOR$?



BEAT THE TEST!

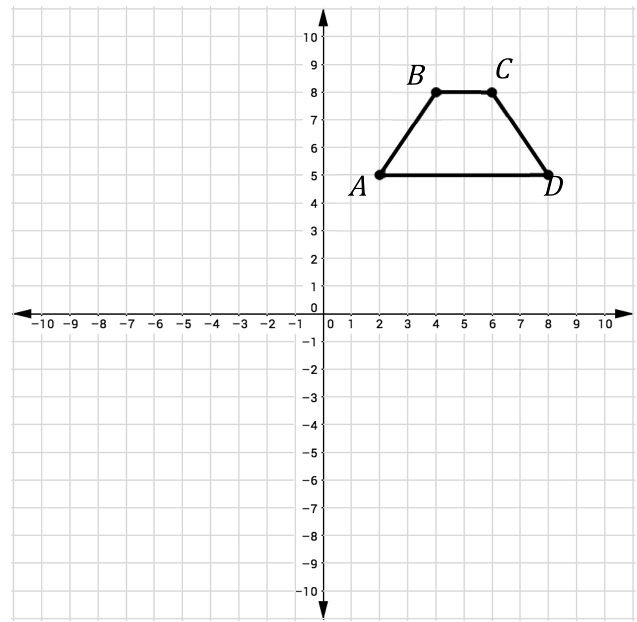
1. Draw the line(s) of reflection of the following figures.





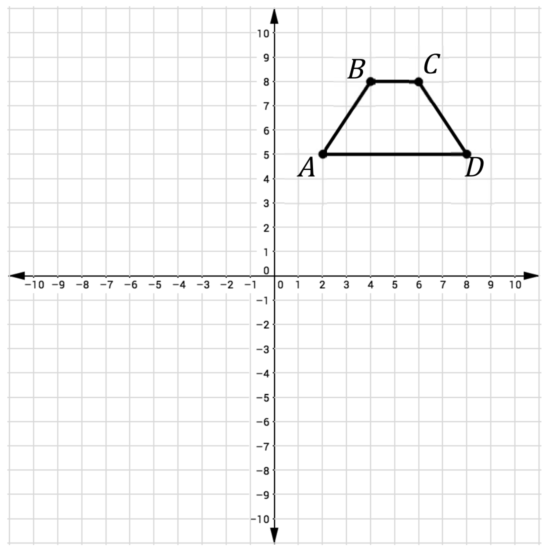
Section 4 – Topic 6
Rotation of Polygons – Part 1

Consider polygon $ABCD$ and the transformed polygon that is rotated $90°$, $180°$, $270°$, and $360°$ clockwise about the origin.



Vertices of $ABCD$	Vertices of $90°$ rotation	Vertices of $180°$ rotation	Vertices of $270°$ rotation	Vertices of $360°$ rotation
$(2, 5)$				
$(4, 8)$				
$(6, 8)$				
$(8, 5)$				

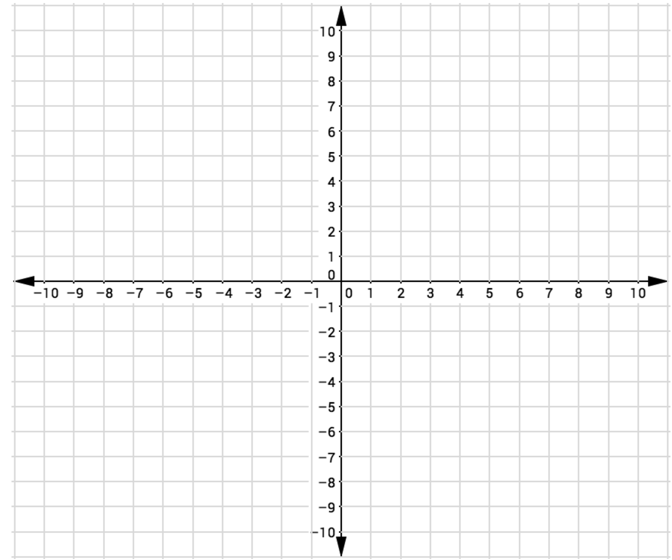
Consider polygon $ABCD$ and the transformed polygon that is rotated $90°$, $180°$, $270°$, and $360°$ counterclockwise about the origin.



Vertices of $ABCD$	Vertices of $90°$ rotation	Vertices of $180°$ rotation	Vertices of $270°$ rotation	Vertices of $360°$ rotation
$(2,5)$				
$(4,8)$				
$(6,8)$				
$(8,5)$				

Let's Practice!

1. Rotate and draw $COMA$ $90°$ counterclockwise about the origin if the vertices are $C(1,-2), O(0,2), M(3,2), A(3,-3)$.



What are the coordinates of $C'O'M'A'$?

Try It!

2. Samuel rotated $AMEN$ $270°$ clockwise about the origin to generate $A'M'E'N'$ with vertices at A'$(1,5)$, $M'(1,0)$, $E'(-1,-1)$, and $N'(-3,2)$.

   What is the sum of all y-coordinates of $AMEN$?

3. What happens if we rotate a figure around a different center point instead of rotating it around the origin?

Section 4 – Topic 7
Rotation of Polygons – Part 2

Use the following steps to rotate polygon $ABCD$ $155°$ clockwise about C. Use the figure on the following page.

Step 1. Extend the line segment between the point of rotation, C, and another vertex on the polygon, towards the opposite direction of the rotation.

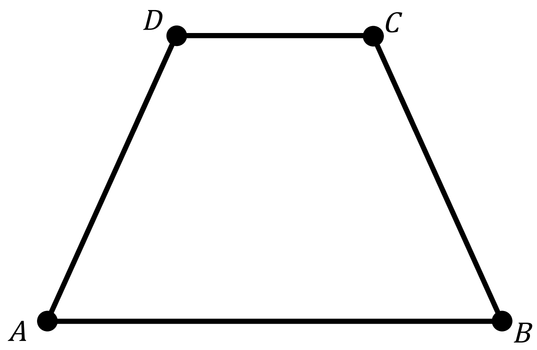
Step 2. Place the center of the protractor on the point of rotation and line it up with the segment drawn in step 1. Measure the angle of rotation at C. Mark a point at the angle of rotation and draw a segment with your straightedge by connecting the point with the center of rotation, C.

Step 3. Use a compass to measure the segment used in step 1. Keeping the same setting, place the compass on the segment drawn in step 2 and draw an arc where the new point will be located. Label the new point with a prime notation.

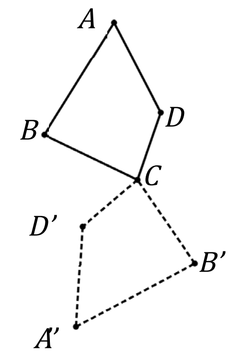
Step 4. Copy the angle adjacent to the angle of rotation. Mark a point at the copied angle in the new figure. Draw a segment by connecting the point at the new angle with the point created in step 3.

Step 5. Use a compass to measure the segment adjacent to the one used in step 1. Keeping the same setting, place the compass on the segment drawn in step 4 and draw an arc where the new point will be located. Label the new point with a prime notation.

Step 6. Repeat steps 4-5 with the two other angles to complete the construction of the rotated polygon.



Consider the figure below.



Which is the point of rotation? How do you know?

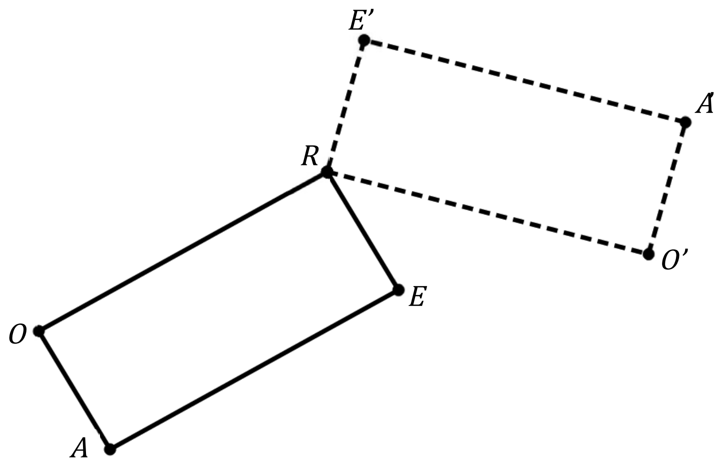
Do you have enough information to determine if the rotation is clockwise or counterclockwise?

If the rotation is clockwise, should it be in between $0° - 90°$, $90° - 180°$, $180° - 270°$, *or* $270° - 360°$?

If the rotation is counterclockwise, should it be in between $0° - 90°$, $90° - 180°$, $180° - 270°$, *or* $270° - 360°$?

Let's Practice!

1. Consider the figure below.

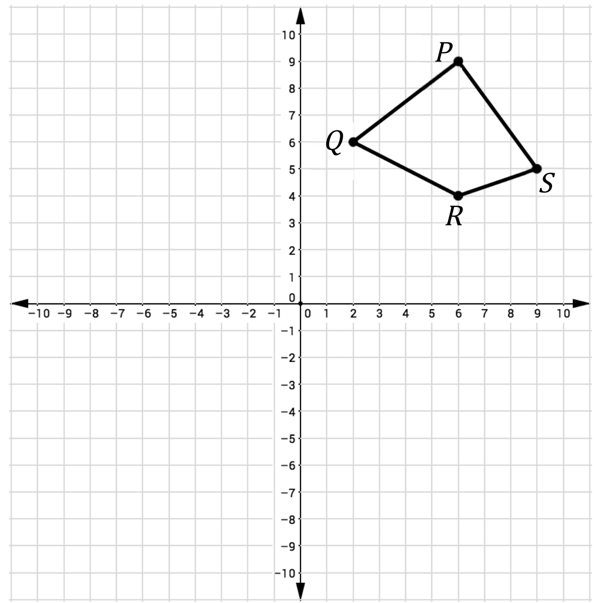


Which of the following statements is true?

Ⓐ The figure shows quadrilateral $AERO$ rotated $45°$ counterclockwise about R and $90°$ clockwise about R.
Ⓑ The figure shows quadrilateral $AERO$ rotated $45°$ clockwise about R and $90°$ counterclockwise about R.
Ⓒ The figure shows quadrilateral $AERO$ rotated $135°$ clockwise about R and $225°$ counterclockwise about R.
Ⓓ The figure shows quadrilateral $AERO$ rotated $135°$ counterclockwise about R and $225°$ clockwise about R.

Try It!

2. Consider quadrilateral $PQRS$ on the coordinate plane below.



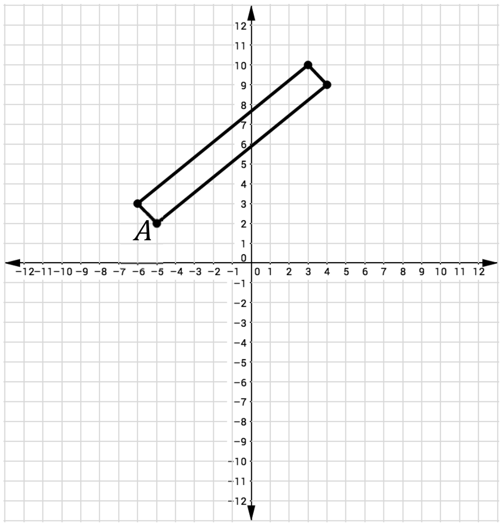
After a rotation of $PQRS$ $90°$ clockwise about the origin, answer each of the following questions.

a. Which vertex will be at point $(-9, 6)$?

b. What will be the coordinates of point R'?

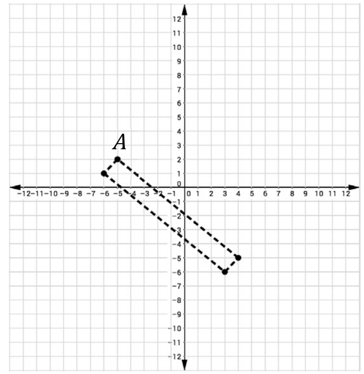
BEAT THE TEST!

1. Consider the quadrilateral below, in which A is the point for rotation.

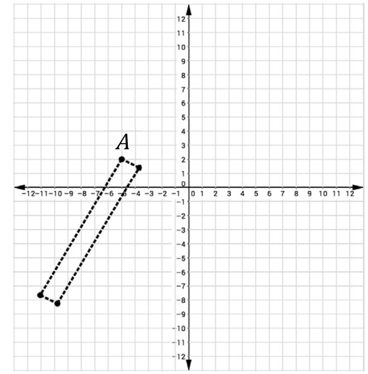


Which figure on the following page shows the quadrilateral rotated $80°$ counterclockwise about A?

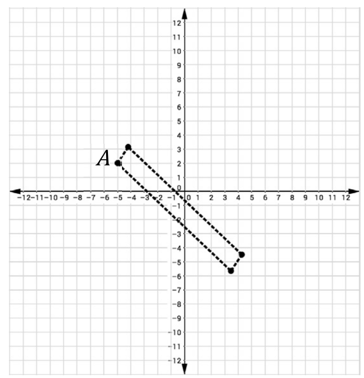
Ⓐ



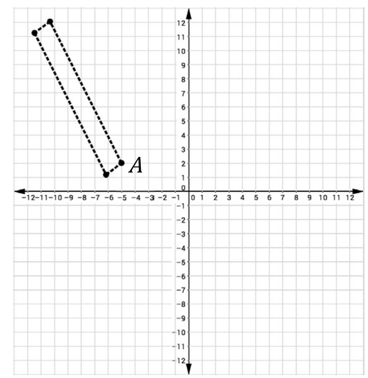
Ⓑ



Ⓒ



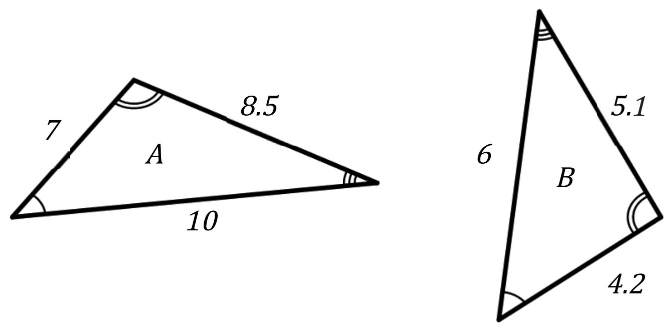
Ⓓ



Section 4 – Topic 8
Dilation of Polygons

How is a ***dilation*** different from a translation, reflection, and rotation?

Consider the figures below.



Is Figure B a dilation of Figure A? Justify your answer.

What is the scale factor?

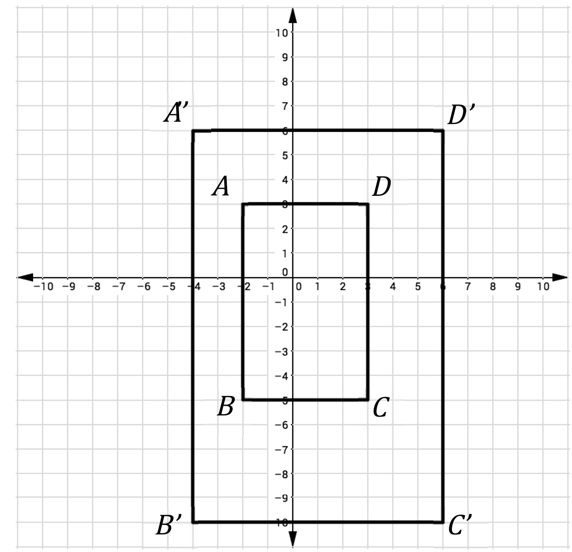
Is Figure A a dilation of Figure B? Justify your answer.

What is the scale factor?

We often represent a dilation with the following notation:

$$D_k = k(x, y)$$

Consider the dilation of quadrilateral $ABCD$ below.



What do you notice about the dilation represented in the figure above?

Let's Practice!

1. Pentagon $PENTA$ has coordinates $P(0,0), E(4,4), N(8,4)$, $T(8,-4)$, and $A(4,-4)$ and is dilated at the origin with a scale factor of $\frac{3}{4}$.

   What are the coordinates of $P'E'N'T'A'$?

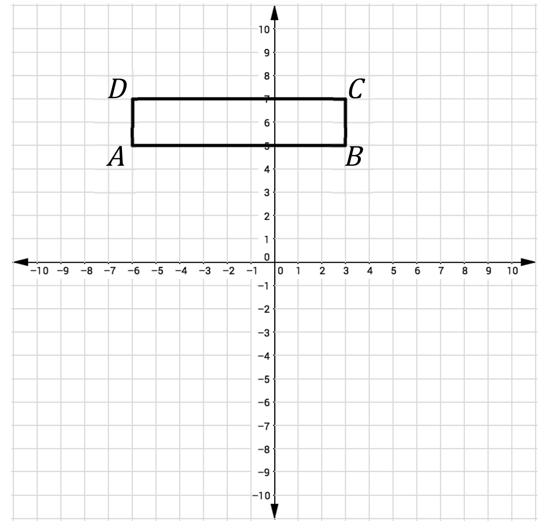
Try It!

2. Quadrilateral $PINT$ is dilated at the origin with a scale factor of $\frac{5}{3}$.

   Describe Quadrilateral $PINT$ and Quadrilateral $P'I'N'T'$ by filling in the table below with the most appropriate answer.

Quadrilateral $PINT$	Quadrilateral $P'I'N'T'$
(x, y)	$(\quad , \quad)$
$P(3,3)$	$P'(\quad , \quad)$
$I(\quad , \quad)$	$I'(10,15)$
$N(\quad , \quad)$	$N'(15,-5)$
$T(-3,-6)$	$T'(\quad , \quad)$

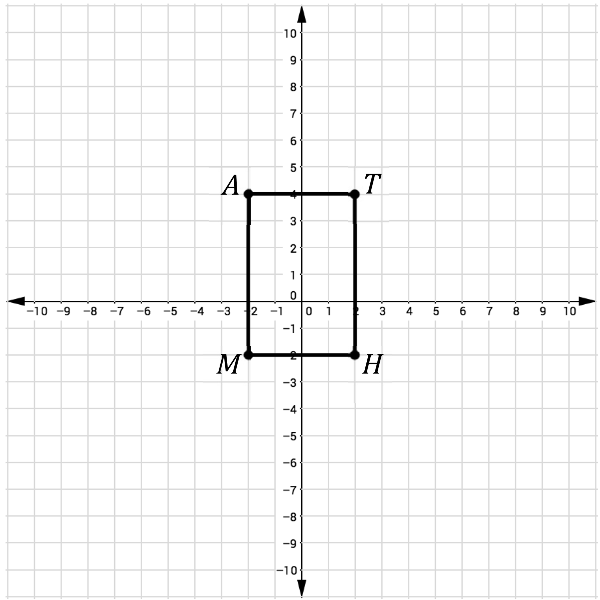
3. Consider rectangle $ABCD$.



   Dilate $ABCD$ by a scale factor of $\frac{1}{2}$ using a center of dilation of $(1,1)$. Draw $A'B'C'D'$ on the same coordinate plane.

BEAT THE TEST!

1. Consider Quadrilateral $MATH$ on the figure below.



Quadrilateral $MATH$ is dilated by a scale factor of 0.5 centered at $(-2,-2)$ to create quadrilateral $M'A'T'H'$.

What is the difference between the y-coordinate of A' and the y-coordinate of T'?

The difference is ☐ units.

2. Triangle PRA was dilated by a scale factor of 3 centered at the origin to create triangle $P'R'A'$, which has coordinates $P'(-6,-12), R'(-18,-6), A'(-6,-6)$. Write the coordinates of the vertices of triangle PRA in the spaces provided below.

$P($______, ______$)$

$R($______, ______$)$

$A($______, ______$)$